AQTR 2004 (THETA 14)
2004 IEEE-TTTC - International Conference on Automation, Quality and Testing, Robotics
May 13 – 15, 2004, Cluj-Napoca, Romania

STUDYING AND MODELLING NON-LINEAR CIRCUIT ELEMENTS

Agoston Katalin

"Petru Maior" University, Tg.Mures, Str. Nicolae Iorga Nr. 1, kagoston@upm.ro

Abstract: Circuit elements with non-linear characteristic modify the voltage or current curve, causing faults and supplementary loses in the circuit. Knowing the level and the grade of current and voltage harmonics can help at their decreasing. In this work simple models are searched for some circuit elements with non-linear characteristic, like the coil or the capacitor. These models are searched to make the study of distorting state easier which is caused by these non-linear elements. Models are controlled and simulated in SIMULINK. Using these models voltage picks and current shocks can be defined as well harmonics, and circuits can be planed to filter or to eliminate them.

Key words: iron cored coils, system model, distorting state.

1. INTRODUCTION

Non-linear characteristics of some elements in electrical circuits result to the appearance of son-sinusoidal currents, even if the wave form of the system's sources have perfect sinusoidal form. Passing of non-sinusoidal current through linear elements produces voltage loses and this ends to the fact that in different points of the system voltages will be distorted [3],[4],[5].

Ferromagnetic materials which magnetically circuits of iron cored coils or power transformers of electrical circuits are made of have their permeability dependent of the magnetically field's intensity, so they are dependent of the intensity of the current which passes through these elements. The characteristics of these materials are non-linear so magnetically circuits and elements of circuits which have such materials in their structure have non-linear characteristics, too. The analyze of distorting state caused by iron cored coils or by transformers can be made graphical or analytical, according to the way in witch permeability variation is given, depending of the current intensity in the winding. The u=u(i) relation for a coil doesn't show a directly proportionality of quantities because of the flux-current characteristic $\varphi=\varphi(i)$ which has the same form like the magnetisation characteristic B=B(H).

In practice application is essential the case when the supplying voltage at the ends of the coil is sinusoidal and of industrial frequency $u(t)=U\sin\omega t$. The flux which determinates the current is: $\varphi_{fu}=(U/N\omega)\sin(\omega t-\pi/2)$. Knowing the flux-current characteristic, using a simple graphical construction we can find the form of the current form in the circuit. Even if the magnetization cycle is thin and it can be reduced to a first magnetization curve, the current is distorted. The analyze of the current curve evidences an important count of the 3rd harmonic shifted with a $\pi/3$ angle compared to the fundamental. If the histeresis phenomena cannot be neglected the current curve will be even more distorted. To define the energy dissipation witch is caused by distortion state as a supplementary consume, as well to obtain the spectrum of the current it's analytical form or model is searched.

2. ANALYTIC MODEL OF DISTORTION STATE CAUSED BY COIL

Analytic analyse of distortion state needs an expression which beastly approximates the real flux-current characteristic.

For materials with thin histeresis cycle, which is reducable to the first magnetization curve, flux-current relation can be approximated in many ways[7]:

$$i = a_0 + a_1\varphi + a_2\varphi^2 + a_3\varphi^3 + ...$$
$$\varphi = \alpha \cdot arctg(\beta \cdot i) + \gamma \cdot i$$
$$i = a \cdot shb\varphi \quad (1)$$

where a_j $j=0\div3$ as well α, β, γ and a, b are constants. To describe the curve, just the proper coefficients need to be found, but this is quite a difficult work. Approximation the magnet sating curve with straight segments permits to obtain simple analytic expressions which are correct enough. If the magnetizating curve is approximated with three segments, current depending of the flux can be expressed[1]:

$$\varphi_{fu} = m_1 \cdot i + \mu(m_2 - m_1)(i - \upsilon i_1) \quad (2)$$

where μ and υ are two operators with the next values, $\mu=0$ for $-i_1 \le i \le i_1$ $\mu=1$ for $i>i_1$ and $i<-i_1$ $\upsilon=1$ for $\varphi_{fu}>0$ $\upsilon=-1$ for $\varphi_{fu}<0$. In the relation i_1 is the current value at the separation point between linear and saturation domain, with coefficient angle m_1 of the straight which goes through the origo, and m_2 is the coefficient angle of the straight which corresponds to the saturation domain. μ and υ operators are square functions with different values according to different parts of the current curve.

If the histeresis fenomen cannot be neglected it can be linearised using analytical expressions, and this results to rather good approximation. In the condition when the cycle is symmetrical compared to the origo of the axes and the approximation is made with four straights, the analytical expression of the current can be written:

$$\varphi_{fu} = (1-\mu)m_1(1 - \upsilon \cdot i_1) + \mu[m_2(i - \upsilon \cdot i_2) + \upsilon\Phi_2] \quad (3)$$

where μ and υ are square functions with values 0 and 1 as well 1 and –1. The equations of the straights are:

d_1	$\mu=0$	$\upsilon=1$	$\varphi_{fu}=m_1(i-i_1)$
d_3	$\mu=0$	$\upsilon=-1$	$\varphi_{fu}=m_1(i+i_1)$
d_2	$\mu=1$	$\upsilon=1$	$\varphi_{fu}=m_2(i-i_2)+\Phi_2$
d_4	$\mu=1$	$\upsilon=-1$	$\varphi_{fu}=m_2(i+i_2)-\Phi_2$, see the figure 1.

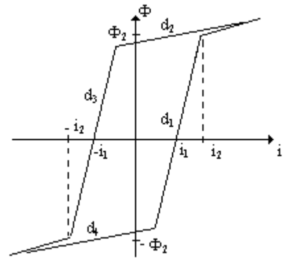

Figure. 1. Histeresis cycle approximated with straights.

Taking into consideration that the voltage at the ends of the coil is sinusoidal results the flux and from linearised relation results the expression of the current in the circuit:

$$i = \frac{\varphi_{fu} + \mu(m_2 - m_1)\cdot \upsilon \cdot i_2 + m_1 \cdot \upsilon \cdot i_1}{m_1 + \mu(m_2 - m_1)} \tag{4}$$

Determining the frequency spectrum of the current curve from this expression is very difficult and needs lot of work.

These linearisations are useful, they approximate the flux-current curve quite well, but they do not render correctly the distortion produced in the current curve.

3. SIMULINK MODELS OF THE IRON CORED COIL

An other possibility of modelling the iron cored coil is to use the differential equations and/or the transfer function written for the real coil. Non-linearity appears in the coefficients of the differential equations which are dependents of the circuit's current.

Using an equivalent serie scheme for real coil with L inductivity which depends of the current intensity of the circuit and of resistance R, the equation is the following::

$$\frac{di(t)}{dt} = -\frac{R}{L} i(t) + \frac{1}{L} u(t) \tag{5}$$

This equation doesn't show the properties of the magnetical material which the iron core is made of. Modelling of this equation in Simulink [2] has this form:

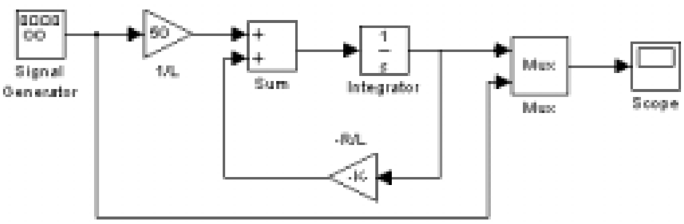

Figure 2. Modelling the circuit with ideal coil and resistance.

At the visualisation of the applied voltage and the current of the circuit a delay of $\pi/2$ is observed compared to the voltage, but without distortion, figure 3.

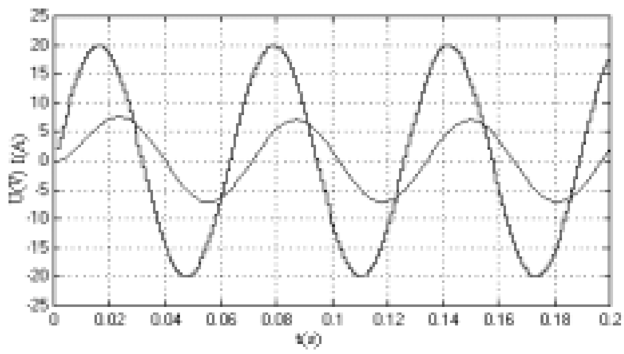

Figure 3. Current and voltage forms through the coil

If the magnetical material of the iron core has a thin histeresis cycle, this can be reduced to the magnetisation curve. To evidence this fenomen the model of sinh is inseried into the model of the coil, which has a similar form to the magnetisation curve as well to the flux-current dependent curve given at analytic modelling too.

$$i = a \cdot shb\varphi \qquad (6)$$

The obtained model is shown by figure 4.

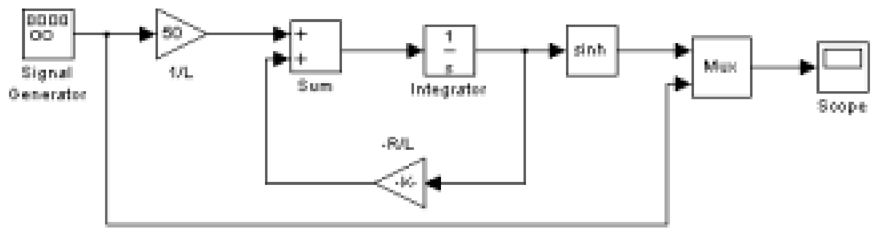

Figure 4. Coil model when the core is made of magnetical material with thin histeresis cycle

In this case the current distorsion can be observed in figure 5.

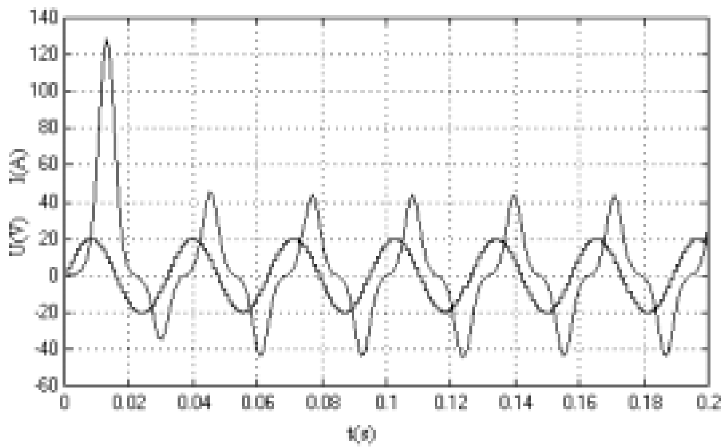

Figure 5. Voltage and current through the iron cored coil

The current curve has picks, which evidences the existence of the 3-rd armonics as it shown by the frequency spectrum on figure 6.

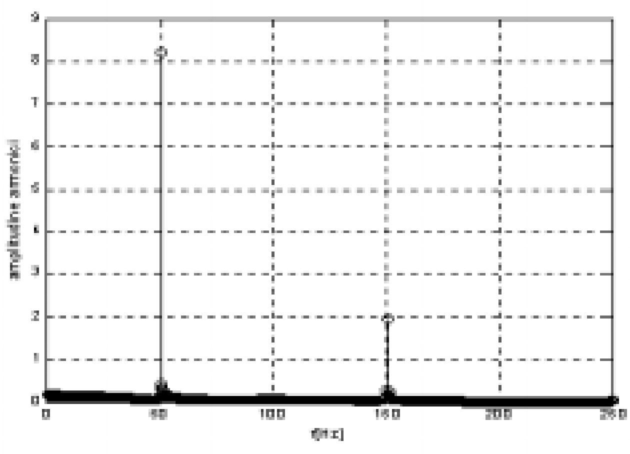

Figure 6. Frequency spectrum of the current through the coil.

If the histeresis fenomen cannot be neglected at the at the magnetisation characteristic of the material, the flux-current dependence is expressed with the equation:

$$i = sh\varphi_{fu} \pm i_c\sqrt{1-\frac{\varphi_{fu}^2}{\Phi^2}} \quad (7)$$

The model of the circuit with iron cored coil is modified in this way:

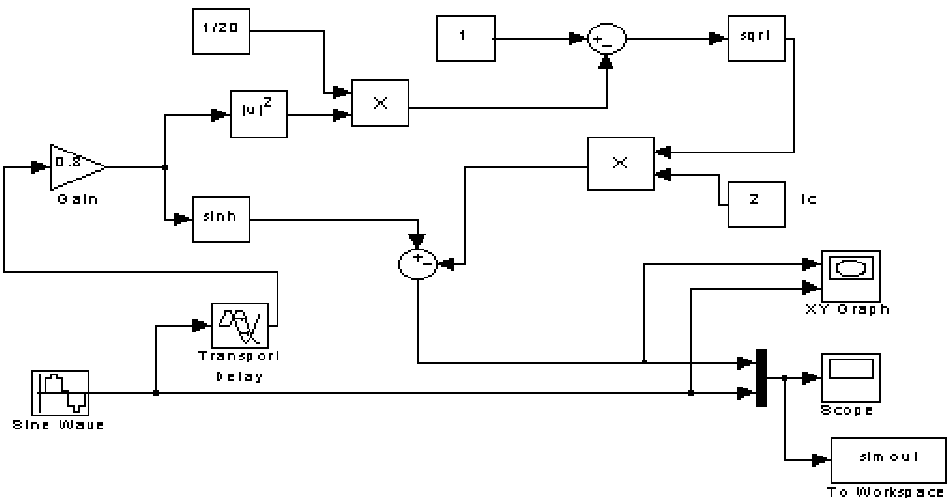

Figure 7. Coil model when the core is made of magnetic material with large histeresis cycle.

When the histeresis cycle of the magnetic material cannot be neglected, the current and voltage form is the following, figure 8.

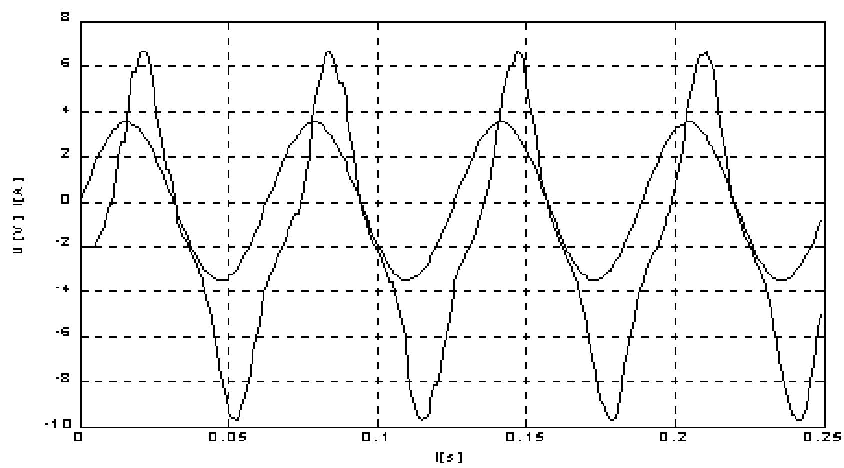

Figure 8. Voltage and current through the iron cored coil with large histeresis.

Frequency spectrum of the current is shown at figure 9.

We can obtain the values of the coefficients from the equation which describe the flux-current characteristic and which cause the distortion of the current curve in this way: we egalate the real characteristics with those obtained by simulation using the models.

The model can be changed also easily by adding or removing functional blocks until these describe correctly the iron-cored coil system.

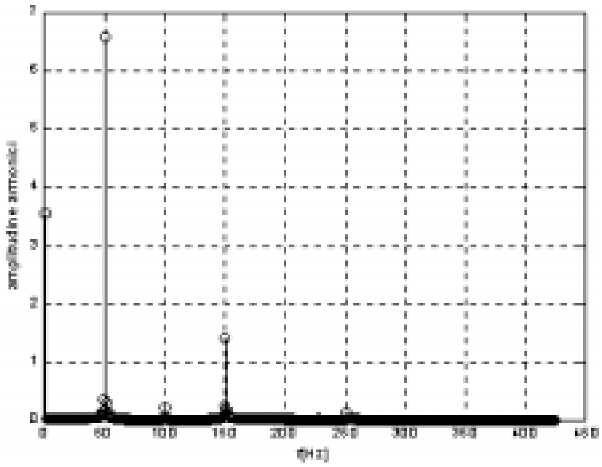

Figure 9. Frequency spectrum of the current through the coil.

In this case can be observed the presence of other armonics, not only those with the third rang and a constant component.

4. CONCLUSIONS

With modelling in Simulink the definition and the analyse of the form and the spectrum of the current through the iron cored coil is faster and easier, as well the changing of input parameters, like the amplitude of the applied voltage, the width of the histeresis cycle is very simple.

After we have got the model, it can be easily adjusted to particular situations by simply modifying the parameters and without calculations. The same way can be analysed by simulation the distortion of the current curve and the spectrum of armonics.

The realised models can be improved and used for modelling some filter to decrease the effects of the harmonics.

REFERENCES

1. A. Arie, C. Neguş, C. Golovanov, M. Golovanov, (1994) Poluarea cu armonici ai sistemelor electrice funcţionând în regim permanent simetric, Bucureşti, Ed. Ac. Rom. ISBN 973-27-0378-4
2. *** (2000) Using Simulink, Dynamic SystemSimulation for MATLAB, The Math Works Inc.
3. A. Kelemen, M. Imecs, (1980) Mutatoare. Aplicații, E.D.P. Bucureşti
4. A. Kelemen, M. Imecs, (1983) Electronică de putere. E.D.P. Bucureşti
5. G. Savin, H. Rosman, (1971) Circuite electrice neliniare şi parametrice, Ed.Tehnica, Bucuresti
6. M. Ghinea, V. Firețeanu, (1999) Matlab. Calcul numeric, grafică, aplicații., Ed. Teora. ISBN 973-601-275-1
7. K. Selmeczi, A. Schnöller, (1978) Villamosságtan II. Műszaki könyvkiadó, Budapest, vol.2