THE SIMULATION OF TIME-OPTIMAL CONTROL FOR ROBOTIC ARMS

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Abstract: This paper presents the simulation results for time-optimal control of robotic arms along specified paths. The implementation of time-optimal control consists of several problems: (1) the control is generally discontinuous (Bang-Bang), (2) the actuator dynamics are usually ignored to reduce the order of the system, and (3) the optimal control leaves no control authority to compensate for tracking errors caused by unmodelled dynamic and the delays introduced by the on-line feedback controller. To overcome these difficulties the motor dynamics are compensated for using a simplified friction model, and the dynamics of the feedback controller are accounted for using trajectory preshaping. The simulation results show the advantages of time-optimal control for reducing motion duration as well as for increasing tracking accuracy.

Key word: time-optimal control , trajectory Bang-Bang , robotic arm , feedback controller

1. INTRODUCTION

The time-optimal control method is studied on a robotic arm with two rotational joints as shown in figure 1. The goal is to obtain a minimum time interval in which point M travels from the initial position P_i to the final position P_f.

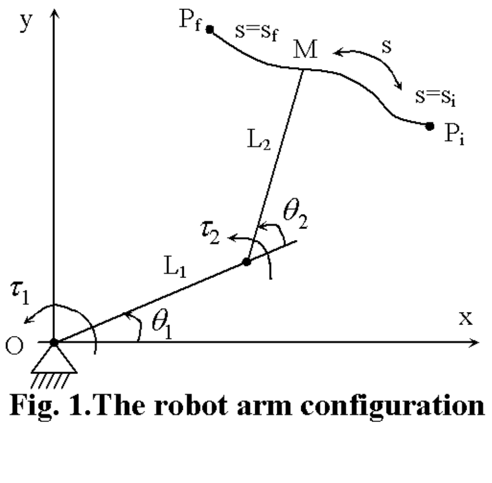


Fig. 1.The robot arm configuration

The parameters marked above represent:

L_i-the lengths of the elements 1 and 2, which are the parts of the kinematic chain of the robotic arm;
- C_i – the center of gravity of element i;
- L_{ci}- the length of rotational joint i from i to ci;
- m_i – the mass of element i;
- I_i – the moment of inertia of element i.
- x,y– rectangular reference axis

The dynamics of the final effector will be neglected. It is supposed that the robotic arm operates horizontally without the influence of gravitational pull. The dynamic movement equations of the robotic arm are derived using Lagrange's formula and have the following form:

$$M(\theta)\cdot\ddot{\theta}+V(\theta,\dot{\theta})=\tau \qquad (1)$$

in which the parameters of the rotational joints are the angles θ_1 and θ_2 and the speeds of τ_1 and τ_2 as shown in figure 1. In equation (1) following variables are used:

$$M(\theta)=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix};\ V(\theta)=\begin{bmatrix}V_1\\ V_2\end{bmatrix}$$

where: $M_{11}=m_1\cdot L_{c1}^2+I_1+m_2\cdot\left[L_1^2+L_{c2}^2+2\cdot L_1\cdot L_{c2}\cdot\cos\theta_2\right]+I_2$

$$M_{12}=M_{21}=m_2\cdot L_1\cdot L_{c2}\cdot\cos\theta_2+m_2\cdot L_{c2}^2+I_2$$

$$M_{22}=m_2\cdot L_{c2}^2+I_2$$

$$V_1=-h\cdot\dot{\theta}_2^2-2\cdot h\cdot\dot{\theta}_1\cdot\dot{\theta}_2$$

$$V_2=h\cdot\dot{\theta}_1^2$$

$$h=m_2\cdot L_1\cdot L_{c2}\cdot\sin\theta_2$$

The parameters of the rotational joints $\theta=[\theta_1 \quad \theta_2]^T$ are given in relation with a single control variable, the length of the arch S. The speed $\dot{s}$ and the acceleration $\ddot{s}$, when the point M moves along the arch s have the following expressions:

$$\theta=\theta(s);\ \dot{\theta}=\dot{\theta}(s,\dot{s});\ \ddot{\theta}=\ddot{\theta}(s,\dot{s},\ddot{s}) \qquad (2)$$

The position of the $\overline{OM}$ vector is defined as follows:

$$r=r(\theta)=\tilde{r}(s) \qquad (3)$$

Deriving expression (3) with respect to time one can obtain:

$$[r_\theta]\cdot\dot{\theta}=\tilde{r}_s(s)\cdot\dot{s} \qquad (4)$$

where: $[r_\theta]$ – is the Jacobi matrix of the position vector $\overline{OM}$;

$\tilde{r}_s(s)$ – is the unitary vector, tangent to the trajectory.

If the robotic arm doesn't operate in single configuration mode, the Jacobi matrix is reversible.

$$\theta(s,\dot{s})=[r_\theta]^{-1}\cdot\tilde{r}_s(s)\cdot\dot{s} \qquad (5)$$

Deriving (4) with respect to time we obtain:

$$[r_\theta]\cdot\ddot{\theta}+[\dot{r}_\theta]\cdot\dot{\theta}=\tilde{r}_s(s)\cdot\ddot{s}+\tilde{r}_{ss}(s)\cdot\dot{s}^2 \qquad (6)$$

where: $\tilde{r}_s(s)\cdot\ddot{s}$ - is the tangent acceleration;

$\tilde{r}_{ss}(s)\cdot\dot{s}^2$ - is the normal acceleration of point M along the trajectory;

Refining expression (6) the following expression is obtained:

$$\ddot{\theta}(s,\dot{s},\ddot{s})=[r_\theta]^{-1}\cdot\left[\tilde{r}_s(s)\cdot\ddot{s}+\tilde{r}_{ss}(s)\cdot\dot{s}^2+[\dot{r}_\theta]\cdot[r_\theta]^{-1}\cdot\tilde{r}_s(s)\cdot\dot{s}\right] \qquad (7)$$

The restrictions for speed are:

$$\tau_{i\max}\le\tau_i\le\tau_{i\min} \qquad (8)$$

where: τ_i - is the speed of the rotational joint i;

The expression of the maximal acceleration and maximal deceleration are obtained by replacing (7) and (8) into (1):

$$C_1(s)\cdot\ddot{s}+C_2(s,\dot{s})=\tau \qquad (9)$$

where: $C_1(s) = M(\theta)\cdot[r_\theta]^{-1}\cdot\tilde{r}_s$ (10)

$$C_2(s,\dot{s}) = M(\theta)\cdot[r_\theta]^{-1}\cdot\left[\tilde{r}_{ss}\cdot\dot{s}^2 + [\dot{r}_\theta]\cdot[r_\theta]^{-1}\cdot\tilde{r}_s\cdot\dot{s}\right] + V(\theta,\dot{\theta}) \quad (11)$$

For each element one can write the following:

$$\tau_i = C_{1i}(s)\cdot\ddot{s} + C_{2i}(s,\dot{s})\ ,\ i = 1,2 \quad (12)$$

The acceleration restrictions are:

$$\tau_{i\min} - C_{2i}(s,\dot{s}) \le C_{1i}(s)\cdot\ddot{s} \le \tau_{i\max} - C_{2i}(s,\dot{s});\ i = 1,2 \quad (13)$$

For $c_{1i}(s) \ne 0$ equation (13) becomes:

$$f_i(s,\dot{s}) \le \ddot{s} \le g_i(s,\dot{s}) \quad (14)$$

where: $$f_i(s,\dot{s}) = \begin{cases} \dfrac{\tau_{i\min} - C_{2i}}{C_{1i}}; C_{1i} \ge 0 \\ \dfrac{\tau_{i\max} - C_{2i}}{C_{1i}}; C_{1i} \le 0 \end{cases} \quad (15)$$

$$g_i(s,\dot{s}) = \begin{cases} \dfrac{\tau_{i\max} - C_{2i}}{C_{1i}}; C_{1i} \ge 0 \\ \dfrac{\tau_{i\min} - C_{2i}}{C_{1i}}; C_{1i} \le 0 \end{cases} \quad (16)$$

The condition (14) is necessary for keeping point M on the imposed trajectory without effecting equation (1) and restriction (8). Next, it is supposed that such acceleration $\ddot{s}$ exists that:

$$f(s,\dot{s}) \le \ddot{s} \le g(s,\dot{s}) \quad (17)$$

where: $f(s,\dot{s}) = \underbrace{\max}_{i} f_i(s,\dot{s})$ (18)

and:

$$g(s,\dot{s}) = \underbrace{\min}_{i} g_i(s,\dot{s}) \quad (19)$$

Under these conditions the solving the time-optimal control is to maintain the following condition:

$$f(s,\dot{s}) \le g(s,\dot{s}) \quad (20)$$

2. THE SOLUTION FOR THE PROBLEM

The time-optimal control requests the minimization of:

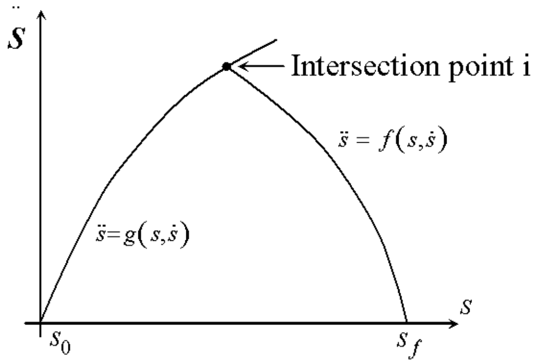
$$t_f = \int_{s_0}^{s_f} \frac{ds}{\dot{s}} \quad (21)$$

so that the following boundary restrictions are achieved:

$$s(0) = s_0;\ s(t_f) = s_f \quad (22)$$
$$\dot{s}(0) = \dot{s}(t_f) = 0$$

and keeping the restriction (20) valid.

In order to minimize (21) we have to choose the acceleration $\ddot{s}$ and increase the speed $\dot{s}$ without changing expression (20). One can choose either $\ddot{s} = g(s,\dot{s})$ for maximal acceleration, or $\ddot{s} = f(s,\dot{s})$ for maximal deceleration so that the time for positioning the object to be minimum. In order to get the coordinates of the intersection point of $\ddot{s} = f(s,\dot{s})$ and $\ddot{s} = g(s,\dot{s})$ curves, function "Ode45" from Matlab is applied to resolve the differential equations using Runge-Kutta method of 4, respectively 5 orders. In figure 2 .a,b a simple case of application the time-optimal control is show.



s - space

$\ddot{s}$ - acceleration /deceleration

$\dot{s}$ -speed

Fig 2.a Acceleration and deceleration diagram

To obtain the exact moment when the intersection between the two curves occurs one has to:

- integrate $\ddot{s} = g(s,\dot{s})$ by a normal time scale between $s = s_0$ and $\dot{s} = 0$;
- then integrate $\ddot{s} = f(s,\dot{s})$ backwards in time between $s = s_f$ and $\dot{s} = 0$.

For a general case more points of intersection have to be taken into consideration in order for the speed not to exceed a certain level and thus altering the validity of expression (20). An example of how the time is obtained when the intersections take place is shown in figure 3.a, b

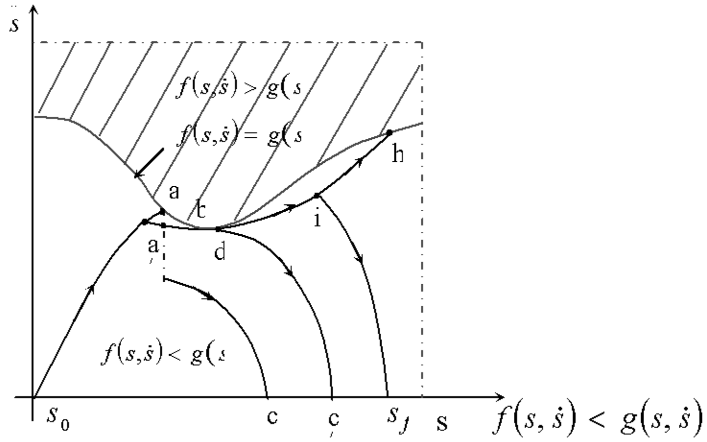


Fig.3.a Acceleration and deceleration diagram , general case ,more points of intersection

The steps that one has to follow in order to get the profile shown above are:

a) integrate $\ddot{s} = g(s,\dot{s})$ following the time scale until there is an intersection between the two functions $g(s,\dot{s}) = f(s,\dot{s})$ in point a;
b) reduce the value of $\dot{s}$ iteratively and integrate $\ddot{s}$ with respect to time until the curve is tangentially intersected $f(s,\dot{s}) = g(s,\dot{s})$ in point d, which is the second point of changing the direction. The low limit of integration in this case is a'.
integrate $\ddot{s} = f(s,\dot{s})$ backwards in time from point a' in order to find
the first point e;

c) integrate $\ddot{s} = g(s, \dot{s})$ by the normal time scale from d point until it reaches the intersection $f(s, \dot{s}) = g(s, \dot{s})$ in point h;

d) integrate $\ddot{s} = f(s, \dot{s})$ backwards in time from s_f. Then one has to return to the operation from step d. If an intersection occurs, then point i is the final changing point, otherwise the operations continue starting from step b until the final changing point is found.

The time-optimal control was applicated on a robotic arm with two rotational joints. The simulation results for time-optimal control is rerezented in figures 2b and 3b.

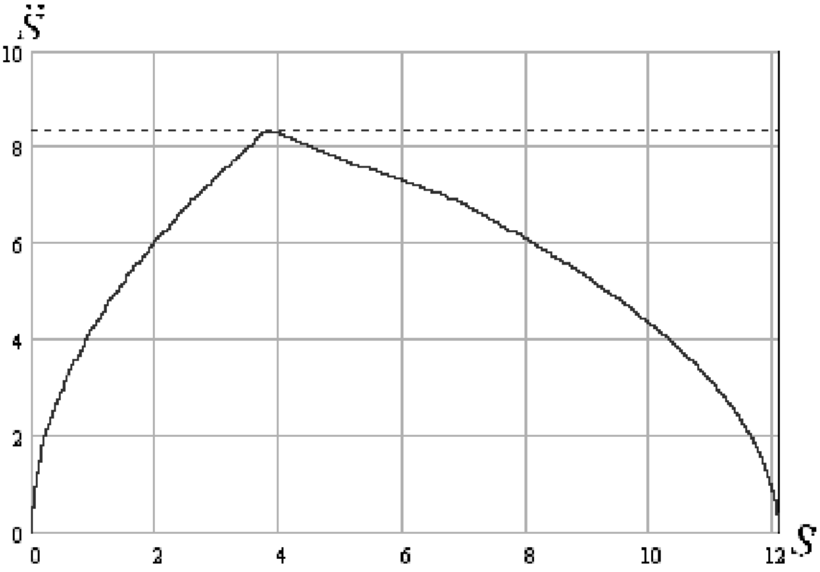


Fig 2.b Simulation results for time-optimal control of robotic arm with two rotational joints

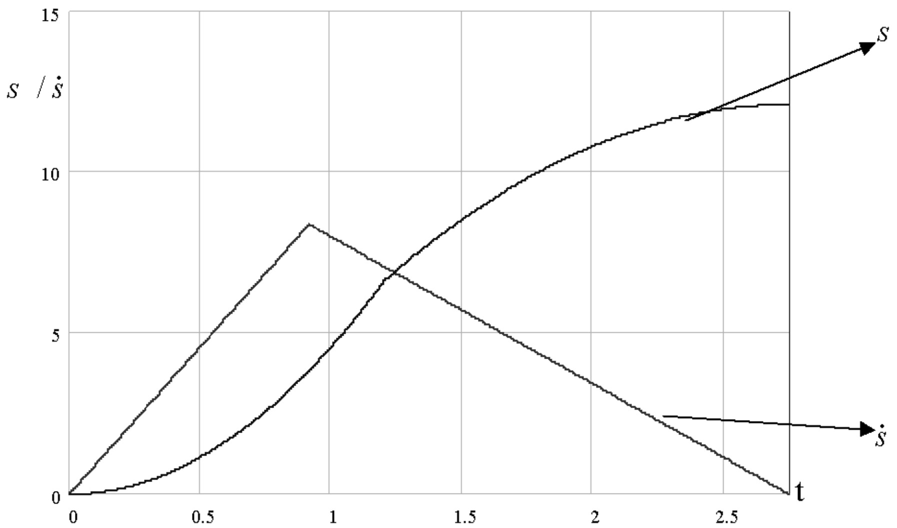


Fig 3.b Simulation results for time-optimal control , type bang-bang ,of robotic arms

3. CONCLUSIONS

The time-optimal control method is studied on a robotic arm with two rotational joints and requests the minimization of:

$$t_f = \int_{s_0}^{s_f} \frac{ds}{\dot{s}}$$

so that the following boundary restrictions are achieved:

$$s(0) = s_0;\ s(t_f) = s_f\ ,\ \dot{s}(0) = \dot{s}(t_f) = 0.$$

Function “Ode45” from Matlab is applied to resolve the differential equations using Runge-Kutta method of 4, respectively 5 orders.

For a general case more points of intersection have to be taken into consideration in order for the speed not to exceed a certain level

The simulation results show the advantages of time-optimal control for reducing motion duration as well as for increasing tracking accuracy.

4. REFERENCES

[1] Canudas de Wit, C., Fixot, N. (2002), Trajectory tracking in robot manipulators via nonlinear estimated state feed-back, *IEEE Transactions on Robotics and Automation*, 8(1), 138-140

[2] Egri Angela, (2002) , Flexibile manufacturing and robots**,** E. D. P.Bucharest , ISBN 973-30-2593-3, 83-90

[3] Ivanescu M., (1994), Industrial robots. . Algorithms and control systems, Editura Universitaria Craiova, 92-94

[4] Negrean I., Negrean D.C.,(2002) , The acceleration energy to robot dynamics, *International Conference, A&QT-R 2002 (THETA 13),* Cluj-Napoca, Romania, ISBN 973-9357-11-1, 59-61

[5] Manolea Ghe.,Botenu N., Ravigan F .(2002), Software Packages for Industrial Robots simulation *International Symposium ‘UNIVERSITARIA ROPET’* ,Petrosani, Romania, ISBN 973-8435-23-4 ,211-213

[6] Pop E., Egri A., Leba M., (2001), Control systems of industrial robots, E.D.P.,Bucharest , ISBN 973-30-2735-9, 223-227

[7] Trasca M., Lazea GHE., (2002),Adaptive controller for robot arms *International Conference, A&QT-R 2002 (THETA 13*), Cluj-Napoca, Romania, ISBN 973-9357-11-1, 90