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DYNAMICAL NONLINEAR MODEL OF THE WHEEL-RAIL SLIP IN RAILWAY TRACTION BY DIESEL-ELECTRIC LOCOMOTIVES WITH DC TRACTION MOTORS

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Abstract: Due to the lessening of the ratio weight/ power of the actual Diesel-locomotives, the slip of the system wheel-rail may arise in frequent start process.
In order to limit or even to present this phenomenon, the first step of the new equipment design is the conceiving of the mathematical nonlinear model, based on the algebraic and differential specific equation and on the empirical knowledge. The results will be used in subsequent works, to conceive the anti slip equipment analysis and improvement.

Key-words: Diesel electric railway traction, DC-traction motors, slip phenomenon

1. INTRODUCTION

The actual technological progress led to a continuous diminution of the locomotive weight despite the growth of the locomotive tractive force and power. The problem of the wheel-rail slip becomes more stringent and the disastrous consequences, more effective.

The wheel-rail slip is a nonlinear, intricate process, hence a dynamical mathematical model is a difficult task one.

The authors of the paper use the general algebraic and differential equations governing the electrical and mechanical processes in order to infer a (general) nonlinear model of the wheel-rail slipping phenomenon. Based on this model, a numerical simulation program in MATLAB-SIMULINK was set in order to analyze the "magnitude" of the slip. Further, this model is used in slip control systems [1, 3].

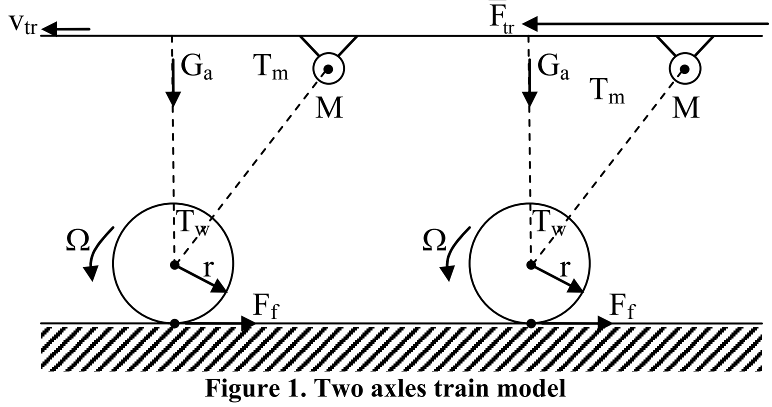

Figure 1. Two axles train model

2. MATHEMATICAL BACKGROUND

For the analysis of the wheel-rail slip phenomenon analysis, a two axles simplified model, as shown in figure 1 is assumed and a series of differential and algebraic equations are proposed [2, 3, 4].

Traction force $\overline{F}_{tr}$ ("no slip) is given by:

$$F_{tr} = \psi \cdot G_a \tag{1}$$

where (ψ) is the adhesion coefficient (as the maximum value of the friction coefficient μ) and (G_a) is the axel load.

If (Ω) is the angular (rotational) speed of the wheel of radius (r) and (v_{tr}) is the linear train speed, the slip-speed is given by:

$$u_s = \Omega \cdot r - v_{tr}\,.$$

The adhesion coefficient is a nonlinear function of (v_{tr}) and (u_S) as in the figure 2, as a results of experimental data [3, 4]. It is proposed the approximation:

$$\psi^*(u_S) = \begin{cases} \dfrac{\psi_1}{1 + a \cdot v_{tr}} \cdot \sin\left(\dfrac{\pi \cdot U_S}{2 \cdot U_S^*}\right), \text{for } U_S \le U_S^* \\ \dfrac{\psi_1 - \psi_2 \cdot \exp\left[-\, k \big/ \left(U_S - U_S^*\right)\right]}{1 + a \cdot v_{tr}}, \text{for } U_S > U_S^* \end{cases} \tag{2}$$

assuming a constant speed train during the slip process.

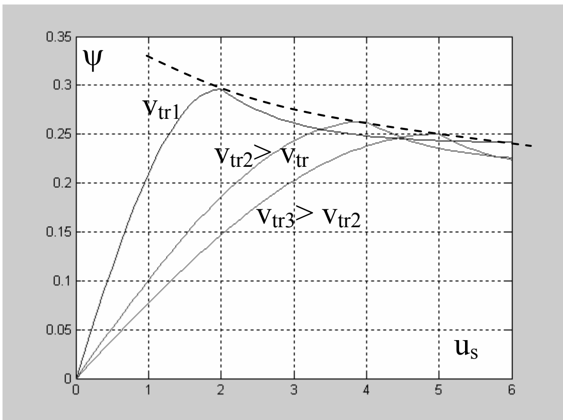

Figure 2. Curves of the adhesion coefficient

From the propulsion system is selected a group of two series connected motors, (M_1) and (M_2), belonging to two different bogies, figure 3. It is supposed, like in the general case, only one motor is affected by slip phenomenon. The slip may arise when the torque given by the adhesion force is smaller than the wheel torque (T_w):

$$\psi \cdot G_a \cdot r < T_w \tag{3}$$

Hence, (ψ) is given by the wheel-rail quality (rail wet or dry, clean or not, etc.) and (G_a), the instantaneous vertical load corresponding to the rail wheel imperfections.

During the start process, the motors operates at high currents, so the motor coils are saturated. For the dependence $\Phi = f(I)$ it is proposed the approximation:

$$\Phi = k_{\Phi 1} \cdot I + k_{\Phi 2} \cdot \sqrt{I} \ \text{[p.u.]} \tag{4}$$

so that the electromotive forces are given by:

$$E_1 = k_{Ev}^* \cdot \Phi \cdot v_w = k_{Ev} \cdot \Phi \cdot \Omega_1 = k_{Ev}^* \cdot \left(k_{\Phi 1} \cdot I + k_{\Phi 2} \cdot \sqrt{I}\right) \cdot r \cdot \Omega_1 \tag{5}$$

$$E_2 = k_{Ev} \cdot \Phi \cdot \Omega_2 = k_{Ev}^* \cdot \left(k_{\Phi 1} \cdot I + k_{\Phi 2} \cdot \sqrt{I}\right) \cdot r \cdot \Omega_2 \tag{5'}$$

For the traction motors in the slip process:

$$\begin{aligned} E_t &= E_1 + E_2 = k_{Ev}^* \cdot \left(k_{\Phi 1} \cdot I + k_{\Phi 2} \cdot \sqrt{I}\right) \cdot \left(2 \cdot v_{tr} + U_S\right) = \\ &= k_{Ev}^* \cdot \left(k_{\Phi 1} \cdot I + k_{\Phi 2} \cdot \sqrt{I}\right) \cdot 2 \cdot v_{tr} + k_{Ev}^* \cdot \left(k_{\Phi 1} \cdot I + k_{\Phi 2} \cdot \sqrt{I}\right) \cdot u_S = 2E_N + E_S \end{aligned} \tag{6}$$

and

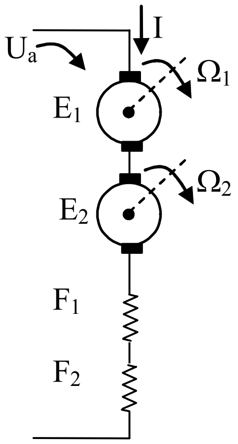

Figure 3. Two motor traction model

$$U_a = L_t \frac{dI}{dt} + r_t \cdot I + E_t \tag{7}$$

where (L_t, r_t) are the total, equivalent circuit inductance and resistance so that:

$$I(s) = \frac{U_a(s) - E_t(s)}{s \cdot L_t + r_t} \tag{7'}$$

The motor torque is given by the equation:

$$T_m = k_m \cdot \Phi \cdot I = k_m \cdot \left(k_{\Phi 1} \cdot I^2 + k_{\Phi 2} \cdot I^{3/2}\right).$$

The mechanical transmission motor-wheel has an equivalent system "mass-spring-damping"[2]:

$$k_{spring}\left(\theta_m - \theta_w\right) = J_g^* \cdot \frac{d\Omega}{dt} + f_{fr} \cdot \Omega = J_g^* \cdot \frac{d^2\theta_w}{dt^2} + f_{fr} \cdot \frac{d\theta_w}{dt}$$

$$k_{spring} \cdot \theta_m = \frac{J_g^*}{k_{sp}} \cdot \frac{d^2}{dt^2}\left(k_{sp} \cdot \theta_w\right) + \frac{f_{fr}}{k_{sp}} \cdot \frac{d}{dt}\left(k_{sp} \cdot \theta_w\right) + \left(k_{sp} \cdot \theta_w\right)$$

with: $k_{sp} \cdot \theta_m = T_m$; $k_{sp} \cdot \theta_w = T_w$ results:

$$T_w(s) = \frac{1}{\frac{J_g^*}{k_{sp}} s^2 + \frac{f_{fr}}{k_{sp}} s + 1} \cdot T_m(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1} \cdot T_m(s) \tag{8}$$

The traction force is generated by a "normal" friction force [3, 4], but if condition (3) is carried out, one motor and one axle start to rotate with a greater speed (supplementary slip speed u_S). The wheel angular speed will be:

$$\Omega_{ws} = \frac{u_S + v_{tr}}{r} \tag{9}$$

The ground of the slip speed (of course, in the following the authors understand only the **supplementary** slip speed) is the diminishing of the wheel load torque:

$$T_{L_1} \rightarrow T_{L_2} = T_L = T_{L_0} - \Delta T_L - T_B \tag{10}$$

where $\left(T_{L_0}\right)$ states for the load torque before the slip start and:

$$\begin{aligned} \Delta T_L &= \Delta\left[k_\Psi \cdot \Psi \cdot G_a\right] = \left(k_\Psi \cdot \Delta\Psi\right) \cdot \left[G_{a0} + f^*(G_a)\right] = \\ &= \left[k_\Psi \cdot \Delta\Psi_r + \Psi^*(u_S)\right] \cdot \left[G_{a0} + f^*(G_a)\right] \end{aligned}$$

where:

$\Delta\psi_r$ is the suddenly lessening of the adhesion coefficient due to the rail-wheel local quality degradation;

$\psi^*(u_S)$ is the evolution of the adhesion coefficient, equation (2);

G_{a0} is the constant component of the vertical load;

$f^*(G_a)$ is an empirical function, dependent on the wheel eccentricity, rail differences in vertical level, etc.

ΔT_B is the braking torque, given by the "anti-slip system" [1, 5].

If (J_{wt}) is moment of inertia of the system: traction motor + transmission system + axle + wheels, the dynamics in the slip progress is given by the equation:

$$T_w - T_{L_2} = T_w - T_L \cong J_{wt} \cdot \frac{d\Omega_S}{dt} = \frac{J_{wt}}{r} \cdot \frac{du_S}{dt} = J^*_{wt} \cdot \frac{du_S}{dt} \qquad (11)$$

3. STRUCTURE AND PRELIMINARIES OF THE MATLAB SIMULINK SIMULATION PROGRAM

The program put together and connects the Simulink-blocks, based on the equation (2-11). As an example, the equation $f^*(G_a)$ highlights the wheel eccentricity:

$$f^*(G_a) = K_1 + K_2 \cdot \sin(\Omega_{ws} \cdot t)$$

The simulation scheme uses linear and nonlinear blocks: four multipliers, one square-root block, function generator $\Psi^*(u_s)$, $f^*(G_a)$, etc., figure 4.

The detailed simulation program is given in figure 5.

The simulated example presented in this program supposes a simple strategy:

a) the locomotive driver sets a starting acceleration value of 0.2 so that, $v_{tr}(t) = 0.2 \cdot t$ which is possible if the motor voltage rises also linearly: $U_a(t) = 10 \cdot t$;
b) the friction coefficients on all wheels have the rated value, so that no initially slip occurs;
c) at the time-point $t^*=5$, the friction coefficient ("adhesion") decreases with ($\Delta\psi_r$), so that the motor M1 starts to slip, figure 6
d) without the anti-slip system, $\Delta T_B=0$;
e) some important "output" variables like: motor current (I), supplementary slip speed (u_s), angular speed of the wheel "slipping" (Ω_{ws}) are accessible, figure 6
f) the influence of the locomotive parameters (L_t, r_t, k_Φ, J_{wt}, etc.) upon the output variables can be studied.

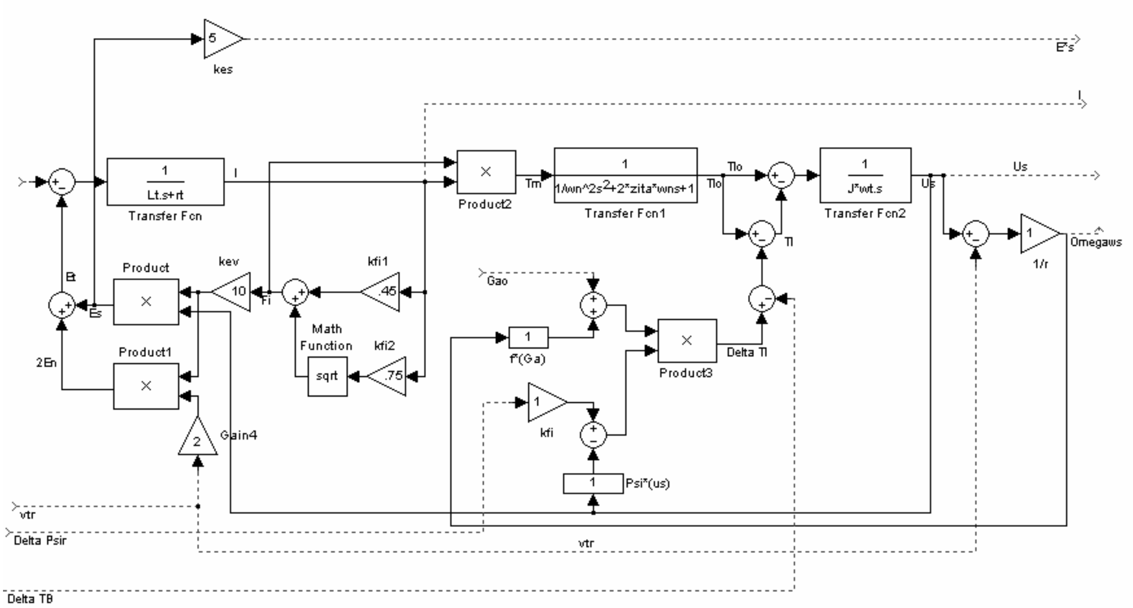

Figure 4. SIMULINK simulation scheme for the slip process

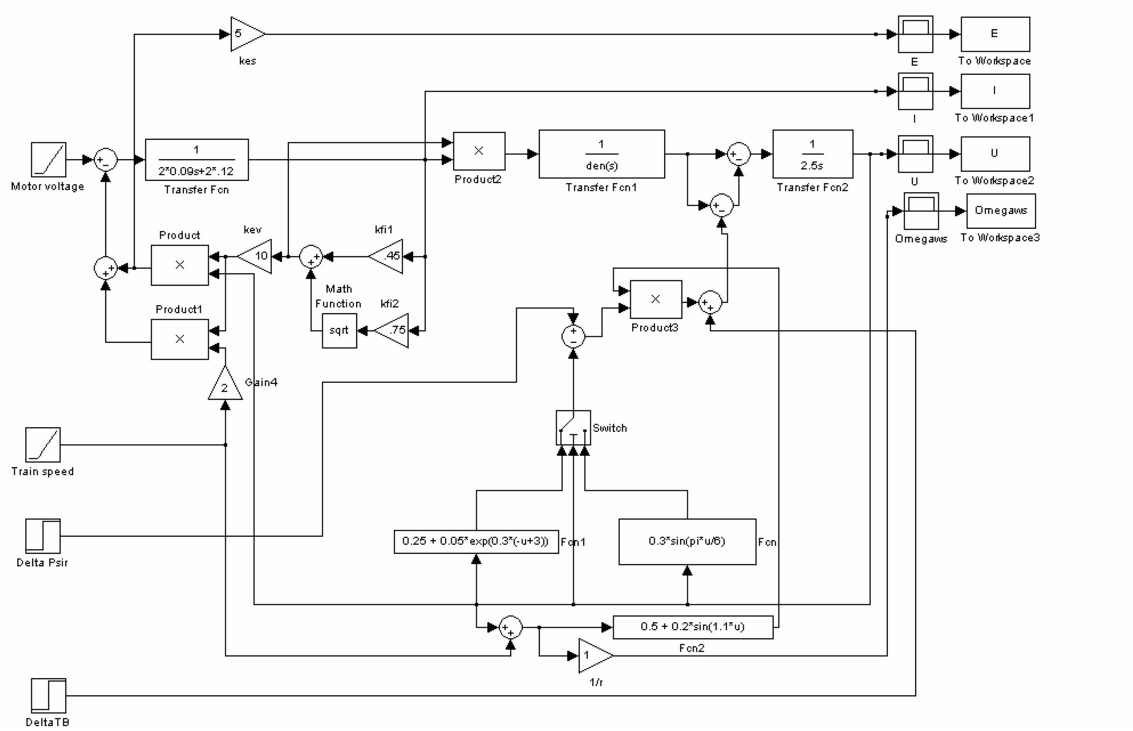

Figure 5. The detailed simulation scheme

The evolution of the “output” variables is given in figure 6.

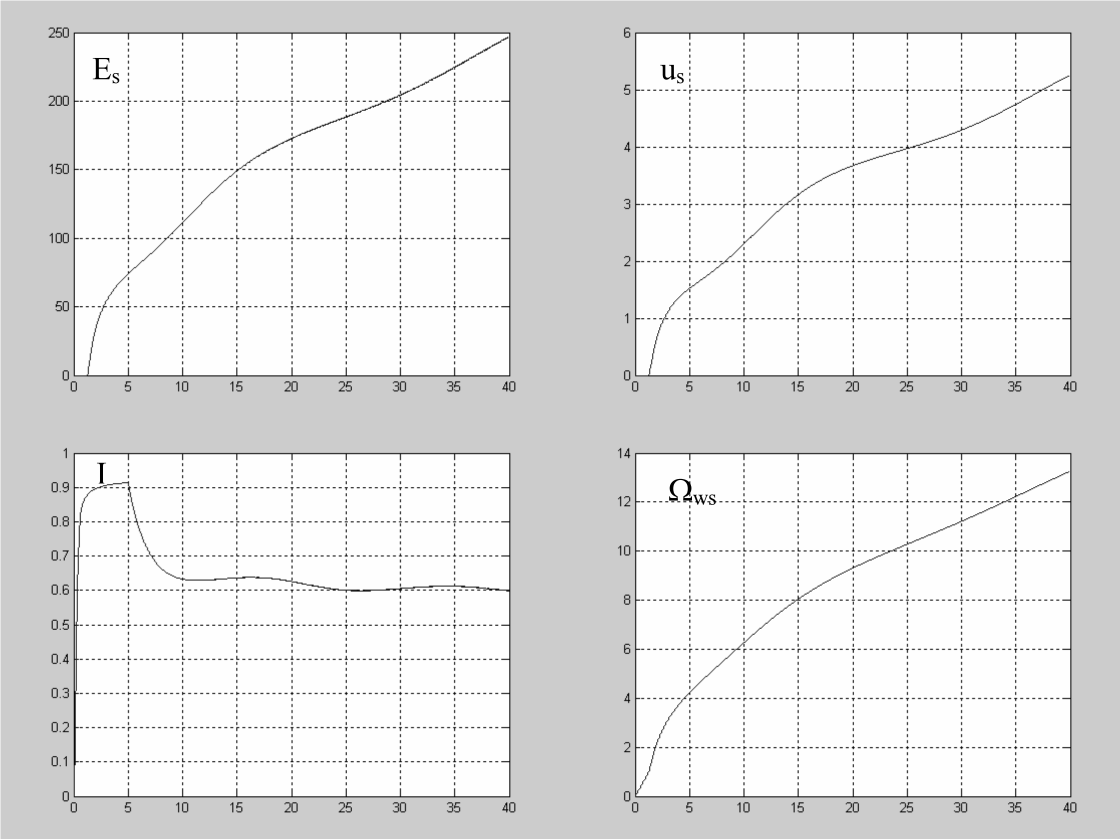

Figure 6. Simulation results

4. CONCLUSIONS

The authors consider as an important step forward in the anti-slip systems analyze and improvement, the conceiving of the simulation program of the slip-phenomenon, based on the fundamental equation, governing the locomotive running process.

The SIMULINK program gives the possibilities to interconnect some **linear** and **nonlinear** blocks in order to obtain a general "board" of the slip phenomenon.

This program will be integrated in a more complex anti-slip control system.

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