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NEW FORMULATIONS ABOUT DYNAMIC ACCURACY IN ROBOTICS

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Abstract: The dynamic accuracy in Robotics is estimated by means of the dynamics errors. On the one hand they are referring to the locating, velocities and accelerations. On the other hand the dynamic errors are extended about the generalized dynamics forces. In this paper, new approach of the dynamic accuracy based on the application of the matrix exponentials will be presented.

Keywords: Robotics, Modeling, Mechanics, Dynamics and Accuracy.

1. INTRODUCTION

The assessment of dynamic accuracy performances, according to [1], is achieved by means of the dynamic errors. According to [1] and [3], the dynamic errors refer to the operational dynamics parameters leading to so-called locating (position-orientation) dynamics errors, as well as the dynamics errors of operational velocities and accelerations. Also, the dynamic errors refer to the generalized variables and dynamic control functions.

2. THE ROBOT DYNAMICS ACCURACY

The generalized equations of the dynamic accuracy are, symbolically, described below by means of a few expressions, in keeping with the *SimMEcROb Simulator*. The main symbols of the kinematics and dynamics errors are shown in the Fig.1.

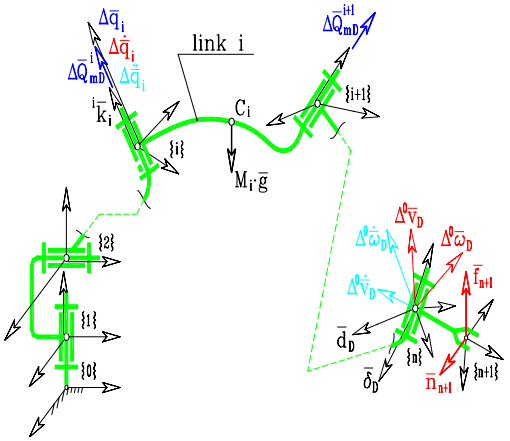

Fig. 1 Dynamics Errors

At first, the generalized matrix of the *MD-type errors* is determined, as follows:

$$E_{MDQ}=\begin{Bmatrix} E_{MDQk} \\ (nx10) \\ \bar{\varepsilon}_{MDQk} \end{Bmatrix}=\left[\begin{bmatrix} \Delta M_{ik} & \left(\Delta^i \bar{r}_{CiQk}\right)^T & \Delta^i I_{\delta Qk} \end{bmatrix}\right]^T . \quad (1)$$

$$\left\{E_{MDQk} \subset \Delta^i I_{psiQk} = \underset{(4\times4)}{Matrix}\left(\begin{matrix} \Delta I_{jl\,k} = \Delta I_{l\,jk} \\ j=1\rightarrow 4 \;\; l=1\rightarrow j \end{matrix}\right)\right\};$$

where $\Delta^i I_{iQk}$ and $\Delta^i I_{psiQ\,k}$ are the error matrices of the inertial and pseudo-inertial tensor, while $\bar{\varepsilon}_{MDQ}$ is the column vector of the *MD-type errors (mass distribution errors)*.

The generalized matrix of dynamic errors is determined and symbolically below shown as:

$$\Delta M^{xy} = \left\{ \begin{matrix} \Delta M^{xy}_{Q} = \{\Delta M^{xy}_{L}; \Delta M^{xy}_{D}\ ; \Delta M^{xy}_{G}; \Delta M^{xy}_{ME}\} \\ \{\ \Delta M^{xy}_{Qzk}; z = \{\Delta_m\ ;\ \Delta_\theta\}; k = 1 \to m\ \} \end{matrix} \right\}; \Delta M^{xy}_{Qzk} = \left\{ \begin{matrix} \Delta M(\bar{\theta})^{xy}; \Delta \bar{V}(\bar{\theta}; \dot{\bar{\theta}})^{xy}; \Delta B(\bar{\theta})^{xy} \\ \Delta C(\bar{\theta})^{xy}; \Delta Q_g(\bar{\theta})^{xy}; \Delta \bar{Q}^{xy}_{SU}; \Delta \bar{Q}^{xy}_{fd} \end{matrix} \right\}; \quad (2)$$

where ΔM^{xy}_{Qzk} is the differentials of the dynamic matrices with respect to $\bar{\varepsilon}_{yQk}$ and $\bar{\varepsilon}_{MDQ}$; According to [1], the matrix equation for the inverse model of the dynamics errors is:

$$\bar{Y}^{xy}_{dQk} = \{\ \Delta M^{xy}_{DEQzk}(\tau);\ \Delta M^{xy}_{XDFQzk}(\tau)\} = E^{xy}_{YdQk} \cdot \bar{\varepsilon}_{ydQK}; \quad (3)$$

$$\bar{Y}_{dQ} = \{\Delta \bar{Q}_{mQ}\ ; \Delta \bar{X}_{dQ}\}; \quad \Delta \bar{X}_{dQ} = \left[\Delta \dot{\bar{v}}^T_{dQ}\ \ \Delta \dot{\bar{\omega}}^T_{dQ}\right]^T; \ \bar{\varepsilon}_{ydQk} = \left[\bar{\varepsilon}^T_{yQk}\quad \bar{\varepsilon}^T_{MDQk}\right]^T. \quad (4)$$

The symbol $\{E^{xy}_{YdQk}\}$ is devoted to transfer matrix of the dynamics errors, while $\bar{Y}_{dQ}$ is answerable to $\Delta \bar{Q}_{mQ}$ generalized driving forces and $\Delta \bar{X}_{dQ}$ operational variables.

➢ For estimate the dynamic accuracy the *maximum-likelihood method* is applied, according to [1]. So, the maximum-likelihood function in a matrix form is written as:

$$ln L(\bar{Y}_{dQ}\ ; V_{YdQ}) = -\frac{3}{2} m \cdot ln(2\pi \cdot \sigma^2_{YdQ}) + \frac{1}{2} ln|P_{YdQk}| - \frac{1}{2\sigma^2_{YdQ}} \cdot \sum_{k=1}^{m} (\bar{Y}_{dQk} - \bar{Y}_{dQ})^T \cdot P_{YdQk} (\bar{Y}_{dQk} - \bar{Y}_{dQ}). \quad (5)$$

The covariant matrix V_{YdQk} answerable to the dynamic errors is determined as follows:

$$V_{YdQk} = \left\{ \begin{matrix} E_{YdQk} \cdot V_{\varepsilon dQ} \cdot E^T_{YdQk} \\ Matrix_{(3\times 3)}(v_{ijk} = v_{jik}\ ;\ i = 1 \to 3\ ;\ j = 1 \to i) \end{matrix} \right\}; \text{ where: } V_{\varepsilon dQ} = Diag(\sigma^2_{jdQ}\ ;\ j_{dQ} = i_{dQ}). \quad (6)$$

The mean square error σ_{YdQ}, respectively the mean square weighted error $\sigma_{\bar{\bar{Y}}dQ}$ typical of the mean of kinematic errors is calculated, as follows:

$$\sigma_{YdQ} = \pm \sqrt{\frac{1}{6m-6} \sum_{k=1}^{m} (\bar{Y}_{dQk} - \bar{\bar{Y}}_{dQ})^T \cdot P_{YdQk} (\bar{Y}_{dQk} - \bar{\bar{Y}}_{dQ})}; \quad \sigma\left\{ M(Y_{dQj})_{(j=1\to 6)} \right\} = \sigma_{YdQ} \Big/ \left(\sum_{k=1}^{m} p_{jjk} \right). \quad (7)$$

Considering (23) the following expressions are written:

$$T_{1YdQ} \le Y_{dQ} \le T_{2YdQ};\ T_{jYdQ} = |-Y_{dQ}| + \Delta_j \cdot \tau_{YdQ} \cdot \sigma_{YdQ};\ \tau_{YdQ} = \tau_{YdQ}\{m; \gamma; (1-\alpha)_Q\};\ \Delta_j = \{\{-1, j=1\}; \{1; j=2\}\}. \quad (8)$$

They characterize *the tolerance interval for the dynamic errors* $\bar{Y}_{dQ}$. Therefore, the expression (8) leads to the optimization of the dynamic accuracy to whatever robots.

3. CONCLUSIONS

The dynamic accuracy in Robotics is assessed by means of the dynamic errors. They are referring to the locating, velocities and accelerations. On the other hand the dynamic errors have been extended about the generalized dynamics forces. This paper is devoted to new modeling based onto matrix exponentials in the dynamics accuracy. In the same paper an optimization model based on maximum-likelihood method was applied.

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