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ANALOG MODELING VARIANT OF A RAILWAY TRACTION BY MEANS OF ASYNCHRONOUS MOTORS

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Abstract: Certain significant aspects of the traction by means of 4 asynchronous squirrel cage motors are detailed in this study, motors that are part of a 3.6 MW power EC-43 type electric locomotive. An analog model specific to the railway traction based on three condition variables is developed for an imposed condition of nominal magnetic field flux.
The results thus obtained will be used in the work "Numeric simulation variant of a railway traction by means of asynchronous motors."

Keywords: railway traction, asynchronous motor, analog modeling, state variables.

1. THE ANALOG MODEL WITH INPUT-OUTPUT RELATIONS OF THE RAILWAY TRACTION BY MEANS OF ASYNCHRONOUS MOTORS

It is based on the asynchronous motor' T-shape quadruple in figure 1, where u_1, i_1, i_m and i_2' are the instantaneous phase values of the supply voltage, of the magnetization stator current and of the rotor's reduced to stator, respectively.

The symbols R_1, L_1, R_m, L_m, R_2' si L_2' correspond to the electrical parameters for the magnetization stator circuit and the magnetization rotor circuit reduced to stator, respectively. The electromagnetic balance in instantaneous signals can be approximated through the following system of equations:

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + R_m i_m + L_m \frac{di_m}{dt} \quad (1)$$

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{R_2'}{S} i_2' + L_2' \frac{di_2'}{dt} \quad (2)$$

$$i_1 = i_m + i_2' \quad (3)$$

$$m_M = \frac{m_1 \cdot i_2'^2 \cdot R_2'}{\Omega_1 - \Omega} \quad (4)$$

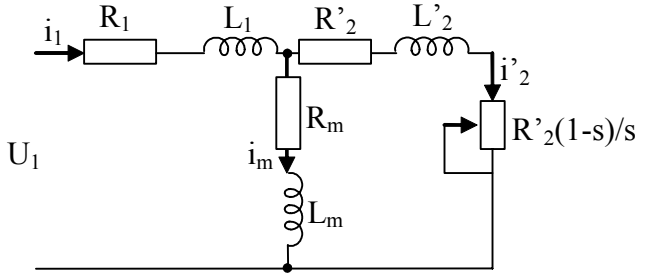

Figure 1

$$f_T = NRMOT \cdot \frac{m_M}{r} = M \cdot r \cdot \frac{d\Omega}{dt} + \rho_0 + \rho_1 \cdot r \cdot \Omega + \rho_2 r^2 \Omega^2 \quad (5)$$

where f_T, m_M, r, Ω and $v = r\Omega$, represents traction force [N], motor torque [Nm], radius of drive wheel [m], rotor angular speed [rad/sec] and respectively speed [m/s].

As for the case being examined, the number of traction motors is NRMOT = 4 while the angular speed of the stator magnetic field is:

$$\Omega_1 = \frac{2\pi}{60} n_{1N} = \frac{2\pi}{60} 1200 = 40 \cdot \pi [\frac{rad}{sec}]$$

The total mass is $M = \frac{1}{g}(G_L + G_R) = \frac{1}{g} G_T$ in [Kgm], where g = 9,80665$\left[\frac{m}{sec^2}\right]$, while G_T=G_L+G_R is the total mass [Kgf] of the train, composed by the weight of the locomotive (G_L) and towed weight (G_R).

With

$$\rho_0 = g \cdot G_T \cdot (\sin\alpha + 1.5 \cdot 10^{-3} \cdot \cos\alpha), \qquad (6)$$

where $\rho_0[N]$ is the component of the traction force independent of velocity (v) , where for the horizontal, upward and downward running regime, correspond as follows: $\alpha = 0, \alpha > 0,$ respectively $\alpha < 0$. The symbol $g \cdot G_T \cdot 1.5 \cdot 10^{-3}$ corresponds to the component (F_T) in [N] , necessary for the horizontal movement, at low speed, of the total weight (G_T) in [Kgf].

The term ($\sin\alpha$) in (6) corresponds to the railway path declivity, which do not exceed usually the limits (±25/1000), where the signs "+" and "–" correspond to the upward and downward movement. The gravity acceleration g = 9.80665m/sec^2.

A possible approximation of the coefficients (ρ_1 and ρ_2) in (5) results from the traction force in steady state (f_{Tst}) in [N] considered for $\frac{dv}{dt} = r \cdot \frac{d\Omega}{dt} = 0$ and horizontal running, namely $f_{Tst} = \rho_0 + \rho_1 \cdot v + \rho_2 \cdot v^2$.

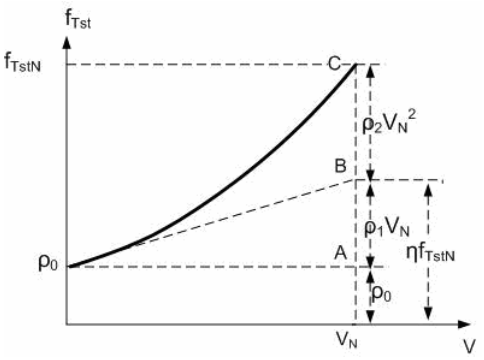

Figure 2

For a running velocity (v) [m/sec] chosen at random, for instance the nominal velocity (v_N) shown in figure 2, we can determine as follows:

$$(\frac{df_{Tst}}{dv})_{v=0} = \rho_1 = \frac{AB}{v_N}$$

where the straight-line segment AB = $\rho_1 \cdot v_N$

In case $\rho_0 + AB = \eta \cdot f_{TstN}$

where $0 < \eta < 1$ then $\rho_1 = \frac{1}{v_N}(\eta \cdot f_{TstN} - \rho_0)$

and $\rho_2 = \frac{1}{v_N^2} \cdot (1 - \eta) \cdot f_{TstN}$.

Certainly (ρ_0) , (f_{TstN}) and (η) are depending on the total weight (Gt) in [Kgf] as well as on certain constructive operating characteristics of the rolling stock..

From (1) (2) and (5), the following can be defined:

-the time constant of the stator impedance $T_{Z1} = \frac{L_1}{R_1}$, with usual values of 10^{-2} sec order;

-the time constant of the magnetization impedance $T_{Zm} = \frac{L_m}{R_m}$, with usual values of 10^{-2} sec order;

-the time constant of the rotor impedance reduced to stator $T_{Z_2'} = \frac{L_2'}{R_2'} \cdot S = \frac{L_2'}{R_2'} \cdot \frac{\Omega_1 - \Omega}{\Omega_1}$, with usual values of ($10^{-2}$ .. 10^{-4}) sec order as the slide (s) gradually decreases after start;

-the mechanical time constant $T_\Omega = \frac{M}{\rho_1}$, which accounts for the highest usual values of (10^2 … 10^3) sec order;

-the sinusoidal (three-phase) supply voltage period, $u_1 = \sqrt{2} \cdot U_{1ef} \cdot \sin(2\pi \cdot f_1 \cdot t)$ respectively $T_{u1} = \frac{1}{f1}$, that are within the limits (0.014-1) sec.

The systems of equations (1), (2), … (5) set out in a simplified form a possible analog modeling in open circuit of the railway traction, with input/output relations exemplified in figure 3.

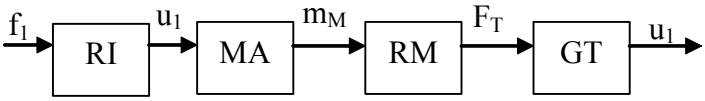

Figure 3

The rectifier inverter (RI) controlled in (f_1) frequency generates at output the sinusoidal (three-phase) power voltage system

$$u_1(t) = \sqrt{2} \cdot U_1(f_1, s) \cdot \sin 2\pi f_1 \cdot t$$

having the real value $U_1(f_1,s)$ connected according to (11). The asynchronous motor (MA) generates the engine torque (m_M) that develops, when applied to the driving wheels (RM), a traction force (F_T) able to move the set of railcars (GT) with the velocity (v) .

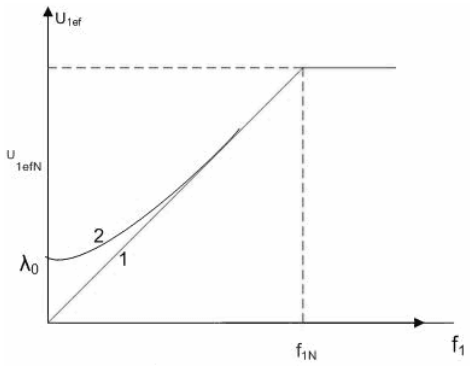

Figure 4

2. THE NOMINAL MAGNETIC FIELD FLUX STATE

The change of the real value (U_{1ef}) of the sinusoidal three-phase supply voltage system of the asynchronous motor is able to provide either a linear function, in accordance with the line (1) in figure 4, namely

$$U_{1ef} = \frac{f_1}{f_{1N}} \cdot U_{1efN}, \qquad (7)$$

where the index (N) correspond to the nominal values, or a function linked (2) particularly to low frequencies, of polynomial forms, for instance

$$U_{1ef} = \lambda_0 + \lambda_1 \cdot f_1 + \lambda_2 \cdot f_1^2 \qquad (8)$$

Although the relations (7) and (8) have an extensive use, they do not take into account the considerable influence of the slide (s) on the magnetic field flux (Ψ) that is recommended to be maintained at nominal values (Ψ_N). This recommendation ensures superior operating characteristics for the asynchronous motor, which is subjected in the

railway traction to large variations of frequency (f_1), slide (s) and, implicitly, of the angular velocity (Ω).

Therefore, it is intended to ensure in actual values the state $\psi = \xi \cdot \psi_N$, where $\xi = 1$ if $f_1 \le f_{1N}$, fact that ensures (ψ_N), while $0 < \xi < 1$ if $f_1 > f_{1N}$, ensuring thus the diminution of the magnetic field flux (ψ) at super-synchronous rotations.

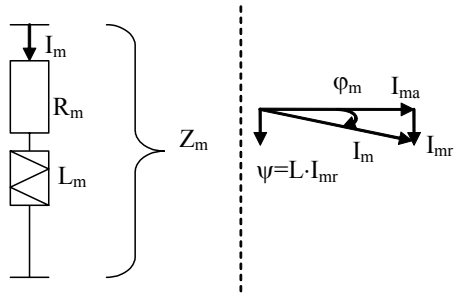

Figure 5

Considering the magnetization impedance (Z_m) covered by the current (I_m) in figure 5, the magnetization impedance will decompose in an active component (I_{ma}) and a reactive one (I_{mr}).

Consequently, we obtain real values – magnetization fluxes:

$$\psi = L_m \cdot I_{mr} = L_m I_m \sin\varphi_m$$
$$\psi_N = L_m \cdot I_{mrN} = L_m I_{mN} \sin\varphi_{mN} \qquad (9)$$

where the index (N) corresponds to the nominal values of the frequency (f_{1N}) and slide (s_N), while $\sin\varphi_m = \dfrac{2\pi f_1 L_m}{\sqrt{R_m^2 + (2\pi f_1 L_m)^2}}$

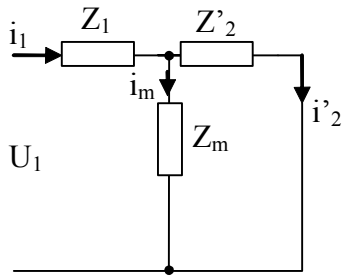

Figure 6

$$\sin\varphi_{mN} = \frac{2\pi f_{1N} L_m}{\sqrt{R_m^2 + (2\pi f_{1N} L_m)^2}}.$$

According to figure 5, the current (I_m) is obtained from the system of equations, in real values: $U_1 = Z_1 I_1 + Z_m I_m$; $U_1 = Z_1 I_1 + Z_2' I_2'$; $I_1 = I_m + I_2'$ we obtain thus the following results from calculations:

$$I_m = \frac{Z_2'}{Z_1 Z_m + Z_m Z_2' + Z_2' Z_1} \cdot U_1, \qquad (10)$$

where

$$Z_1 = \sqrt{R_1^2 + (2\pi f_1 L_1)^2}; \qquad Z_m' = \sqrt{R_m^2 + (2\pi f_1 L_m)^2}; \qquad Z_2' = \sqrt{(\frac{R_2'}{S})^2 + (2\pi f_1 L_2')^2}.$$

The nominal values (I_{mN}), (Z_{1N}), (Z_{mN}) and (Z'_{2N}) correspond to the nominal frequency $f_1 = f_{1N} = 40$ Hz and the nominal slide $s = s_N = 0{,}033$.

In accordance with (9), resulting from the state $\Psi = \xi \cdot \Psi_N$, namely $I_m \sin\varphi_m = \xi I_{mN} \sin\varphi_{mN}$

We obtain after replacement:

$$U_1(f_1,s) = \frac{Z_1 Z_m + Z_m Z_2' + Z_2' Z_1}{Z_{1N} Z_{mN} + Z_{mN} Z_{2N}' + Z_{2N}' Z_{1N}} \cdot \frac{Z_{2N}'}{Z_2'} \cdot \frac{\sin\varphi_{mN}}{\sin\varphi_m} \cdot \xi \cdot U_{1N}(f_{1N}, s_N) \qquad (11)$$

result that depends on the frequencies (f_1), (f_{1N}) ad the slides (s), (s_N).

Figure 7 shows the qualitative function U_1 (f_1, s) according to the relation (11), that ensures the constant level of the nominal magnetic field flux, for $\xi = 1$.

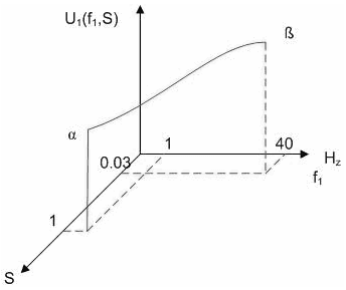

Figure 7

In running regime, the start corresponds to the movement from point (α), U_1 (1,1), respectively, to point (β), U_1 (40, 0.03), respectively assuming that the frequency (f_1) raises from 1Hz up to 40Hz, while the slide (s) decreases from (1) to (0,03).

For every working and braking mode studied herein – through numeric modeling and simulation – condition (11) shall be observed, namely $\Psi = \xi \cdot \Psi_N$, ensuring thus the efficient operation of the 4 asynchronous motors.

3. THE NON-LINEAR ANALOG MODEL THROUGH STATE VARIABLES

From (1), (2) and (5) the condition variables are defined as follows $x_1 = i_1$, $x_2 = i_1'$ and $x_3 = \Omega$, resulting the non-linear analog model of the railway traction by means of asynchronous motors, in form of three condition variables:

$$\dot{x}_1 = b_1 u_1 + a_{11} x_1 + \left(a_{120} + a_{121} \cdot \frac{\Omega_1}{\Omega_1 - x_3} \right) \cdot x_2 \qquad (12)$$

$$\dot{x}_2 = b_2 u_1 + a_{21} x_1 + \left(a_{220} + a_{221} \cdot \frac{\Omega_1}{\Omega_1 - x_3} \right) \cdot x_2 \qquad (13)$$

$$\dot{x}_3 = a_{333} \cdot \frac{x_2^2}{\Omega_1 - x_3} - \left(a_{330} + a_{331} x_3 + a_{332} x_3^2 \right) \qquad (14)$$

The coefficients ($a_{...}$) and ($b_{...}$) are constants depending on the electrical ($R_{...}$, $L_{...}$) and mechanical (M,r, $\rho_{...}$) parameters in (1), (2), …, (5).

The (12), (13), (14) system can be written in a more compact manner as follows:

$$\dot{x}_1 = N_1(u_1, x_1, x_2, x_3), \qquad (15)$$

$$\dot{x}_2 = N_2(u_1, x_1, x_2, x_3), \qquad (16)$$

$$\dot{x}_3 = N_3(x_2, x_3), \qquad (17)$$

for which the vector form of the analog model is corresponding, more precisely

$$\dot{\mathbf{x}} = \mathbf{N}(u_1, x_1, x_2, x_3). \qquad (18)$$

By repeatedly deriving in relation to the time, we obtain the vectors:

$$\ddot{\mathbf{x}} = \dot{\mathbf{N}}\left(\dot{u}_1, \dot{x}_1, \dot{x}_2, \dot{x}_2, \dot{x}_3, \dot{x}_3 \right), \qquad (19)$$

$$\dddot{\mathbf{x}} = \ddot{\mathbf{N}}\left(\ddot{u}_1, \ddot{x}_1, \dot{x}_2, \ddot{x}_2, \dot{x}_2, \dot{x}_3, \ddot{x}_3, \ddot{x}_3 \right), \qquad (20)$$

$$\dddot{\mathbf{x}} = \dddot{\mathbf{N}}\left(\ddot{u}_1, \ddot{x}_1, \dot{x}_2, \ddot{x}_2, \dddot{x}_2, \dot{x}_2, \dot{x}_3, \ddot{x}_3, \dddot{x}_3, \dot{x}_3 \right). \qquad (21)$$

The numeric integration was rendered evident by Taylor series limited to four derivatives, in particular: $\mathbf{x}_k = \mathbf{x}_{k-1} + \sum_{i=1}^{4} \frac{\Delta t^i}{i!} \overset{(i)}{\mathbf{x}}_{k-1}$, where $\mathbf{x}_k = \mathbf{x}(t_k)$, $\mathbf{x}_{k-1} = \mathbf{x}(t_{k-1})$, $t_k = k \cdot \Delta t$, $t_{k-1} = (k-1) \cdot \Delta t$, while (k) and (k-1) are the current sequence, and the regressive

sequence, respectively. The integration step (Δt) is recommended to be smaller than the smallest value of the constants (T_{Z1}), (T_{Zm}), ($T_{Z'2}$), (T_Ω) and (T_{U1}) defined above.
The initial condition at (t_0), namely at k=1, in particular $\mathbf{x}_0 = \mathbf{x}(t_0)$ is considered to be known, in order to provide the start of the calculations. The final moment (t_1) of completion of the calculations can be chosen either by a preset velocity or if $t_f \geq 2T_\Omega$.

4. CONCLUSIONS

4.1. The asynchronous squirrel cage motors, powered by variable frequency and corrections of the values of the three-phase inverter-generated voltages, are a particularly efficient, flexible and reliable actuator in the control systems' structure, with a variety of possible applications, from the most simple to the most complex and diversified ones.

4.2. In the case supposed by the usual hypothesis of neglecting the harmonics of the supply voltage and of certain possible electromechanical dissymmetry, the asynchronous motor's analog model, by means of T-shape, per-phase quadru-pole and three state variables, is a reasonable compromise between theoretical simplicity and the precision of the results.

4.3. The above mentioned analog model, by three state variables, emphasizes the non-linear structure of the asynchronous motor, justifying the use of the analog modeling followed by numeric simulation in order to solve the system of three non-linear differential equations.

4.4. Additionally to the period $T_U = 1/f_1$ of the three-phase sinusoidal voltage, are introduced the time constants (T_{Z1}), (T_{Zm}), ($T_{Z'_2}$) and (T'_Ω), since they have to be taken into consideration by choosing of the numeric integration step (Δt).

4.5. It is operated with a second degree polynomial as function of the angular velocity (Ω), for the static friction force, to which the dynamic accelerating [$(d\Omega/dt) > 0$] or retarding force [$(d\Omega/dt) < 0$] in (5) is added.

4.6. The importance of maintaining the magnetic field flux (Ψ) at the rated value (Ψ_N) is emphasized, for $f_1 \leq f_{1N}$. Although the relations (7) and (8) are extensively used, they do not take into consideration the significant influence of the slide (s) on the magnetization flux (Ψ). By setting and using the relation $U_1(f_1, s)$ in (11), a more realistic and efficient correction of the asynchronous motor's supply voltage is provided, in order to maintain $\Psi = \Psi_N$.

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