APLICATIONS OF DISTRIBUTION THEORY AND PROPERTIES IN POWER ELECTRONIC CONTROLLER

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Abstract: In this paper distributions theory and their remarcable properties are first considered. Using the definition of distribution, the mathematical models for discrete elements are determined. The derivation and integration operators extended to discrete elements allow determining the interesting mathematical equations of complicated structure of digital control systems. This approach allows describing the hybrid systems containing analog and digital elements with the same equations on distributions. Based on distribution equations there can be modeled, simulated and designed flexible and reliable systems. The complex PWM software-oriented controller for power electronics inverter is presented. The results are similar to classical PWM inverter, but much simpler.

Key-words: distributions, digital control, PWM inverter, applications

1. General considerations

Some complex phenomena can't be represented by a functions based mathematical model because of their discontinuities, severe nonlinearities, non-derivation etc. In the real function theory this regions are avoided with the partitioning method, eliminating the discontinuities and never using the derivation operator in this critical points.

This approach has two disadvantages: first, the representation is complicated and unusual and second, just the discontinuities are eliminated where some systems work, like digital systems, commutations relay, logical transducers and hybrid elements.

It is possible to extend the function theory using the theory of distributions, which generalizes the mathematical operators applied on the whole domain.

On the other hand by this method it is possible to define new generalized distributions not existing in the real function class, but very usual in the phenomena analyzing.

Now, consider the most usual "function" sign(x) which returns the sign of the argument (fig.1.a). This can be defined as follows:

$$\operatorname{sign}(x) = \begin{cases} 1; & x \geq 0 \\ -1; & x < 0 \end{cases} \qquad [\operatorname{sign}(x)]' = \begin{cases} 0; & x \neq 0 \\ 2 \cdot \delta(x); & x = 0 \end{cases} \tag{1}$$

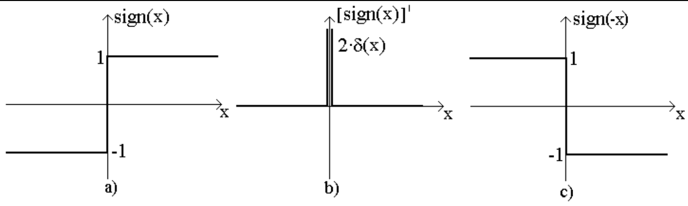

Fig.1. a) Sign distribution; b) Sign derivation; c) Sign invert

If sign(x) is treated as a function, its derivate $[\mathrm{sign}(x)]' = 0$, but this is not true because of the origin discontinuity which produce a short pulse (fig.1.b). It is possible to define: $\mathrm{sign}(-x) = -\mathrm{sign}(x)$ as an odd function (fig.1.c.). Switching a switch is a very known phenomenon; this can be represented as a "function" $\theta(x)$ (fig.2.a). This is a Heaviside distribution defined as follows:

$$\theta(x) = \begin{cases} 1; & x \geq 0 \\ 0; & x < 0 \end{cases} \; ; \; \theta(-x) = \begin{cases} 1; & x \leq 0 \\ 0; & x > 0 \end{cases} ; \; [\theta(x)]' = \delta(x); \; [\theta(-x)]' = -\delta(x) \quad (2)$$

$$\int_{R^+} \theta(x) dx = \theta(x) \cdot x = x_+ = r(x)$$

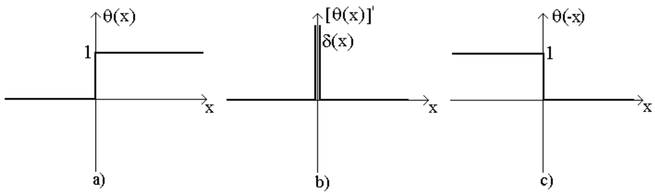

Fig.2. a) Heaviside distribution; b) Heaviside derivation; c) Heaviside invert

If $\theta(x)$ is a function then $[\theta(x)]' = 0$ and if it is a distribution then $[\theta(\pm x)]' = \pm\delta(x)$ (fig.2.b,c). The relations between these distributions are:

$$\theta(x) + \theta(-x) = 1; \qquad \mathrm{sign}(x) = \theta(x) - \theta(-x) = 2 \cdot \theta(x) - 1 \quad (3)$$

Based on this simple distribution is possible to define hystheresis distribution (fig.3.a):

$$h(x) = \begin{cases} \left.\begin{matrix} 1; & x \geq a \\ -1; & x < a \end{matrix}\right\} \text{if } \dfrac{dx}{dt} \geq 0 \\ \left.\begin{matrix} 1; & x \geq -a \\ -1; & x < -a \end{matrix}\right\} \text{if } \dfrac{dx}{dt} \leq 0 \end{cases} \quad (4)$$

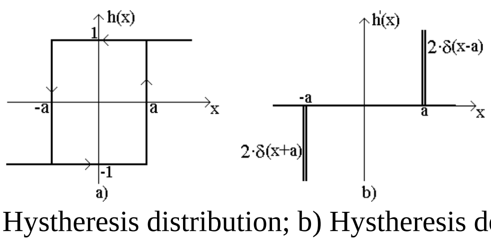

Fig.3. a) Hystheresis distribution; b) Hystheresis derivation

The distribution $h(x)$ can be written:

$$h(x) = \mathrm{sign}(x - a) \cdot \theta\left(\dot{x}\right) + \mathrm{sign}(x + a) \cdot \theta\left(-\dot{x}\right) \quad (5)$$

Results a new formula:

$$h(x) = \mathrm{sign}\left(x - a \cdot \mathrm{sign}\left(\dot{x}\right)\right) \quad (6)$$

If is calculated $[h(x)]'$ results (fig.3.b):

$$[h(x)]' = \left[x - a \cdot \mathrm{sign}\left(\dot{x}\right)\right] \cdot \theta\left[x - a \cdot \mathrm{sign}\left(\dot{x}\right)\right] = \dot{x} \cdot \delta\left[x - a \cdot \mathrm{sign}\left(\dot{x}\right)\right] = \pm\delta[x \mp a] \qquad (7)$$

In the case of the rectifier function $d(x)$ applied for $\sin(x)$ results:

$$d(x) = \sin_+(x) - \sin_-(x) = \mathrm{sign}[\sin(x)] \cdot \sin(x) \qquad (8)$$

$$[d(x)]' = \cos(x) \cdot \mathrm{sign}[\sin(x)] \qquad (9)$$

This approach opens a good perspective to analyze and design in an accurate manner, complex discreet elements like digital systems controller.

2. Mathematical support for distributions and their properties

We consider the n-dimensional Euclidian real space R^n, organized as a linear space. The elements of this space are n-coordinates vectors: $x = (x_1, x_2, ..., x_n)$, $y = (y_1, y_2, ..., y_n)$ and scalar product, vector norm and metrics are defined as follows:

$$\langle x, y\rangle = \sum_{i=1}^{n} x_i \cdot y_i \,; \qquad |x| = \sqrt{\sum_{i=1}^{n} {x_i}^2} \,; \;\; d(x,y) = |x-y| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2} \qquad (10)$$

K^m is called a fundamental space because it is formed out of C^m class (having continuous derivates till the m order) real argument functions $\varphi(x)$, equal with zero together with all their derivates till the m order in and outside the coresponding limited domains. Functions $\varphi(x) \in K^m$ form a linear space.

The name of distribution represents an extension of the function name, as follows: the distribution is a linear and continuous functional defined on a fundamental space that satisfies the following conditions:

- To the function $\varphi(x)$ corespondes a real number (f, φ) by the f law: $\varphi \xrightarrow{f} (f, \varphi)$;
- $\forall \lambda_1, \lambda_2 \in R$ and $\varphi_1, \varphi_2 \in K^m$: $(f, \lambda_1 \cdot \varphi_1 + \lambda_2 \cdot \varphi_2) = \lambda_1 \cdot (f, \varphi_1) + \lambda_2 \cdot (f, \varphi_2)$;
- If $\varphi_k \to \varphi$ for $k \to \infty$, then $\lim\limits_{k\to\infty}(f, \varphi_k) = (f, \varphi)$.

Elementary distributions

The pulse distribution or Dirac pulse is defined as follows (fig.4.a):

$$\delta : K^m \to R^n : \;\; \delta[\varphi(x)] = \varphi(0) = \begin{cases} 0; & x \neq 0 \\ -\infty; & x = 0 \end{cases}; \qquad \varphi(x) \in K^m; \quad \int_R \delta[\varphi(x)]dx = 1 \;\; (11)$$

The most important properties of Dirac distribution are:

1. $\delta(x - x_0) = \varphi(x_0)$ (fig.4.b); $a(x) \cdot \delta(x - x_0) = a(x_0) \cdot \delta(x - x_0)$ (fig.4.c)
2. $\delta(-x) = \delta(x)$; $k \cdot \delta(x - x_0) = k \cdot \varphi(x_0)$; $\delta[f(x)] = 0$; $f(x) \neq 0$ (fig.4.d)
3. $\dfrac{d[f(x)]}{dx} = f(x)' + \sum_{i=1}^{p} s_i \cdot \delta(x - x_i)$; $s_i = f(x_i + 0) - f(x_i - 0)$
4. $\delta[f(x)] = \sum_{i=1}^{p} \dfrac{1}{|f'(x)|} \cdot \delta(x - x_i)$; x_i=simple solutions of $f(x) = 0$.
5. $\delta[\sin(x)] = \sum_{k=-\infty}^{\infty} \delta(x - k \cdot \pi)$; $k \in Z$ (fig.4.g)

6. $\delta[\cos(x)] = \sum_{k=-\infty}^{\infty} \delta\left(x - (2\cdot k+1)\cdot\frac{\pi}{2}\right)$; $k \in Z$ (fig.4.h)

7. $f(x) = x - 1 + \theta(x+1) - \theta(x-1)$ (fig.4.e); $\frac{d[f(x)]}{dx} = 1 + \delta(x+1) - \delta(x-1)$ (fig.4.f).

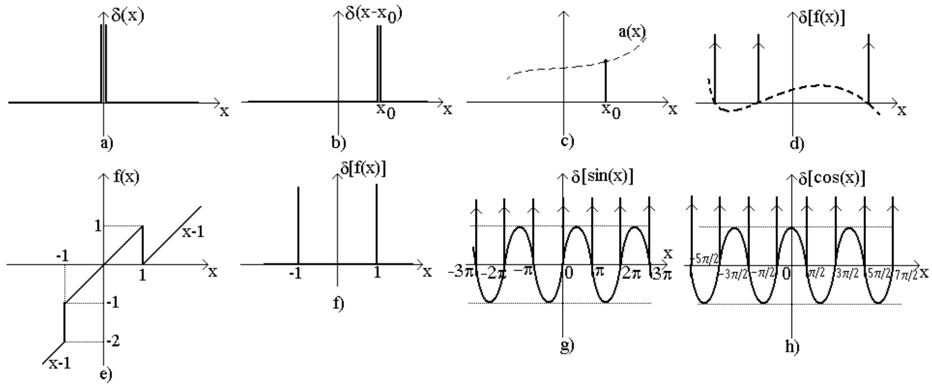

Fig.4. Dirac distribution and properties

The step distribution or Heaviside distribution is defined bellow:

$$\theta : K^m \to R; \qquad \theta[\varphi(x)] = \begin{cases} 1; & x \geq 0 \\ 0; & x < 0 \end{cases} \qquad \text{(fig.5.a)}$$

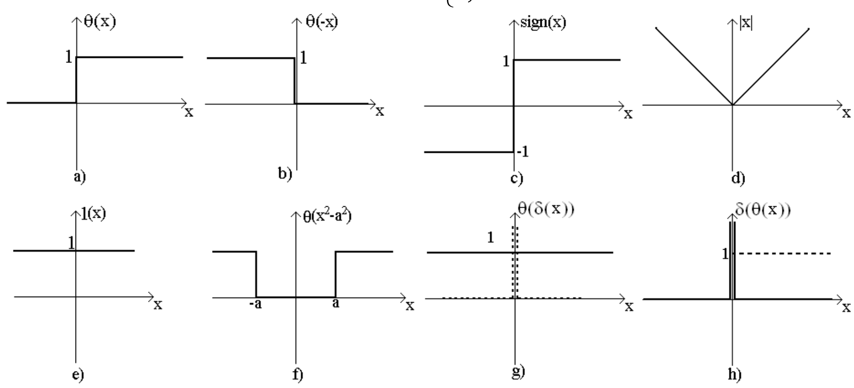

Fig.5. Heaviside distribution and properties

The important properties of $\theta(x)$ are:

1. $\theta(-x) = 1 - \theta(x)$ (fig.5.b); $\theta(x) - \theta(-x) = \text{sign}(x) = \begin{cases} 1; & x \geq 0 \\ -1; & x < 0 \end{cases}$ (fig.5.c)
2. $x \cdot \text{sign}(x) = |x|$ (fig.5.d); $\theta(x) + \theta(-x) = 1(x)$ (fig.5.e)
3. $\theta(x^2 - a^2) = 1 - \theta(x+a) + \theta(x-a)$ (fig.5.f)
4. $\theta(\delta(x)) = 1(x)$ (fig.5.g); $\delta(\theta(x)) = \delta(x)$ (fig.5.h)

New important distributions

There can be defined a new composed distributions using the two elementary ones:

1. Square wave distribution: $s(x) = \text{sign}[\cos(x)]$ (fig.6.a)
2. Tooth wave distribution: $\Delta(x) = r[\text{sign}[\cos(x)]]$ (fig.6.b)
3. Pulse Width Modulation distribution: $\text{PWM}(\alpha) = \theta[\sin(\alpha) - \Delta(p\cdot\alpha)]$ (fig.6.c)

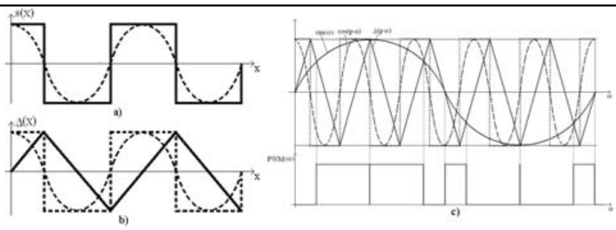

Fig.6. Composed distributions

3. Simulation and software implementation

In this section the elementary distributions will be simulated and major properties will be tested for a software implementation.

In fig.7 the model of $\delta(t), \theta(t), \mathrm{sign}(t)$ was designed and simulated.

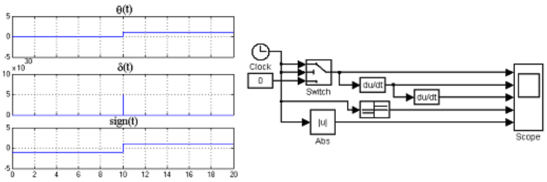

Fig.7. Simulation of $\delta(t), \theta(t), \mathrm{sign}(t)$ distributions

It is possible to implement by software the distributions presented above. First was written in assembly language the software for elementary distributions: $\delta(t), \theta(t) \text{and}\, r(t)$. In fig.8 is presented the subroutine for $\theta(t)$ and the results on the screen after running.

```
grafic_treapta proc near
; based on the ramp
distribution determined
previously is calculated the
step distribution
    mov di,offset RampComplet
     inc di
     inc di
    mov si,offset StepComplet
     mov word ptr [si],0
     inc si
     inc si
; step is graphic displayed
     mov x1,350
     mov y1,20
     mov x2,435
     mov y2,20
     mov culin,29
     call nilinie
     mov x1,430
     mov x2,430
     mov y2,780
     mov culin,29
     call nilinie
; step is saved in the
StepComplet buffer
     mov x1,430
     mov y1,20
     mov culin,32
     mov index,1
repTr: mov ax,[di]
       cmp ax,0
       je Tzero
       mov word ptr [si],1
       inc si
       inc si
       mov ax,index
       mov y2,20
       add y2,ax
       mov x2,360
       jmp trT
Tzero: mov word ptr [si],0
       inc si
       inc si
       mov ax,index
       mov y2,20
       add y2,ax
       mov x2,430
trT:   call nilinie
       mov ax,x2
       mov x1,ax
       mov ax,y2
       mov y1,ax
       inc word ptr index
       inc di
       inc di
       cmp index,700
       jae outT
       jmp repTr
outT: ret
grafic_treapta endp
```

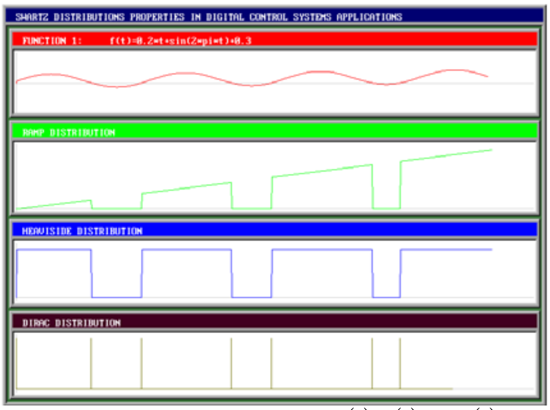

Fig.8. Software for elementary distributions: $\delta(t), \theta(t) \text{and}\, r(t)$

4. Experimental results for power inverter controller

The power inverter controller must produce six appropriate PWM pulses that are 120 degrees unphased. This is a complicated process as there must be solved by approximations some nonlinear transcendental equations in order to determine the pulses.

$$R \cdot \sin\left(\alpha + \frac{2 \cdot (i-1) \cdot \pi}{3}\right) = (-1)^{k-1} \cdot \frac{2 \cdot p}{\pi} \cdot \left(\alpha + \frac{2 \cdot (i-1) \cdot \pi}{3}\right) + \frac{(k-1) \cdot \pi}{p} \qquad (12)$$

Using the distributions theory this problem can be solved very simple and precisely. First the rectangular distribution will be generated by formula: $\theta[\sin(\omega \cdot t)] = \sum_{i=0}^{2 \cdot p} \delta(t - k \cdot \pi) \cdot$ After this, using the integration, applying $\delta(t)$ the pulses will be obtained as shown in fig.9.

The software was written in assembly language. There are different routines. One is to model the equation (12) at sample time and other to apply PWM distribution to sine-delta function. This software was tested and compared with the pulses produced by an electronic PWM inverter based on the IPM module FUJI 120RA100.

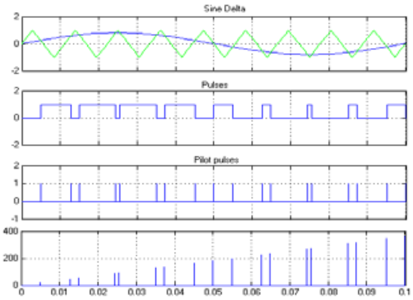

Fig.9. Simulation results of PWM-pulses

In fig.10 is presented PWM distribution software main screen and experimental results.

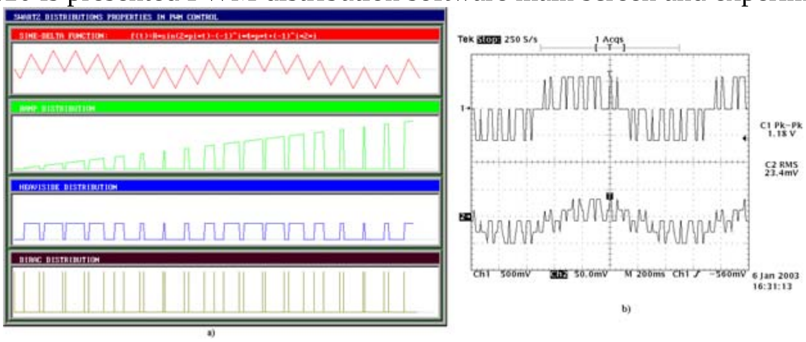

Fig.10. a) PWM software; b) Experimental results

5. Conclusions

1. The distributions are very useful for discreet signals like those in digital control.
2. Using the properties of distributions many complex problems of digital control can be solved easy and precisely.
3. The distributions are suitable for software implementation and offer very good solution for digital controller of PWM power inverter.
4. The experimental results are very closed to the classical ones.

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