RESEARCHES REGARDING THE GENERALIZED MOTOR FORCES FOR THE TTRTR PORTAL ROBOT

Ioan Ovidiu COSTIN*

** Technical University of Cluj-Napoca, Dept. Machine Tools and Industrial Robots, Blvd. Muncii 103-105, Cluj-Napoca, Romania*

Abstract. For a robot that has to execute a working task, being determined the geometrical and kinematic model, we determine the dynamic model for analyzing its functioning. In this paper, using the Maple application, we plot the variation of the generalized motor forces, determined based on the Newton-Euler formalism.

Keywords: portal robot, dynamic model, generalized motor forces, trajectory.

1. INTRODUCTION

The model of the robot with the TTRTR structure is represented in figure 1.

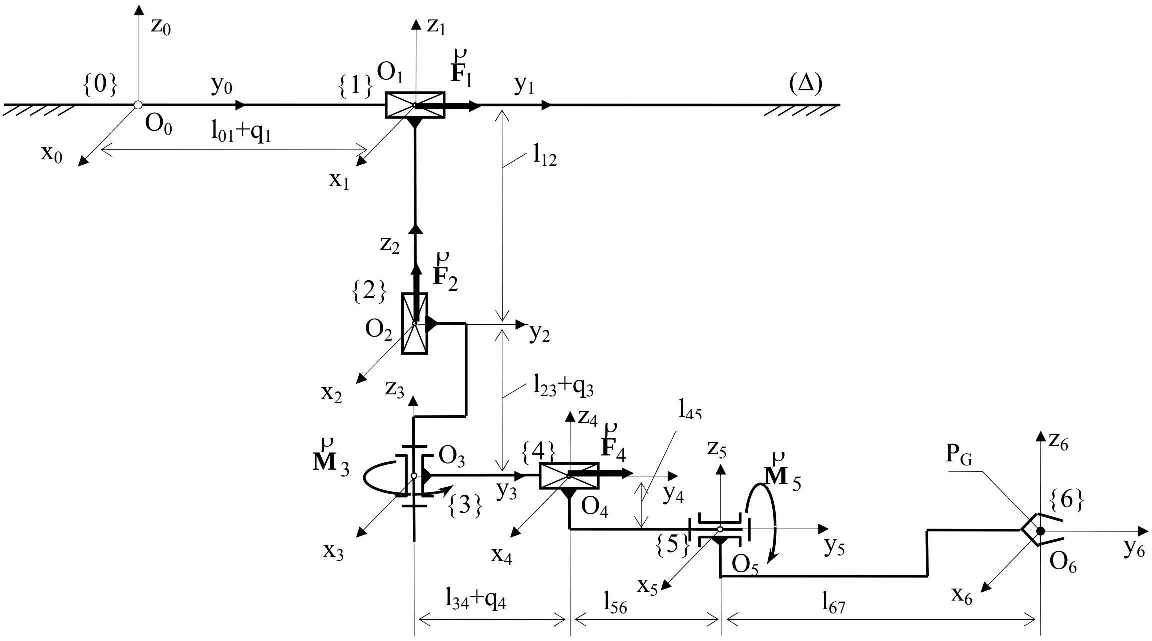


Fig.1. The model of the robot with the structure TTRTR

The manipulated object is moving in the workspace, on the trajectory requested by the application. The dynamical performances of the robot are established by the values of some parameters such as: the bounds for the velocity and acceleration, the accuracy, the repeatability, the resolution of the motion, the ability of controlling the motion, etc. ([1, 2, 6]). By trajectory planning, one generates the variation laws with respect to the time of the generalized coordinates, which make up the reference entries for the control system of the mechanical structure. Thus, every component module performs some motion, the variation law of the angular velocity of the engine axle being known.

2. DYNAMIC MODELING

We consider known: the position vectors ${}^{i}\bar{r}_{C_i}$ of the mass centers of the modules i with respect to the reference system $\{i\}$, $i=\overline{1,5}$, the inertial tensors ${}^{i}I_i^*$ of the modules i with respect to the reference system $\{i\}$, $i=\overline{1,5}$ and the external force ${}^{6}\bar{f}_6$ and the torque ${}^{6}\bar{n}_6$ that characterizes the effective manipulated charge.

$$ {}^{i}\bar{r}_{C_i} = \begin{bmatrix} rcx_i \\ rcy_i \\ rcz_i \end{bmatrix} \qquad {}^{i}I_i^* = \begin{bmatrix} Ixs_i & 0 & 0 \\ 0 & Iys_i & 0 \\ 0 & 0 & Izs_i \end{bmatrix}, \; i=\overline{1,5} \tag{1} $$

$$ {}^{i}\bar{n}_i = \begin{bmatrix} ni_x \\ ni_y \\ ni_z \end{bmatrix} \qquad {}^{i}\bar{f}_i = \begin{bmatrix} fi_x \\ fi_y \\ fi_z \end{bmatrix}, \; i=\overline{1,6} \tag{2} $$

Based on the kinematic model and according to [4, 5], we determine the accelerations of the centers of mass, the external forces, the torques of the external forces, the linking forces and torques, and the generalized motor forces, respectively. The generalized motor forces have the following expressions:

$$ Q^1_m = \Bigg(\Big(\frac{1}{2} rcx_5 \cos(q_3(t)+q_5(t)) - \frac{1}{2} rcx_5 \cos(q_3(t)-q_5(t)) + \frac{1}{2} rcz_5 \sin(q_3(t)-q_5(t)) + $$
$$ + \frac{1}{2} rcz_5 \sin(q_3(t)+q_5(t)) \Big) \left(\frac{d^2}{dt^2} q_5(t) \right) + \left(\frac{d^2}{dt^2} q_1(t) \right) + \Big(-\frac{1}{2} rcz_5 \sin(q_3(t)-q_5(t)) + $$
$$ + \frac{1}{2} rcz_5 \sin(q_3(t)+q_5(t)) + (-q_4(t) - l_{3,4} - l_{5,6} - rcy_5) \sin(q_3(t)) + \frac{1}{2} rcx_5 \cos(q_3(t)+q_5(t)) + $$
$$ + \frac{1}{2} rcx_5 \cos(q_3(t)-q_5(t)) \Big) \left(\frac{d^2}{dt^2} q_3(t) \right) + \cos(q_3(t)) \left(\frac{d^2}{dt^2} q_4(t) \right) - 2 \sin(q_3(t)) \left(\frac{d}{dt} q_3(t) \right) \left(\frac{d}{dt} q_4(t) \right) + $$
$$ + \Big(-\frac{1}{2} rcx_5 \sin(q_3(t)-q_5(t)) - \frac{1}{2} rcx_5 \sin(q_3(t)+q_5(t)) + \frac{1}{2} rcz_5 \cos(q_3(t)+q_5(t)) - $$
$$ - \frac{1}{2} rcz_5 \cos(q_3(t)-q_5(t)) + (-q_4(t) - l_{3,4} - l_{5,6} - rcy_5) \cos(q_3(t)) \Big) \left(\frac{d}{dt} q_3(t) \right)^2 + $$
$$ (rcx_5 \sin(q_3(t)-q_5(t)) + rcz_5 \cos(q_3(t)-q_5(t)) + rcz_5 \cos(q_3(t)+q_5(t)) - $$
$$ - rcx_5 \sin(q_3(t)+q_5(t))) \left(\frac{d}{dt} q_5(t) \right) \left(\frac{d}{dt} q_3(t) \right) + \Big(-\frac{1}{2} rcx_5 \sin(q_3(t)+q_5(t)) $$
$$ + \frac{1}{2} rcz_5 \cos(q_3(t)+q_5(t)) - \frac{1}{2} rcx_5 \sin(q_3(t)-q_5(t)) - \frac{1}{2} rcz_5 \cos(q_3(t)-q_5(t)) \Big) $$
$$ \left(\frac{d}{dt} q_5(t) \right)^2 \Bigg) M_5 + (M_3 + M_4 + M_1 + M_2) \left(\frac{d^2}{dt^2} q_1(t) \right) + (((-q_4(t) - l_{3,4} - rcy_4) M_4 - $$
$$ - M_3 \, rcy_3) \sin(q_3(t)) + (M_3 \, rcx_3 + M_4 \, rcx_4) \cos(q_3(t))) \left(\frac{d^2}{dt^2} q_3(t) \right) $$
$$ + \cos(q_3(t)) M_4 \left(\frac{d^2}{dt^2} q_4(t) \right) - 2 \sin(q_3(t)) M_4 \left(\frac{d}{dt} q_3(t) \right) \left(\frac{d}{dt} q_4(t) \right) + ((-M_4 \, rcx_4 - $$
$$ - M_3 \, rcx_3) \sin(q_3(t)) + ((-q_4(t) - l_{3,4} - rcy_4) M_4 - M_3 \, rcy_3) \cos(q_3(t))) \left(\frac{d}{dt} q_3(t) \right)^2 $$

$$+\frac{1}{2} f6_x \sin(q_3(t)+q_5(t)) + \frac{1}{2} f6_x \sin(q_3(t)-q_5(t)) + \frac{1}{2} f6_z \cos(q_3(t)-q_5(t)) - \\ -\frac{1}{2} f6_z \cos(q_3(t)+q_5(t)) + \cos(q_3(t))\, f6_y \tag{3}$$

$$Q^2{}_m = \left((-rcx_5 \cos(q_5(t)) - \sin(q_5(t))\, rcz_5) \left(\frac{d^2}{dt^2} q_5(t)\right) + \left(\frac{d^2}{dt^2} q_2(t)\right) + (rcx_5 \sin(q_5(t)) - rcz_5 \cos(q_5(t))) \\ \left(\frac{d}{dt} q_5(t)\right)^2 + g \right) M_5 + (M_2+M_3+M_4)\left(\frac{d^2}{dt^2} q_2(t)\right) - \sin(q_5(t))\, f6_x + \cos(q_5(t))\, f6_z + M_4\, g + (M_2+M_3)\, g \tag{4}$$

$$Q^3{}_m = \Bigg(((rcy_5\, rcx_5 + q_4(t)\, rcx_5 + l_{5,6}\, rcx_5 + l_{3,4}\, rcx_5)\sin(q_5(t)) + (-q_4(t)\, rcz_5 - l_{5,6}\, rcz_5 - l_{3,4}\, rcz_5 \\ - rcy_5\, rcz_5)\cos(q_5(t)))\left(\frac{d^2}{dt^2} q_5(t)\right) + \Big(-\frac{1}{2} rcz_5 \sin(q_3(t)-q_5(t)) + \frac{1}{2} rcz_5 \sin(q_3(t)+q_5(t)) + \\ + (-q_4(t) - l_{3,4} - l_{5,6} - rcy_5)\sin(q_3(t)) + \frac{1}{2} rcx_5 \cos(q_3(t)+q_5(t)) + \frac{1}{2} rcx_5 \cos(q_3(t) \\ - q_5(t))\Big)\left(\frac{d^2}{dt^2} q_1(t)\right) + \Big(rcz_5\, rcx_5 \sin(2\,q_5(t)) + \left(-\frac{1}{2} rcz_5{}^2 + \frac{1}{2} rcx_5{}^2\right)\cos(2\,q_5(t)) + \\ + l_{5,6}{}^2 + \frac{1}{2} rcz_5{}^2 + 2\,l_{5,6}\,l_{3,4} + q_4(t)^2 + 2\,rcy_5\,q_4(t) + 2\,rcy_5\,l_{5,6} \\ + 2\,l_{5,6}\,q_4(t) + \frac{1}{2} rcx_5{}^2 + l_{3,4}{}^2 + 2\,rcy_5\,l_{3,4} + rcy_5{}^2 + 2\,q_4(t)\,l_{3,4}\Big)\left(\frac{d^2}{dt^2} q_3(t)\right) + (rcx_5 \cos(q_5(t)) + \\ + \sin(q_5(t))\, rcz_5)\left(\frac{d^2}{dt^2} q_4(t)\right) + (2\,l_{5,6} + 2\,rcy_5 + 2\,q_4(t) + 2\,l_{3,4})\left(\frac{d}{dt} q_3(t)\right)\left(\frac{d}{dt} q_4(t)\right) \\ + ((-rcx_5{}^2 + rcz_5{}^2)\sin(2\,q_5(t)) + 2\,rcz_5\,rcx_5 \cos(2\,q_5(t)))\left(\frac{d}{dt} q_5(t)\right)\left(\frac{d}{dt} q_3(t)\right) + \\ ((l_{3,4}\, rcz_5 + rcy_5\, rcz_5 + q_4(t)\, rcz_5 + l_{5,6}\, rcz_5)\sin(q_5(t)) \\ + (rcy_5\, rcx_5 + q_4(t)\, rcx_5 + l_{5,6}\, rcx_5 + l_{3,4}\, rcx_5)\cos(q_5(t)))\left(\frac{d}{dt} q_5(t)\right)^2 \Bigg) M_5 \\ + (((-q_4(t) - l_{3,4} - rcy_4)\, M_4 - M_3\, rcy_3)\sin(q_3(t)) + (M_3\, rcx_3 + M_4\, rcx_4)\cos(q_3(t)))\left(\frac{d^2}{dt^2} q_1(t)\right) + \\ \Big(\left(-\frac{1}{2} Ixs_5 + \frac{1}{2} Izs_5\right)\cos(2\,q_5(t)) + (2\,rcy_4\,q_4(t) + 2\,q_4(t)\,l_{3,4} + 2\,rcy_4\,l_{3,4} + rcy_4{}^2 + l_{3,4}{}^2 + rcx_4{}^2 + \\ + q_4(t)^2)\, M_4 + rcx_3{}^2\, M_3 + \frac{1}{2} Izs_5 + \frac{1}{2} Ixs_5 + Izs_4 + rcy_3{}^2\, M_3 + Izs_3\Big)\left(\frac{d^2}{dt^2} q_3(t)\right) + M_4\, rcx_4\left(\frac{d^2}{dt^2} q_4(t)\right) \\ + (2\,q_4(t) + 2\,l_{3,4} + 2\,rcy_4)\, M_4\left(\frac{d}{dt} q_3(t)\right)\left(\frac{d}{dt} q_4(t)\right) + (Ixs_5 - Izs_5)\sin(2\,q_5(t))\left(\frac{d}{dt} q_5(t)\right)\left(\frac{d}{dt} q_3(t)\right) \\ + (-q_4(t)\, f6_z - n6_x - l_{3,4}\, f6_z - l_{5,6}\, f6_z - l_{6,7}\, f6_z)\sin(q_5(t)) \\ + (-l_{6,7}\, f6_x - q_4(t)\, f6_x - l_{3,4}\, f6_x + n6_z - l_{5,6}\, f6_x)\cos(q_5(t)) \tag{5}$$

$$Q^4{}_m = M_5\Bigg(\cos(q_3(t))\left(\frac{d^2}{dt^2} q_1(t)\right) + (\cos(q_5(t))\, rcx_5 + \sin(q_5(t))\, rcz_5)\left(\frac{d^2}{dt^2} q_3(t)\right) + \left(\frac{d^2}{dt^2} q_4(t)\right) \\ + (2\,rcz_5 \cos(q_5(t)) - 2\,rcx_5 \sin(q_5(t)))\left(\frac{d}{dt} q_3(t)\right)\left(\frac{d}{dt} q_5(t)\right) + (-l_{5,6} - q_4(t) - l_{3,4} - rcy_5) \\ \left(\frac{d}{dt} q_3(t)\right)^2\Bigg) + M_4 \cos(q_3(t))\left(\frac{d^2}{dt^2} q_1(t)\right) + M_4\left(\frac{d^2}{dt^2} q_3(t)\right) rcx_4 + M_4\left(\frac{d^2}{dt^2} q_4(t)\right) + (-rcy_4 - \\ - q_4(t) - l_{3,4})\, M_4\left(\frac{d}{dt} q_3(t)\right)^2 + f6_y \tag{6}$$

$$
\begin{aligned}
Q^5{}_m = & \Bigg((rcz_5{}^2 + rcx_5{}^2)\left(\frac{d^2}{dt^2} q_5(t)\right) + \Bigg(\frac{1}{2} rcx_5 \cos(q_3(t)+q_5(t)) - \\
& \frac{1}{2} rcx_5 \cos(q_3(t)-q_5(t)) + \frac{1}{2} rcz_5 \sin(q_3(t)-q_5(t)) + \frac{1}{2} rcz_5 \sin(q_3(t)+q_5(t))\Bigg) \\
& \left(\frac{d^2}{dt^2} q_1(t)\right) + (-rcx_5 \cos(q_5(t)) - \sin(q_5(t))\, rcz_5)\left(\frac{d^2}{dt^2} q_2(t)\right) + \\
& ((rcy_5\, rcx_5 + q_4(t)\, rcx_5 + l_{5,\,6}\, rcx_5 + l_{3,\,4}\, rcx_5) \sin(q_5(t)) \\
& + (-q_4(t)\, rcz_5 - l_{5,\,6}\, rcz_5 - l_{3,\,4}\, rcz_5 - rcy_5\, rcz_5) \cos(q_5(t)))\left(\frac{d^2}{dt^2} q_3(t)\right) \\
& + (2\, rcx_5 \sin(q_5(t)) - 2\, rcz_5 \cos(q_5(t)))\left(\frac{d}{dt} q_3(t)\right)\left(\frac{d}{dt} q_4(t)\right) \\
& + \left(\left(-\frac{1}{2} rcz_5{}^2 + \frac{1}{2} rcx_5{}^2\right) \sin(2\, q_5(t)) - rcz_5\, rcx_5 \cos(2\, q_5(t))\right)\left(\frac{d}{dt} q_3(t)\right)^2 - rcx_5 \cos(q_5(t))\, g \\
& - rcz_5 \sin(q_5(t))\, g \Bigg) M_5 + Iys_5 \left(\frac{d^2}{dt^2} q_5(t)\right) + \left(-\frac{1}{2} Ixs_5 + \frac{1}{2} Izs_5\right) \sin(2\, q_5(t)) \left(\frac{d}{dt} q_3(t)\right)^2 + n6_y \qquad (7)
\end{aligned}
$$

3. NUMERICAL RESULTS

We assume the gripper has to move along a curve that passes through the points $P_1(0.2, 0.1, -0.65)$, $P_2(0.2, 0.4, -0.65)$, $P_3(0.2, 0.7, -0.9)$, $P_4(0.2, 1.2, -0.48)$, and $P_5(0.2, 2.1, -0.9)$. We want to obtain a smooth trajectory for the gripper, variations as smooth as possible for the parameters, and also positioning precision as good as possible in those points. Therefore, we compute a spline function of degree 3 that passes through the five points. The resulting equation (computed using the Maple application [3]), has the following form:

$$
\begin{cases}
x(t) = 0{,}2 \\
y(t) = 0{,}1+0{,}2*t \qquad (t \in [0, 10]) \\
z(y) = \begin{cases}
-\frac{306779}{452250} + \frac{25633}{120600} y + \frac{25633}{24120} y^2 - \frac{25633}{7236} y^3 & y < \frac{2}{5} \\
-\frac{653971}{452250} + \frac{720017}{120600} y - \frac{321559}{24120} y^2 + \frac{61165}{7236} y^3 & y < \frac{7}{10} \\
\frac{496547}{167500} - \frac{2599301}{201000} y + \frac{183197}{13400} y^2 - \frac{17711}{4020} y^3 & y < \frac{6}{5} \\
-\frac{40533}{6700} + \frac{386063}{40200} y - \frac{41069}{8040} y^2 + \frac{5867}{7236} y^3 & \textit{otherwise}
\end{cases}
\end{cases} \qquad (8)
$$

The curve that describes the trajectory of the characteristic point of the gripper (fig.2) is in a plane that is parallel with yOz (x = 0.2), and the values for y vary linearly in the interval [0.1, 2.5].

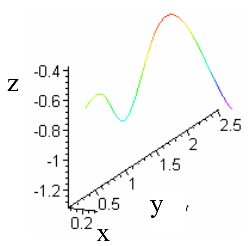


Fig. 2. The trajectory of the gripper

In the figures 3-7 we represented the generalized motor forces for the 5 degrees of mobility of the robot.

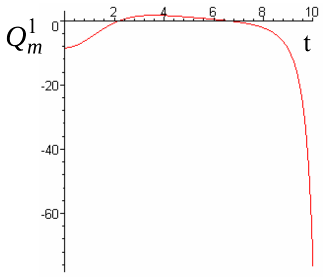


Fig. 3. The plot for Q_m^1 (t)

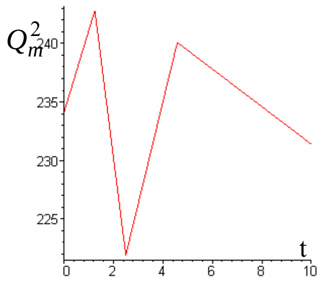


Fig. 4. The plot for Q_m^2 (t)

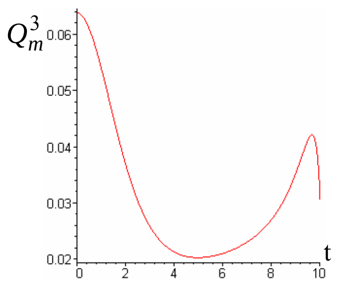


Fig. 5. The plot for Q_m^3 (t)

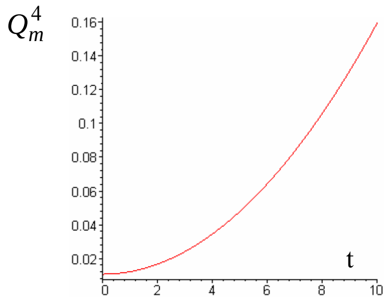


Fig. 6. The plot for Q_m^4 (t)

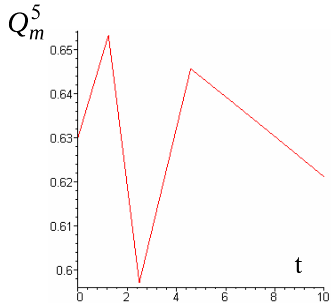
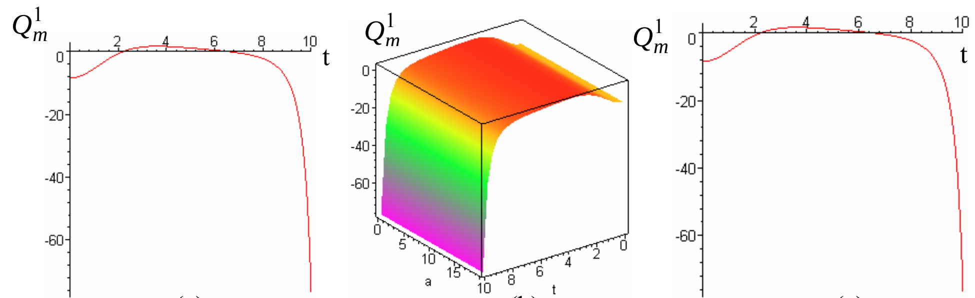


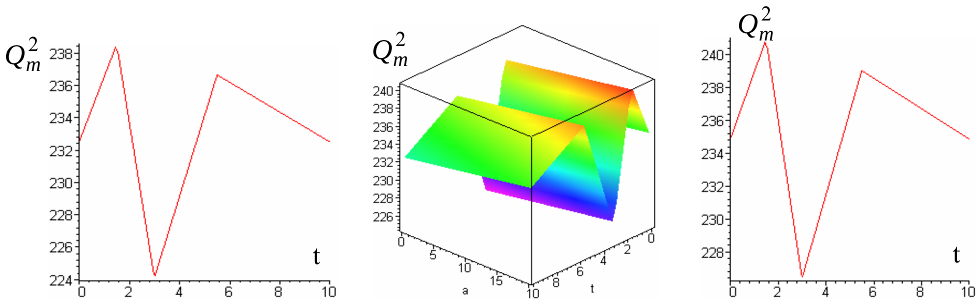
Fig. 7. The plot for Q_m^5 (t)

If, for constructive or functional considerations, the values of the component masses vary, these will influence the values of the generalized motor forces, according to the equations (3)-(7). Thus, for example, if the module 5 has the mass M_5 varying in the interval [0.45, 0.69] (=0.57+/-20%), according to the relation M_5 = 0.45+a*0.012, where a $\in$ [0, 20], we illustrated graphically its influence on the generalized motor forces, as follows: Q_m^1 (in figure 8), Q_m^2 (in figure 9), Q_m^3 (in figure 10), Q_m^4 (in figure 11), and Q_m^5 (in figure 12).



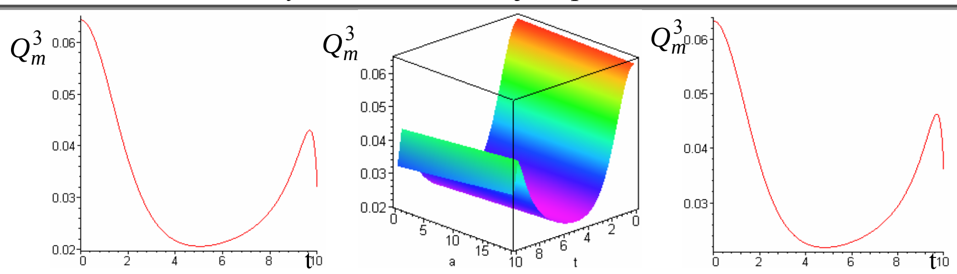
(a) (b) (c)

Fig. 8. The plot for Q_m^1 (t,a), when: (a) a=0; (b) a$\in$[0,20]; (c) a=20



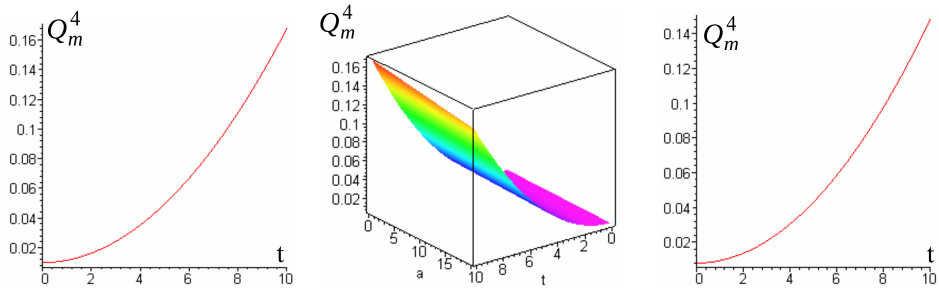
(a) (b) (c)

Fig. 9. The plot for Q_m^2 (t,a), when: (a) a=0; (b) a$\in$[0,20]; (c) a=20



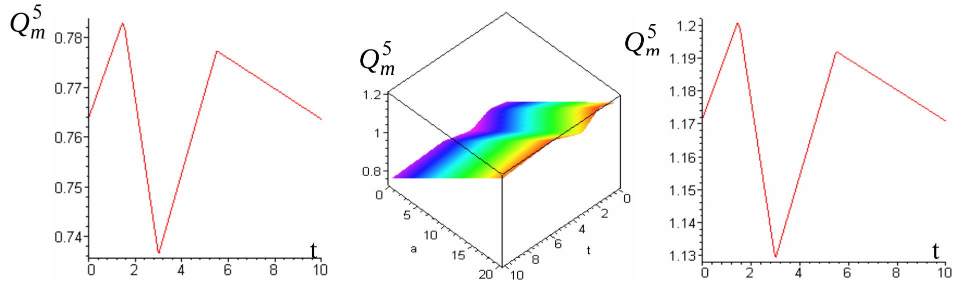
(a) (b) (c)

Fig. 10. The plot for Q_m^3 (t,a), when: (a) a=0; (b) a∈[0,20]; (c) a=20



(a) (b) (c)

Fig. 11. The plot for Q_m^4 (t,a), when: (a) a=0; (b) a∈[0,20]; (c) a=20



(a) (b) (c)

Fig. 12. The plot for Q_m^5 (t,a), when: (a) a=0; (b) a∈[0,20]; (c) a=20

4. CONCLUSIONS

In order that the gripper follows the desired trajectory (having a given manipulation task), in the joints are developing motor forces or torques specific to the joint type (translation or rotation), whose computing relations are complex. From these relations we see, for example, the influence of the variation of the component modules masses on the values of these generalizes motor forces. In the paper we represented the influence of the mass M_5 on the values of $Q_m^i, i = \overline{1,5}$.

5. REFERENCES

[1] Fu, K.S., Gonzalez, R.C., Lee, C.S.G. (1987). *Robotics – Control, Sensing, Vision, and Intelligence*, McGraw-Hill Intl.
[2] Gupta, K.C. (1997). *Mechanics and control of robots*, Springer-Verlag, New York.
[3] Heck, A. (2003). *Introduction to Maple*, Springer-Verlag, New York.
[4] Negrean, I., Vuşcan, I., Haiduc, N. (1998). *Robotics – Kinematic and dynamic modeling*, Ed. Didactică şi Pedagogică, Bucureşti.
[5] Paul, R.P. (1981). *Robot Manipulators: Mathematics, Programming and Control*, MIT Press, Cambridge, Massachussets.
[6] Spong, M.W., Vidyasagar, M. (1989). *Robot Dynamics and Control*, John Wiley & Sons.