CONSTRUCTION OF A TORIC SURFACE BY INTERPOLATING A SET OF PLANAR SECTIONS

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Abstract: The paper discusses some limitations of CAD programs. General-purpose CAD programs are quite restrictive in some situations. This paper proves that such limitations can be overcome by using fundamental methods. The need to develop a computer program is a task that may affect the usual design procedure, so the deadlines of the project should be carefully established.

Key words: CAD program, CATIA, solid modeling, surface modeling, NUBPS.

1. GENERAL TOPICS

Using a general-purpose program to construct a complex surface may lead to some difficulties, especially when the geometry of the surface is very complicated or when the program has not been developed to manipulate such type of surfaces. A very suitable example is the modeling of the toric surfaces with CATIA V5R8 (Fig. 1).

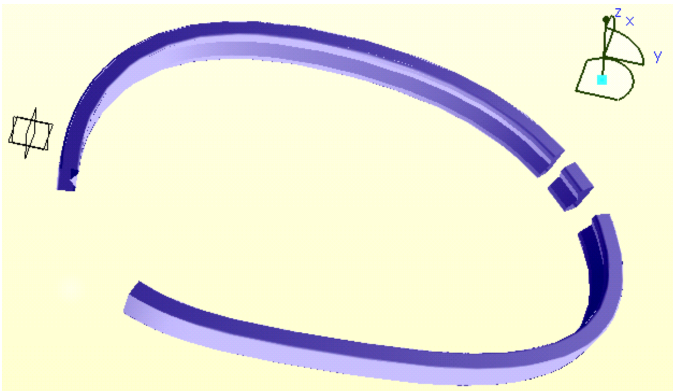


Fig. 1. Toric surface.

The surface is generated using the component called "*Wireframe and Surface Design*". The construction starts from a set of eleven sections placed in planes perpendicular to a closed Spline curve. The program allows the creation of the model only by joining several *"Loft"* surfaces. The junction has been made with the so-called *"Blend surfaces"*.

2. INTERPOLATION OF A TORIC SURFACE USING B-SPLINE PATCHES

In order to overcome this drawback, we have adopted the strategy of interpolating the complex surface with a periodic B-spline surface preserving the same planar sections for definition. After finalizing this operation, we can transfer the result to CATIA for further processing (the export can be made via the STL format).

In each of the initial sections, containing lines and circular arcs, the original curves have been replaced by B-splines. As a consequence, the first step consists in computing the characteristic polygons of the B-splines (Fig. 2).

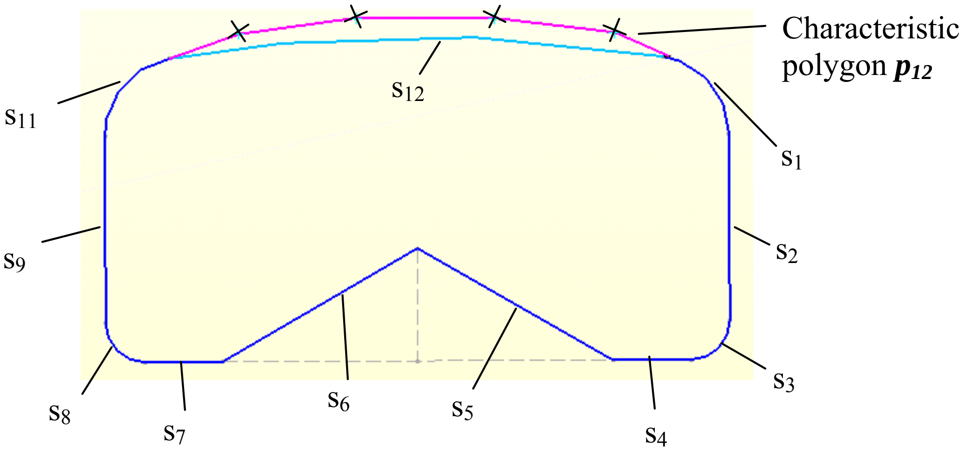


Fig. 2. Approximation of the segments belonging to the current section.

We start from the vector equation defining a point **C** located on a B-Spline curve from the current section:

$$\mathbf{C}(u) = \sum_{i=0}^{n} N_i^m(u) \cdot \mathbf{P}_i \tag{1}$$

We aim to determine the characteristic polygon of a periodic B-spline curve $G(v)$ that interpolates the points **P***i*. This polygon is part of the characteristic mesh associated to the toric surface.

$N_i^m(u)$ are the members of a function basis and may be calculated using recursive formulas [1]:

$$N_i^m(u) = \frac{u - u_i}{u_{i+m} - u_i} N_i^{m-1}(u) \ for \ u < u_{i+1},$$

$$N_i^m(u) = \frac{u - u_i}{u_{i+m} - u_i} N_i^{m-1}(u) + \frac{u_{i+m+1} - u}{u_{i+m+1} - u_{i+1}} N_{i+1}^{m-1}(u) \ for \ u_{i+1} \le u < u_{i+m},$$

$$N_i^m(u) = \frac{u_{i+m+1} - u}{u_{i+m+1} - u_{i+1}} N_{i+1}^{m-1}(u) \ \ for \ u \ge u_{i+m}.$$

$$N_i^0(u) = \begin{cases} 1 \ for \ u_i \le u < u_{i+1} \\ 0 \ for \ u < u_i \ or \ u \ge u_{i+1} \end{cases}. \tag{2}$$

u_i are the elements of the nodal vector V_n. For polynomial base functions having the degree m=3, V_n consists in a series of consecutive integer numbers:

$$V_n = \{0 \quad 1 \quad \ldots \quad n_v \quad n_v+1 \quad n_v+2 \quad n_v+3 \quad n_v+4\}$$

We assume that the toric surface is defined by n_v-2 sections. For the current section k, we know the characteristic polygon as a set of vertices **P***ik*. The condition that the points **P***ik* belong to the interpolating curve **G***(v)* leads to n_v-2 equations [2]:

$$\sum_{j=0}^{n_v} N_j^{m_v}(v_k)\cdot \mathbf{S}_{ij} = \mathbf{P}_{i,k} \quad (k=1,2, \ldots, n_v-2). \tag{3}$$

The points **S***ij* belong to the characteristic mesh of the toric surface. The vector **H** contains discrete values v_k where the periodic curve **G**(v) is coincident with $\mathbf{P}_{ik}$. It has (n_v-2) components chosen as follows:

$$H = \{3 \quad 4 \quad \ldots \quad n_v\}.$$

The set of equations (3) contains n_v+1 unknown values. Due to the fact that the characteristic polygon associated to a periodic B-spline curve is constrained by the conditions $P_0=P_{nv-1}$, $P_1=P_{nv}$ şi $P_2=P_{nv+1}$, the number of unknown values reduces to n_v-2. The set of equation should be solved three times (i.e. for each axis of the coordinate system).

In the case n_v=7, we obtain $H = \{3 \quad 4 \quad \ldots \quad 7\}$ and the matrix of the equation set (3) is:

$$A = \begin{bmatrix} N_0^3(3)+N_5^3(3) & N_1^3(3)+N_6^3(3) & N_2^3(3)+N_7^3(3) & N_3^3(3) & N_4^3(3) \\ N_0^3(4)+N_5^3(4) & N_1^3(4)+N_6^3(4) & N_2^3(4)+N_7^3(4) & N_3^3(4) & N_4^3(4) \\ N_0^3(5)+N_5^3(5) & N_1^3(5)+N_6^3(5) & N_2^3(5)+N_7^3(5) & N_3^3(5) & N_4^3(5) \\ N_0^3(6)+N_5^3(6) & N_1^3(6)+N_6^3(6) & N_2^3(6)+N_7^3(6) & N_3^3(6) & N_4^3(6) \\ N_0^3(7)+N_5^3(7) & N_1^3(7)+N_6^3(7) & N_2^3(7)+N_7^3(7) & N_3^3(7) & N_4^3(7) \end{bmatrix}$$

$$A = \begin{bmatrix} 0.167 & 0.667 & 0.167 & 0 & 0 \\ 0 & 0.167 & 0.667 & 0.167 & 0 \\ 0 & 0 & 0.167 & 0.667 & 0.167 \\ 0.167 & 0 & 0 & 0.167 & 0.667 \\ 0.667 & 0.167 & 0 & 0 & 0.167 \end{bmatrix}$$

By inverting matrix **A**, we get

$$A^{-1} = \begin{bmatrix} -0.455 & 0.091 & 0.091 & -0.455 & 1.727 \\ 1.727 & -0.455 & 0.091 & 0.091 & -0.455 \\ -0.455 & 1.727 & -0.455 & 0.091 & 0.091 \\ 0.091 & -0.455 & 1.727 & -0.455 & 0.091 \\ 0.091 & 0.091 & -0.455 & 1.727 & -0.455 \end{bmatrix}$$

We notice that the rows of matrix **A** , as well as those of the inverse $\mathbf{A}^{-1}$ are obtained by circular permutations of the elements belonging to the first row. This property holds in the cases when the number of sections or the value n_v are increased. It allows to reduce the computational effort when solving the linear set (3).

In the case of our example, if $\mathbf{P}_{ik}$ are defined by the coordinates (related to the corresponding planar section) {(3,4), (14,36), (29,34), (25,-12) and (6,-20)}, we obtain the periodic B-spline curve presented in Figure 3.

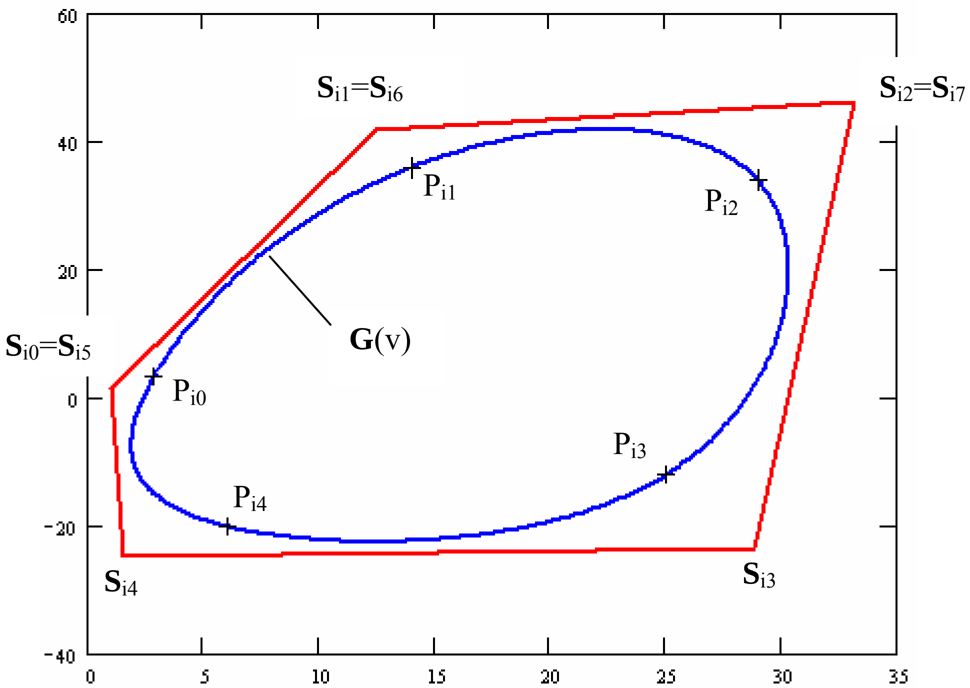
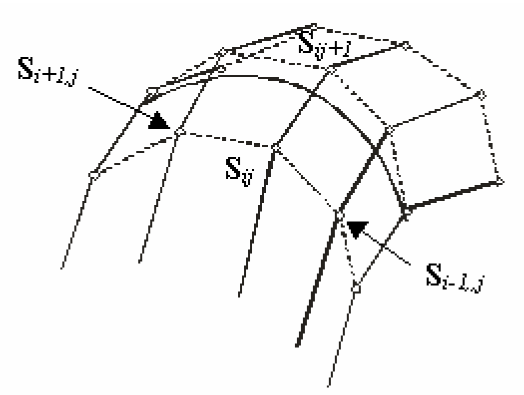


Fig. 3. Periodic B-spline curve interpolating the points P_{i0}, ...,P_{i4}.

Figure 4 shows the characteristic mesh obtained after connecting the points $\mathbf{S}_{ik}$, while Figure 5 shows the toric surface generated from eleven planar section (the image has been generated using OpenGL functions [3]).



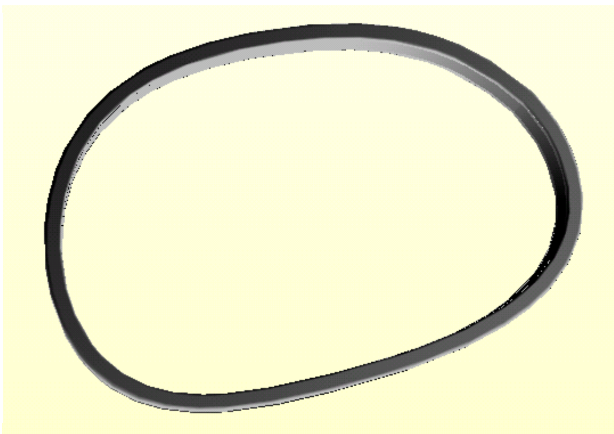


Fig. 4. Characteristic mesh of the surface

Fig. 5. Interpolation surface

3. CONCLUSIONS

General-purpose CAD programs (as the one used in our application) are quite restrictive in some situations. This paper proves that such limitations can be overcome by using fundamental methods. Of course, the need to develop a computer program is a task that may affect the usual design procedure. As a consequence, the deadlines of the project should be carefully established.

4. REFERENCES

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