AQTR 2004 (THETA 14)
2004 IEEE-TTTC - International Conference on Automation, Quality and Testing, Robotics
May 13 – 15, 2004, Cluj-Napoca, Romania

ANALYTICAL METHODS TO QUANTIFY DEPENDENCIES WITH MULTI-STATE COMPONENT MODELS

Vasile DUB, Horatiu GRIF

"Petru Maior" University of Tg-Mures;tel.(fax) 40-0265-162275; e-mail: dubl@upm.ro

Abstract

In [4], [5] we presented an approach to model the failure processes in systems with dependent components. That model comprised algorithms able to generate work periods of the components with a view to obtain time series with the same mean value and standard deviation as these of the real system's components. The cardinal deficiency in estimation systems based on the convergence of the previous two statistical is that the proposed models approximate weakly the dependencies between fault processes. In other words, the found solution was not unique, that leading to errors of the identified dependence intensities , or even in the identification of the dependent components . Hence, a appropriate model must generate identical time distributions with these of the time series of the real components. The paper proposes an analytical method to identify these dependencies, starting from the probability density functions of independent, respectively, system-dependent work of the component.

Key Words –Variable transition rate, state dependent component, failure model.

1.INTRODUCTION

In our problem- the identification of fail processes dependencies- the accurate question which requires an appropriate answer can be formulate like as: "What are the conditions to compute an unique solution, so that describe correctly the fail dependencies, if we have the complete information about the fails history and a partial information on the critical states of some components". A short recapitulation of our results is useful.

We assumed that that the critical states of the real components can be recognized by a state estimator, with a view to collect a database about the concurrent fails of the system's components. These states appear in the component's Markow failure model.

The probability density functions (p.d.f) f_0, f_S of a independent, respectively system-dependent (SD) work are known for each component: they are establish, using a multi-state Markow model, whom parameters are given so to concord with the work periods histogram of the real component, in the two different work modes. If these functions are similar, the component is no system-dependent. We opt for a tree-state Markow chain to model fail processes. Also other model types can be used, which accomplish the following conditions:

- The model is unambiguous, that meaning a unique form of p.d.f. for a certain well defined model parameters. Thus, if $\{\lambda_1^1,\lambda_2^1,...,\lambda_k^1\}\neq\{\lambda_1^2,\lambda_2^2,...,\lambda_k^2\}$, then $f(\lambda_1^1,\lambda_2^1,...,\lambda_k^1)\neq f(\lambda_1^2,\lambda_2^2,...,\lambda_k^2)$;
- It must be able to encode coherently the system dependence. Thus, if the number of connected, in critical state found components increase, the dependence is the stronger.

1. THE TREE-STATE MODEL

The proposed model (fig.1) is a simplified variant of a four-state model presented in [4], [5].The states are defined in the follow:

- The "0" labeled state is a stable state;
- The "1" labeled state is limit stable state, from where the component is able to return to the stable state "0" The come back is not a renewing operation: that is intrinsic property (stability), or is due to a subaltern part with that function (automatic control).
  That is the state having its correspondent with the critical state of the real component, with the attribute to be recognizable by a state estimator.
- The "2" labeled state is the component's fail state.

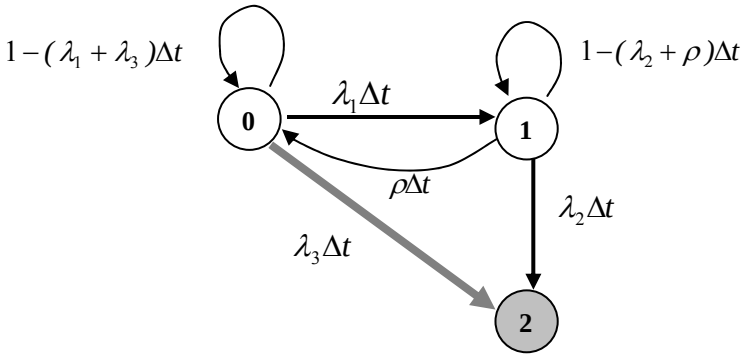

Fig.1 – The 3 -state component's model

In the system-independent work mode,

$$\lambda_1 > \lambda_3;\quad \rho_1 > \lambda_2 \qquad (1)$$

There are two, qualitative different approach for the state's significance of the 3-state model:

a) Consider that there are two fail modes: one, most probable, corresponding to the failure due at the wear, (0,1,2) and the other, more infrequent, modeling a suddenly, catastrophic fault (1,3).

b) Consider that our ability to recognize the critical states of the real component is bounded, thus the (1,3) trajectory measures the stochastic error of the external state estimation system. As consequence, either λ_3 rest constant, either we can renounce at it, if the estimation system is perfectible.

In this paper, the second approach is chosen.

That allows us to simplify the previous model:

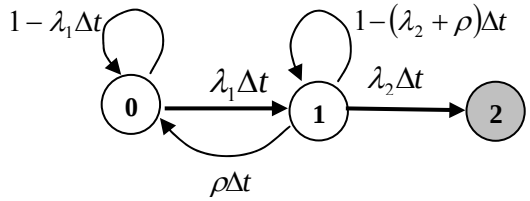

Fig.2 – The simplified 3 -state component

The system dependence is manifest by the simultaneous variance of some, or all parameters, for example:
- the increase of λ_1 and a simultaneous diminution of ρ until the violation of (1);
-the increase of λ_2 until its value become comparable with ρ.
The both, tree- and four-state models has similar form p.d.fs:

$$f(t) = -\sum_{i=1}^{n-1} s_i \cdot A(s_i) \cdot exp(s_i) \tag{2}$$

in which, *n* is equal with the states number.
Some of formulas associated to the 3-state model (proved in [5]) are given:

$$f(t) = -s_1 A(s_1) exp((s_1 t) - s_2 A(s_2) exp(s_2 t) \tag{3}$$

witch:

$$A(s) = \frac{s+a}{2s+a}, \quad s_1 = -\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}, \quad s_2 = -\frac{a}{2} - \sqrt{\frac{a^2}{4} - b} \tag{4}$$
$$a = \lambda_1 + \lambda_2 + \rho, \quad b = \lambda_1 \lambda_2$$

The mean and the standard deviation are:

$$\mu = \frac{a}{b}, \quad \sigma^2 = \frac{a^2 - 2b}{b^2} \tag{5}$$

The unambiguous character of the p.d.f. can proved with elementary methods.

2. COMMUTATIVE TRANSITIONS IN MARKOW CHAINS

The equations associated to the simplified 3-state model (fig.2) are:

$$\begin{aligned} P_0(t+\Delta t) &= [1-\lambda_1\Delta t]P_0(t) + \rho\Delta t P_1(t) \\ P_1(t+\Delta t) &= \lambda_1\Delta t P_0(t) +_1 (t) + [1-(\lambda_2+\rho)\Delta t]P_1(t) \end{aligned} \tag{6}$$

in which P_0, P_1 are the probabilities of "0(1)"labeled states; in matrix form:

$$P(t+\Delta t) = [[1] + \Lambda \cdot \Delta t] \times P(t) \qquad \text{or} \quad P'(t) = \Lambda \times P(t) \tag{7}$$

where:

$$P(t)=\begin{bmatrix}P_0\\P_1\end{bmatrix},\quad \Lambda=\begin{bmatrix}-\lambda_1 & \rho\\ \lambda_1 & -\lambda_2-\rho\end{bmatrix} \tag{8}$$

Λ is the transition matrix and [1] is the identity matrix. From (15), using the Laplace transform, the (2)-form p.d.f is obtain:

$$f(t)=-\left(P_0(t)+P_1(t)\right)' \tag{9}$$

Using the finite differences-form equation (6), for the two, t_1, t_2 time variables, corresponding to Λ_1 and Λ_2 ,the model's evolution can be described by:

$$P(t_1+\Delta t_1)=[[1]+\Lambda_1\cdot\Delta t_1]\times P(t_1),\quad P(t_2+\Delta t_2)=[[1]+\Lambda_2\cdot\Delta t_2]\times P(t_2) \tag{10}$$

Hence, the evolution of P between $(0,\tau_k+\Delta t_2)$ can be write as:

$$\begin{aligned}P(\tau_k+\Delta t_2)=&[[1]+\Lambda_2\cdot\Delta t_2]\times[[1]+\Lambda_1\cdot\Delta t_1]^{nk}\times\ldots\times[[1]+\Lambda_2\cdot\Delta t_2]\times[[1]+\Lambda_1\cdot\Delta t_1]^{ni}\times\ldots\\ &\ldots.\times[[1]+\Lambda_2\cdot\Delta t_2]\times[[1]+\Lambda_1\cdot\Delta t_1]^{n1}\times P(0)\end{aligned} \tag{11}$$

If the multiplication of $[[1]+\Lambda_1\cdot\Delta t_1]$ and $[[1]+\Lambda_2\cdot\Delta t_2]$ is commutative, (11) can be compacted as:

$$P(\tau_k+\Delta t_2)=[[1]+\Lambda_1\cdot\Delta t_1]^{N1}\times[[1]+\Lambda_2\cdot\Delta t_2]^{N2}\times P(0) \tag{12}$$

(12) allow us to rearrange the work periods in a new succession, so that in $[0,t_1)$, the transitions are performed by a Λ_1 transition matrix, and beginning from t_1, the transition matrix changes by leap to Λ_2.

The commutativity of $[[1]+\Lambda_1\cdot\Delta t_1]\times[[1]+\Lambda_2\cdot\Delta t_2]$ can be reduced to:

$$\Lambda_1\times\Lambda_2=\Lambda_2\times\Lambda_1 \tag{13}$$

Now, let divide τ_1,τ_2 in the same number *n* of infinitesimally intervals:

$$\frac{\tau_1}{n}=\Delta\tau_1=\frac{N_1\Delta t}{n},\quad \frac{\tau_2}{n}=\Delta\tau_2=\frac{N_2\Delta t}{n},\ n\to\infty \tag{14}$$

If $\Delta t=\Delta\tau_1+\Delta\tau_2$, we can write two similar equations with (14) for $t+\Delta\tau_1, t+\Delta\tau_1+\Delta\tau_2$, resulting:

$$P(t+\Delta\tau_1+\Delta\tau_2)=[[1]+\Lambda_1\Delta\tau_1]\times[[1]+\Lambda_2\Delta\tau_2] \tag{15}$$

Neglecting in (15) the last term $\Lambda_1\Lambda_2\cdot\Delta\tau_1\Delta\tau_2$,

$$P(t+\Delta t)=[[1]+\Lambda_m\cdot\Delta t]\times P(t) \tag{16}$$

where:

$$\Lambda_m=\frac{\Lambda_1\cdot\Delta\tau_1+\Lambda_2\cdot\Delta\tau_2}{\Delta\tau_1+\Delta\tau_2} \quad \text{or} \quad \Lambda_m=\frac{N_1\Lambda_1+N_2\Lambda_2}{N_1+N_2} \tag{17}$$

So, if the product $\Lambda_1\Lambda_2$ is commutative, Λ_m is a balanced mean of the two transition matrices. The result can be generalized:

Let be a system evolution as result of a finite number of mutually commutative $\Lambda_1,\Lambda_2,...,\Lambda_p$ transition matrices, each appearing with the relative frequencies $N_1/N, N=\sum_p N_i$. The succession of $\Lambda_1,\Lambda_2,...,\Lambda_p$ matrices can be replaced by the equivalent Λ_m matrix:

$$\Lambda_m=\frac{1}{N}\sum_{i=1}^{p}N_i\Lambda_i \tag{18}$$

Let Λ_1,Λ_2 equals with:

$$\Lambda_1=\begin{bmatrix}-\lambda_{11} & \rho_1\\ \lambda_{11} & -\lambda_{21}-\rho_1\end{bmatrix},\quad \Lambda_2=\begin{bmatrix}-\lambda_{12} & \rho_2\\ \lambda_{12} & -\lambda_{22}-\rho_2\end{bmatrix} \tag{19}$$

Developing the commutativity condition (13), results:

$$\rho_1\lambda_{12}=\rho_2\lambda_{11},\quad \lambda_{12}\lambda_{21}=\lambda_{11},\quad \lambda_{22}\rho_1\lambda_{22}=\rho_2\lambda_{21} \tag{20}$$

Consider Λ_1 corresponding to the independent work mode, Λ_2 the transition matrix associate to the temporally action of connected components: Λ_m, as Λ_1 and Λ_2 composition, will be the transition matrix in the system-dependent work

The equations (20) are linear dependents, therefore two of them gives are used. The third equation results from (17), and the solution is unique.

3.APPLICATION: TREE-COMPONENT SYSTEMS

Consider a simplest non-trivial system, with tree components and some of potential dependencies shown in fig.3:

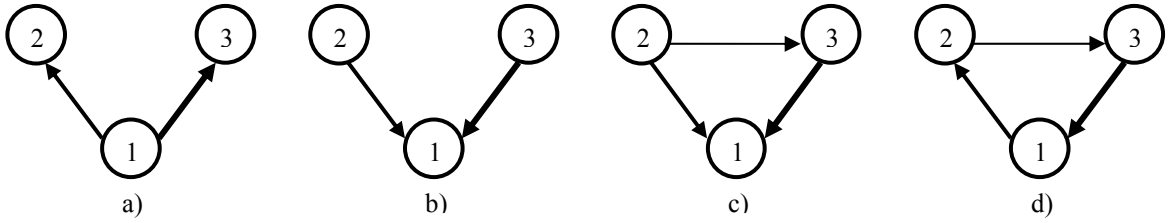

Fig.3 –Dependencies in tree-component systems

In fig.3.a, the components 2,3 are system-dependents and correspond to solution presented at the end of the section 2.
In fig.3.b, only 1 is system-dependents. The transition matrices Λ_0, Λ_m of independent, respectively dependent work mode are known. Also known are the N_2, N_3 numbers of "1" labeled states occurrence of the components 1,2 and the number N_0 of "0" labeled states occurrence of 1.The unknown X_2, X_3, corresponding to the temporally action of connected components (2 and 3), are (16)-form, thus let:

$$\Lambda_0 = \begin{bmatrix} -\lambda_{10} & \rho_0 \\ \lambda_{10} & -\lambda_{20} - \rho_0 \end{bmatrix}, \quad \Lambda_m = \begin{bmatrix} -\lambda_{1m} & \rho_m \\ \lambda_{1m} & -\lambda_{2m} - \rho_m \end{bmatrix}, \quad X_i = \begin{bmatrix} -x_i & z_i \\ x_i & -y_i - z_i \end{bmatrix}, \quad i = 2,3$$

We can write the following equations:
According to (20):

$$\rho_0 x_2 = \lambda_{10} z_2, \quad \lambda_{20} x_2 = \lambda_{10} y_2, \quad \rho_0 x_3 = \lambda_{10} z_3, \quad \lambda_{20} x_3 = \lambda_{10} y_3$$

According to (18):

$$N_2 x_2 + N_3 x_3 = (N_0 + N_2 + N_3)\lambda_{1m} - N_0 \lambda_{10}, \quad N_2 z_2 + N_3 z_3 = (N_0 + N_2 + N_3)\rho_m - N_0 \rho_0$$

Therefore, the equation system is solvable, and the solution is unique.
In fig.3.c, the dependent components are 1 and 3. The (2,3) sub-system analysis is similar with those from fig.1.a. Because 3 depends on 2, and 1 depends on 3, the occurrences number N_3 of the "1" labeled state of 3's must be diminish with the number of simultaneous occurrences of the "1" labeled state of components 2 and 3.

4.CONCLUSIONS

The proposed method is useful in the accurate identification of fail processes in small systems. Beginning yet with dependencies as like in fig.3.d, appears the necessity to process statistically the database of state evolution of all components. In great systems, a possible error source may by those regarding the bounded estimation capacity of state estimators, which must be included in the fail model, e.g. using the tree-state model with four parameters, as like that was discussed in the section 1.

References

[1] Schottl A – A Reliability Model of a System with Dependent Components; *IEEE Transactions on Reliability, vol 45 1996, pag. 267-273*
[2] L.Ensign Johnson, Michael O.Johnson – Some Large Availability Models: Computation&Bounds, *IEEE Transactions on Reliability, vol 46 1997, September, pag.406-420*
[3] Pombuyses B. – Reduction of the markovian system by by the influence graph method: error, bound and reliability computation, *Reliability and System Safety 63 (1999), pag.1-11.*
[4] Dub V. – Failure models with dependent transition rates, Optim 2002, Proceedigs of the 8th International Conference of Electrical and Electronic Equipments, pag.597-600, Braşov, 16-17 mai 2002;
[5] Dub V., Grif H. - Modeling Failure Processes with State Dependent Components, Proceedigs of International Conference Q&A-R, 2000, vol. I, pag.21-25, Universitatea Politehnica Cluj-Napoca