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Model Based Robot Control with Friction and Payload Estimation

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Abstract

The present study deals with control of robotic systems with unknown friction parameters and payload mass. A tracking control algorithm which uses the model of the robotic arm combined with adaptive parameter estimation techniques were developed to solve the proposed control problem. Using Lyapunov method it was shown that the resulting controller achieves guaranteed final tracking accuracy. Simulation results are presented to illustrate the effectiveness and achievable control performance of the proposed scheme.

Keywords - robot control, friction, parameter estimation, Lyapunov stability

1 INTRODUCTION

It is well-known that nowadays the robotic applications require increased transient performances and good path tracking proprieties. To achieve these requirements the introduction of the mathematical model in the control algorithm is necessary. There are analytical methods to determine the exact mathematical model of a robot, for example using the Euler-Lagrange method [1]. The parameters of the model (masses, inertias, length of the arms) are often catalog data which are given by the manufacturer of the robotic system [2].

The friction phenomena which should be considered in every mechanical system can be described with models whose parameters are time varying, depending on external factors. These parameters cannot be determined a-priori. At the other hand there is a tendency in robotic industry to build robots with lightweight arms to avoid unnecessary energy consumption. For this reason in the robot model the mass of the payload cannot be neglected related to the masses of the arms. The mass of the payload in many application is unknown, varies according to the specific task of the robot. These consideration suggests that the friction parameters and the mass of the payload should be estimated on-line. If only the friction forces and the payload are unknown in the model it is unnecessary to use adaptive control algorithms which estimate all the parameters of the robot.

In the past years the adaptive control methods [3] became a wide spread method to control not only the linear but also the nonlinear systems with unknown parameters. Adaptive control of robotic manipulators has also been intensively studied in the control community. Early results can be found in the work of Slotine and Li [4]. The introduction of a neural network in the adaptive control law to handle uncertainties due to payload uncertainty was proposed by Leahy et al. [5]. A decentralized adaptive control algorithm for trajectory tracking was proposed by Fu [6]. The controller can be implemented in a decentralized way i. e. a sub-controller is independently and locally equipped for each joint servo loop. Lewis et al. [7] combine the neural control technique with linear optimal control theory to control rigid manipulators with completely unknown models. Barambones and Etxebarria [8] proposes a controller for generic manipulator with unknown parameters and sliding-mode control which robustifies the design and compensates the neural approximation errors.

The rest of the paper is organized as follows: Section 2 presents the proposed control algorithm. The stability of the closed loop system and the proprieties of the tracking error were analyzed using Lyapunov techniques. In Section 3 simulations were performed using a 3 Degree Of Freedom (DOF) robot to show the applicability of the control law.

2 THE CONTROL ALGORITHM

2.1 *Robot Modelling*

The mathematical model of an open chain, rigid, n degree of freedom robot is given by:

$$H(\underline{q})\underline{\ddot{q}} + C(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D(\underline{q}) = \underline{\tau} - \underline{h}_F(\underline{\dot{q}}) \qquad (1)$$

where $\underline{q}$ is the joint position vector of the robot and the vector $\underline{\tau}$ represents the control signal given by current controlled DC servos. The following notations is used: $H(\underline{q})$ is the generalized inertia matrix, $C(\underline{q}, \underline{\dot{q}})$ shows the effect of the centripetal and Coriolis forces, $D(\underline{q})$ is the gravity effect, $\underline{h}_F(\underline{\dot{q}})$ represents the effect of the friction force which acts on the joints of the robot.

The following properties of the robot model are well known:

1. The matrix $H(\underline{q})$ is positive definite

2. The matrix $\dot{H}(\underline{q}) - 2C(\underline{q}, \underline{\dot{q}})$ is skew-symmetric, namely for any $\underline{x} \in R^n$ we have $\underline{x}^T(\dot{H}(\underline{q}) - 2C(\underline{q}, \underline{\dot{q}}))\underline{x} = 0$

To develop a control law with friction and payload estimation firstly let us consider the following assumptions:

Assumption I (Payload)

Let us assume that the dimensions of the payload can be neglected compared with the dimensions of the robotic arm so its length and inertia is neglected. In this case the terms of the robotic model can be written in the following form:

$$H(\underline{q})) = H_R(\underline{q})) + mH_L(\underline{q})) \qquad (2)$$

$$C(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D(\underline{q}) = C_R(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D_R(\underline{q}) + m(C_L(\underline{q}, \underline{\dot{q}})\underline{\dot{q}} + D_L(\underline{q})) \qquad (3)$$

where m is the mass of the payload. H_R, C_R and D_R are terms of the robotic arm model without payload. H_L, C_L and D_L are known terms which can be determined from the robot model. The mass of the payload is bounded.

Assumption II (Friction Modelling)

The friction force acts separately on each joint of the robot. It can be written in a linearly parameterized form:

$$h_{Fi}(\dot{q}_i) = \mu(\dot{q}_i)\underline{\theta}^T_{PFi}\underline{\xi}_{PFi}(\dot{q}_i) + (1-\mu(\dot{q}_i))\underline{\theta}^T_{NFi}\underline{\xi}_{NFi}(\dot{q}_i) \qquad (4)$$

where $\mu(\dot{q}_i) = 1$ if $\dot{q}_i \geq 0$ and 0 otherwise. The vectors $\underline{\theta}_{Fi}$ contains the unknown parameters of the friction model and $\underline{\xi}_{Fi}(\dot{q}_i)$ are known regressor vectors. This relation means that the friction can be described by two separate linearly parameterized models, one for the positive velocity regime (P) and the other for negative velocity regime (N). The switching between these two models occurs at the zero velocity. If we incorporate the switching function μ in the regressor vector we have:

$$h_{Fi}(\dot{q}_i) = \underline{\theta}^T_{Fi}\underline{\xi}_{Fi}(\dot{q}_i) \qquad (5)$$

where $\underline{\theta}_{Fi} = (\underline{\theta}_{FNi} \;\; \underline{\theta}_{FPi})^T$ and $\underline{\xi}_{Fi}(q_i) = (\mu(\dot{q}_i)\underline{\xi}_{PFi}(\dot{q}_i) \;\; (1-\mu(\dot{q}_i))\underline{\xi}_{NFi}(\dot{q}_i))^T$

2.2 *The control problem*

The problem is to design a control input $\underline{\tau} = (\tau_1 \; \tau_2 \ldots \tau_n)^T$ such that the joint position $\underline{q} = (q_1 \; q_2 \ldots q_n)^T$ track the desired trajectory $\underline{q}_d = (q_{1d} \; q_{2d} \ldots q_{nd})^T$. The desired trajectories q_{id} are known bounded functions of time with bounded known derivatives.

We define the error metric S_i that describes the desired dynamics of the error system for the i'th joint:

$$S_i(t) = (\frac{d}{dt} + \lambda_i)e_i, \quad e_i = q_i - q_{di} \qquad (6)$$

where λ_i is positive constant.

Using this metric the control problem can be reformulated as follows: *design a control law $\underline{\tau}$ such that the tracking error metrics $S_i(t)$ satisfy $|S_i(t)| \to 0$ for $t \to \infty$.*

The error dynamics for the entire robot can be written in the form:

$$\underline{S} = \underline{\dot{e}} + \Lambda\underline{e} \qquad (7)$$

with $\underline{S} = (S_1(t) \; S_2(t) \ldots S_n(t))^T$, $\underline{e} = (e_1 \; e_2 \ldots e_n)^T$, and $\Lambda = diag(\lambda_i)$ is a diagonal matrix with positive elements.

Using the relation (1) and **Assumption I** we get:

$$\begin{aligned} H(\underline{q})\underline{\dot{S}} = &- C_R(\underline{q},\underline{\dot{q}})\underline{\dot{q}} - D_R(\underline{q}) - H_R(\underline{q})(-\underline{\ddot{q}}_d + \Lambda\underline{\dot{e}}) \\ &- m(C_L(\underline{q},\underline{\dot{q}})\dot{q} + D_L(\underline{q}) + H_L(-\underline{\ddot{q}}_d + \Lambda\underline{\dot{e}})) - \underline{h}_F(\underline{\dot{q}}) + \underline{\tau} \end{aligned} \qquad (8)$$

2.3 *Control Law*

The control law is formulated as follows:

$$\underline{\tau} = H_R(\underline{\ddot{q}}_d - \Lambda\underline{\dot{e}}) + C_R(\underline{\dot{q}}_d - \Lambda\underline{e}) + D_R + \widehat{m}\underline{\xi}_m + \widehat{\underline{h}}_F(\underline{\dot{q}}) - K_S\underline{S} \qquad (9)$$

where $\underline{\xi}_F$ is defined in **Assumption II**, $\underline{\xi}_m = H_L(-\underline{\ddot{q}}_d + \Lambda\underline{\dot{e}}) + C_L(-\underline{\dot{q}}_d + \Lambda\underline{e}) + D_L$. $\widehat{m}$ denotes the estimated value of the payload mass and $\widehat{\underline{h}}_F$ denotes the estimated friction force.

The matrix K_S is defined as a diagonal matrix $K_S = diag(K_{Si})$ where $K_{Si} > 0$.

The adaptation laws for the friction parameters ($\underline{\widehat{\theta}}_{Fi}$) in the i'th joint and for the mass of the payload ($\widehat{m}$) are:

$$\dot{\widehat{\theta}}_{ij} = \Gamma_{ij} S_i \xi_{Fij} \qquad \dot{\widehat{m}} = \gamma_m \underline{S}^T \underline{\xi}_m \tag{10}$$

where $\Gamma_i = diag(\gamma_{ij})$ with $\gamma_{ij} > 0$ and $\gamma_m > 0$.

2.4 *Lyapunov analysis*

In this Subsection will be shown that the introduced control law solves the control problem presented in Subsection 2.2. To examine the proposed control law let us consider the following Lyapunov-like function:

$$V(t) = \frac{1}{2}\underline{S}^T H(\underline{q})\underline{S} + \frac{\gamma_m^{-1}}{2}\widetilde{m}^2 + \frac{1}{2}\sum_{i=1}^{n} \underline{\widetilde{\theta}}_{Fi}^T \Gamma_i^{-1} \underline{\widetilde{\theta}}_{Fi} \tag{11}$$

The time derivate of $V(t)$ is given by:

$$\dot{V}(t) = \underline{S}^T H(\underline{q})\dot{\underline{S}} + \frac{1}{2}\underline{S}^T \dot{H}(\underline{q})\underline{S} + \gamma_m^{-1}\widetilde{m}\dot{\widehat{m}} + \sum_{i=1}^{n} \underline{\widetilde{\theta}}_{Fi}^T \Gamma_i^{-1} \dot{\underline{\widehat{\theta}}}_{Fi} \tag{12}$$

where $\widetilde{m} = m - \widehat{m}$ and $\underline{\widetilde{\theta}}_{Fi} = \underline{\theta}_{Fi} - \underline{\widehat{\theta}}_{Fi}$ are the estimation errors for the payload mass and friction parameters, respectively.

By substituting the expression of the control law (9) in the equation of error dynamics (8) and adding $(C_R + mC_L)(-\underline{\dot{q}}_d + \Lambda\underline{e}) - (C_R + mC_L)(-\underline{\dot{q}}_d + \Lambda\underline{e})$ on the right side of the obtained relation we have:

$$H(\underline{q})\dot{\underline{S}} = -C\underline{S} - \widetilde{m}(H_L(-\underline{\ddot{q}}_d + \Lambda\underline{\dot{e}}) + C_L(-\underline{\dot{q}}_d + \Lambda\underline{e}) + D_L) - \widetilde{h}_F(\underline{\dot{q}}) - K_S\underline{S} \tag{13}$$

In the view of (13) and robot arm proriety 2 and introducing the expression of adaptation laws (10) the derivate of Lyapunov function (12) becomes:

$$\dot{V}(t) = -\underline{S}^T K_S \underline{S} \le 0 \tag{14}$$

In the equilibrium we have $S(t) = 0$. According to Lyapunov stability theory the error metric $S(t)$ converge to equilibrium state 0 and the control which uses the control law (9) and the adaptation law (10) is globally and asymptotically stable.

3 Simulation Results

We demonstrate the performance of the proposed control algorithm for a 3 DOF cylindrical manipulator. The first joint represents the rotation around the robot base. The second joint represents the vertical, and the third joint represents the horizontal translation of the payload (see Figure 1). The dynamics of this robotic system is nonlinear with strong coupling between the first and third joints.

The equation of motion in terms of generalized coordinates $\underline{q} = (q_1\ q_2\ q_3)^T$ representing the applied torques $\underline{\tau} = (\tau_1\ \tau_2\ \tau_3)^T$ at these joints is given by:

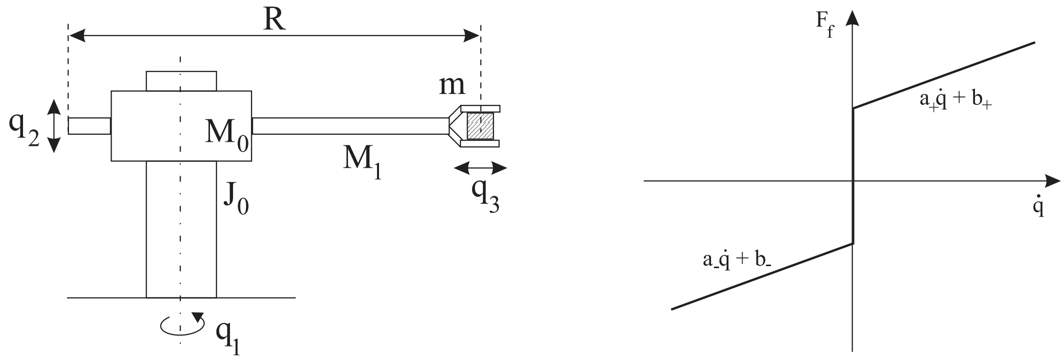

Figure 1: The cylindrical robot and the used friction model

$$\begin{pmatrix} J_0 + M_1\frac{R^2}{3} + M_1(q_3^2 - Rq_3) + mq_3^2 & 0 & 0 \\ 0 & M_0 + M_1 + m & 0 \\ 0 & 0 & M_1 + m \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} + \begin{pmatrix} (2M_1q_3 - M_1R + 2mq_3)\dot{q}_3\dot{q}_1 \\ (M_0 + M_1 + m)g \\ -\frac{1}{2}(2M_1q_3 - M_1R + 2mq_3)\dot{q}_1^2 \end{pmatrix} = \begin{pmatrix} \tau_1 - h_{F1}(\dot{q}_1) \\ \tau_2 - h_{F2}(\dot{q}_2) \\ \tau_3 - h_{F3}(\dot{q}_3) \end{pmatrix} \quad (15)$$

J_0 and M_0 represents the inertia and the mass of the base of the robot, M_0 is the mass of the horizontal link, m is the mass of the payload, R is the length of the horizontal link and g is the gravitational acceleration.

In the friction model the viscous friction term which is proportional to velocity and the constant Coulombic friction term which depends only on the sign of velocity were considered. For each joint the friction term can be written in the following form:

$$h_{Fi}(\dot{q}_i) = \mu(\dot{q}_i)(a_{iP}\dot{q}_i + b_{iP}) + (1 - \mu(\dot{q}_i))(a_{iN}\dot{q}_i + b_{iN}) \quad i = 1 \ldots 3 \quad (16)$$

The switching function μ is defined in **Assumption II**. Note that the parameters (a_{iP} and a_{iN} respectively b_{iP} and b_{iN}) may differ in the positive and negative velocity regimes.

The expressions of the model matrices and vectors needed in the control law ($H_R,\ H_L,\ C_R,\ ,C_L,\ D_R,\ ,D_L$) can easily be obtained from (15).

The numerical values choosen for simulation proposes were choosen as $J_0 = 0.01kgm^2$, $M_0 = 2kg$, $M_1 = 1kg$, $R = 1m$, $g = 9.81m/s^2$, $m = 0.5kg$ $a_{iN} = -a_{iP} = 2N$, $b_{iN} = b_{iP} = 0.2Ns/m$, $i = 1 \ldots 3$. The external disturbances d_i was modelled as an additive random signal with maximum amplitude $d_i \leq d_{iM} = 0.1N$, $i = 1 \ldots 3$. The mass of the payload and the friction parameters were considered totally unknown. For the adaptation laws the initial values for all these parameters were taken 0.

The reference trajectory was chosen in such way that all the joints to have acceleration, deceleration and constant speed regimes. The third joint is tested both in positive and negative velocity regimes.

Simulations were performed with and without adaptation. As it can bee seen the control guarantees precise tracking and tracking error metrics convergence. The performance of the adaptation laws can be seen in the Figure 5. Due to the switching function μ the friction parameters are tuned separately in positive and negative velocities. It can be noticed in the case of the third joint which is used both for negative and positive velocities.

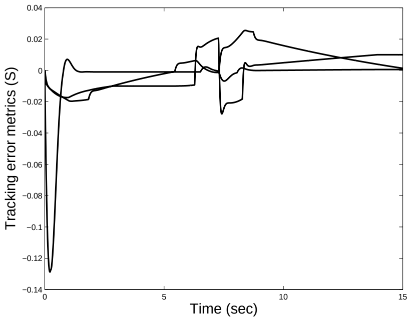

Figure 2: Tracking error metrics

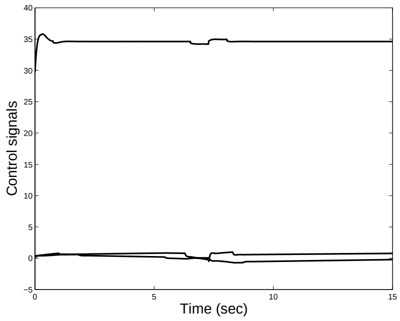

Figure 3: Command signal

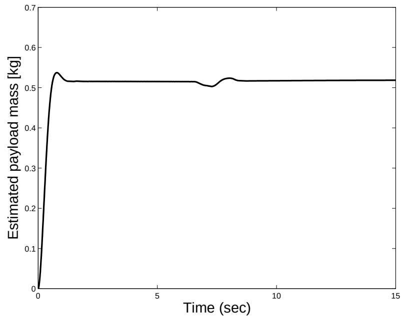

Figure 4: Estimated payload mass

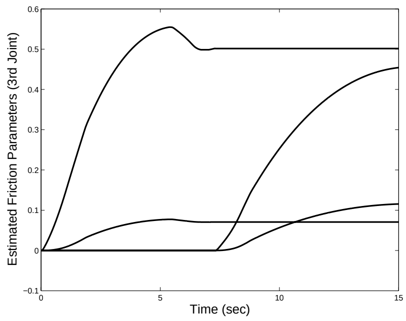

Figure 5: Estimated friction parameters (3'rd joint)

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