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THE STABILITY STUDY OF THE KALMAN FILTER USED IN THE VECTORIAL CONTROL SYSTEM FOR THE SPEED OF AN INDUCTION MOTOR

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Abstract: In the paper we present the stability study of the Kalman estimator used in a vectorial drive system of an induction motor with short-circuited rotor. We consider that the control system for speed is a vectorial system with direct orientation after the rotoric flux (DFOC). Although there are suggestive experimental results, the implementation of such a system containing a Kalman filter generates serious problems because of the tuning of the covariant matrixes of the measurement noise and the process noise. Also the solving of the Riccati equations in real time is difficult. These problems, as well as the simplifier hypothesis from the determination of the mathematical model on the states space of the induction motor and the modification of the rotoric resistance by temperature lead to the decrease of the estimator and control system performances. Therefore you must implement an algorithm for research of the estimator stability. This algorithm is the object of this study. This algorithm has to hold count of the sampling step of sundries processors used in the drives command. The stability study is done by the determination of the transition matrix poles.

Key words: stability, Kalman estimator, vectorial drive system, induction motor.

1. THE KALMAN FILTER

The equations which define the recursive algorithm of the Kalman filter where written on the basis set by the following stochastic model of the induction motor:

$$\begin{cases} x(k+1) = F_k \cdot x(k) + H_k \cdot u(k) + w(k) \\ y(k) = C \cdot x(k) + v(k) \end{cases} \quad (2)$$

where w and v are the process noise vector and the measurement noise vector which are of Gaussian type and have the following properties:

$$\begin{cases} E[w(k)] = E[v(k)] = 0 \\ E[w(i) \cdot w^T(j)] = Q_k \cdot \delta_{ij} \\ E[v(i) \cdot v^T(j)] = R_k \cdot \delta_{ij} \end{cases} \quad (2)$$

where E is a statistic which means:

$$E(x) = \lim_{N \to \infty} \frac{1}{N} \cdot \sum_{k=1}^{N} x(k) \quad (3)$$

and δ_{ij} is a Kronecker function:

$$\delta_{ij} = \begin{cases} 1 & \text{for} \quad i = j \\ 0 & \text{for} \quad i \neq j \end{cases} \quad (4)$$

The F_k and H_k matrixes are obtained from the A and B matrixes of the continuous model of the induction motor after discrimination. In practice two models are used:

- The model obtained from complete discrimination for which the Kalman filter is precise, but a high performance processor is required for implementation :

$$\begin{cases} F_k = I + A_k \cdot T + A_k^2 \cdot \dfrac{T^2}{2} \\ H_k = B \cdot T + A_k \cdot B \cdot \dfrac{T^2}{2} \end{cases} \tag{5}$$

- The model obtained from simplified discrimination for which the Kalman filter is not so precise, but there is no need for a high performance processor for implementation:

$$\begin{cases} F_k = I + A_k \cdot T \\ H_k = H = B \cdot T \end{cases} \tag{6}$$

In (5) and (6) T is the sampling period:

$$T = (200 \div 300)\mu s$$

and A_k and B have forms which depend on the type of the Kalman filter:

a) The robust Kalman filter used for the determination of the rotoric fluxes:

$$A_k = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} \cdot \omega_k \\ 0 & a_{11} & a_{14} \cdot \omega_k & a_{13} \\ a_{31} & 0 & a_{33} & -\omega_k \\ 0 & a_{31} & \omega_k & a_{33} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{11} & 0 & 0 \end{bmatrix}^T \tag{7}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where :

$$a_{11} = -\left(\frac{1}{T_s \cdot \sigma} + \frac{1-\sigma}{T_r \cdot \sigma} \right);\ a_{13} = \frac{L_m}{L_s \cdot L_r \cdot T_r \cdot \sigma};\ a_{14} = \frac{L_m}{L_s \cdot L_r \cdot \sigma};\ a_{31} = \frac{L_m}{T_r};\ a_{33} = -\frac{1}{T_r};$$

$$b_{11} = \frac{1}{L_s \cdot \sigma};\ T_s = \frac{L_s}{R_s};\ T_r = \frac{L_r}{R_r};\ \sigma = 1 - \frac{L_m^2}{L_s \cdot L_r}$$

and L_s, L_r, L_m are the statoric, rotoric and mutual inductance, R_s, R_r are the statoric and rotoric resistance and σ is the mutual coefficient of leakage.

The matrixes (7) were determined for:

- the elements of the input vector are the statoric voltages reported to the orientated axis system d-q

$$u(k) = \begin{bmatrix} u_{ds}(k) & u_{dq}(k) \end{bmatrix}^T \tag{8}$$

- the elements of the state vector are the statoric currents and the rotor fluxes reported to the orientated axis system d-q

$$x(k) = \begin{bmatrix} i_{ds}(k) & i_{dq}(k) & \varphi_{dr}(k) & \varphi_{qr}(k) \end{bmatrix}^T \tag{9}$$

- the elements of the output vector are the statoric currents reported to the orientated axis system d-q

$$y(k) = \begin{bmatrix} i_{ds}(k) & i_{dq}(k) \end{bmatrix}^T \tag{10}$$

b) The extended Kalman filter matrixes used for the determination of the rotoric fluxes and of the rotoric speed are:

$$A_k = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14}\cdot\omega_k & 0 \\ 0 & a_{11} & a_{14}\cdot\omega_k & a_{13} & 0 \\ a_{31} & 0 & a_{33} & -\omega_k & 0 \\ 0 & a_{31} & \omega_k & a_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ 0 & b_{11} & 0 & 0 & 0 \end{bmatrix}^T \qquad (11)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The matrixes (11) were determined for:

- ♦ the elements of the input vector are the statoric voltages reported to the orientated axis system d-q :

$$u(k) = \left[u_{ds}(k) \quad u_{dq}(k)\right]^T \qquad (12)$$

- ♦ the elements of the state vector are the statoric currents and the rotor fluxes reported to the orientated axis system d-q and the rotoric speed :

$$x(k) = \left[i_{ds}(k) \quad i_{dq}(k) \quad \varphi_{dr}(k) \quad \varphi_{qr}(k) \quad \omega(k)\right]^T \qquad (13)$$

- ♦ the elements of the output vector are the statoric currents reported to the orientated axis system d-q:

$$y(k) = \left[i_{ds}(k) \quad i_{dq}(k)\right]^T \qquad (14)$$

In those conditions, the equations which define the Kalman filter are:

a) The robust Kalman filter:

$$\begin{aligned} &\Gamma(k/k-1) = F_{k-1}\cdot P(k-1/k-1)\cdot F_{k-1}^T + Q_{k-1} \\ &K(k) = \Gamma(k/k-1)\cdot C^T\cdot\left[C\cdot\Gamma(k/k-1)\cdot C^T + R_k\right]^{-1} \\ &\hat{x}(k/k-1) = F_{k-1}\cdot\hat{x}(k-1/k-1) + H_{k-1}\cdot u(k-1) \\ &\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)\cdot\left[y(k) - C\cdot\hat{x}(k/k-1)\right] \\ &P(k/k) = \left[I_4 - K(k)\cdot C\right]\cdot\Gamma(k/k-1) \end{aligned} \qquad (15)$$

where :

- $(k/k-1)$ is an estimate at the moment $k\cdot T$ on the basis of the anterior data which is simultaneous at the moment $(k-1)\cdot T$;
- $K(k)$ is the Kalman matrix;
- $\hat{x}(k/k-1)$ is the extrapolated state (provided);
- $\hat{x}(k/k)$ is the estimated state at the moment $k\cdot T$;
- $y(k) - C\cdot\hat{x}(k/k-1)$ is the estimate of the extrapolated state;
- $\Gamma(k/k-1)$ is the covariance a priori matrix of the extrapolated state $\hat{x}(k/k-1)$;
- $P(k/k)$ is the covariance a posteriori matrix of the estimated state $\hat{x}(k/k)$.

The estimate error is:

$$\tilde{x}(k/k) = x(k) - \hat{x}(k/k) \qquad (16)$$

The initial conditions of the algorithm (15) are:

$$P(0/0) = P_0 \quad \text{and} \quad \hat{x}(0/0) = \hat{x}_0 \qquad (17)$$

The covariance matrixes Q and R are constant or are updated at every step. In the paper we consider that the noises do not sensibly modify the process and they could be considered constants. Accordingly, the matrixes Q and R are [4]:

$$R = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$
$$Q = \begin{bmatrix} \sigma_{i_s}^2 & 0 & \rho \cdot \sigma_{\varphi_r} \cdot \sigma_{i_s} & 0 \\ 0 & \sigma_{i_s}^2 & 0 & \rho \cdot \sigma_{\varphi_r} \cdot \sigma_{i_s} \\ \rho \cdot \sigma_{\varphi_r} \cdot \sigma_{i_s} & 0 & \sigma_{\varphi_r}^2 & 0 \\ 0 & \rho \cdot \sigma_{\varphi_r} \cdot \sigma_{i_s} & 0 & \sigma_{\varphi_r}^2 \end{bmatrix} \tag{18}$$

where :
- σ is the leakage introduced by the input variables (u_{ds} and u_{dq});
- σ_{i_s} and σ_{φ_r} are the leakages introduced by the output variables (i_s and φ_r);

Because the PWM inverter introduced the superior harmonics in the voltage and the statoric current components and, implicit, in the rotor flood tides component, we consider for simulation a source of white noise (Gaussian white noise in discreet time) :

$$K(m,n) = \sigma^2 \cdot \delta(m-n) = \begin{cases} \sigma^2 & \text{for} \quad m = n \\ 0 & \text{for} \quad m \neq n \end{cases} \tag{19}$$

On the other side, the vector $\hat{x}_0$ we consider that it is null and the matrix P_0 we determined as solution of discreet Riccati equation of filtration.

b) The extended Kalman filter:

In order for the structural properties of the Kalman filter to be satisfied we will introduce a virtual speed ω_m defined $\omega_m(k+1) = \omega_m(k) + v_6(k)$ so that we'll have:

$$F_{k-1} = \begin{bmatrix} F_{k-1} & G_{k-1} \\ 0_{2x4} & I_2 \end{bmatrix} \tag{20}$$

where:

$$G_{k-1} = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \\ -f_2 & f_1 & -f_4 & f_3 \end{bmatrix}^T;$$

$$\frac{\partial}{\partial \omega}\left(F_{k-1} \cdot x(k-1) + H_{k-1} \cdot u(k-1)\right) = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}^T$$

where: F_{k-1}, H_{k-1} from inside the presented matrix are obtained from (7) by discrimination using (5) and (6).

The covariance matrixes Q and R are:

$$R = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \quad Q = \begin{bmatrix} Q & 0_{4x2} \\ 0_{2x4} & \sigma_\omega \cdot I_2 \end{bmatrix} \tag{21}$$

where: σ_ω is the leakage speed.

2. SIMULATION RESULTS

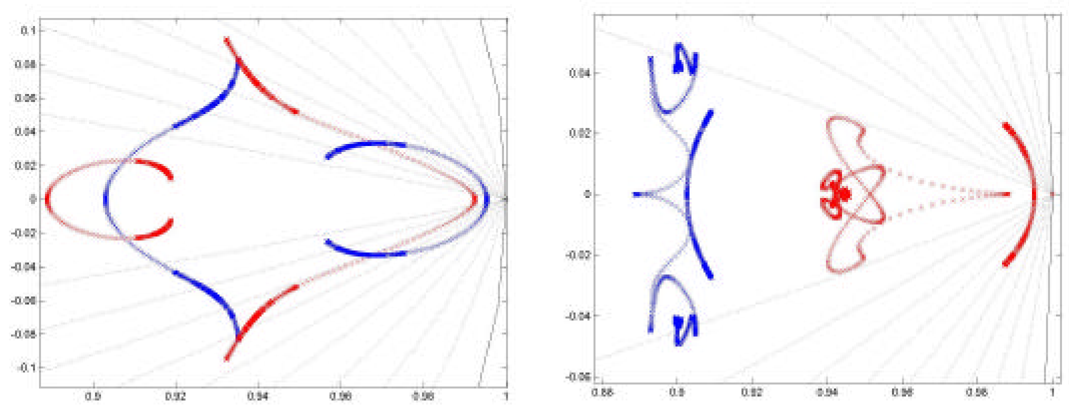
As an example for what was mentioned, an induction motor with the circuited rotor that has the fallowing parameters:

$$P_n = 500[W];\ U_n = 127[V];\ I_n = 2.9[A];\ n_n = 1400[rot/min];\ z_p = 2;\ M_n = 3.41[N \cdot m];$$

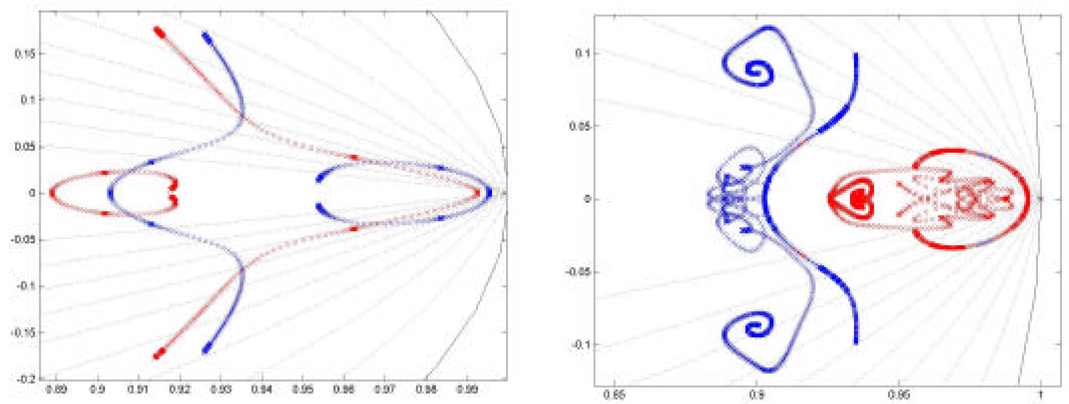
$$R_s = 4.495[\Omega];\ R_r = 5.365[\Omega];\ L_s = 165[mH];\ L_r = 162[mH];\ L_m = 149[mH];$$

$$J = 0.95 \cdot 10^{-3}\left[Kg \cdot m^2\right];\ f_n = 50[Hz]$$

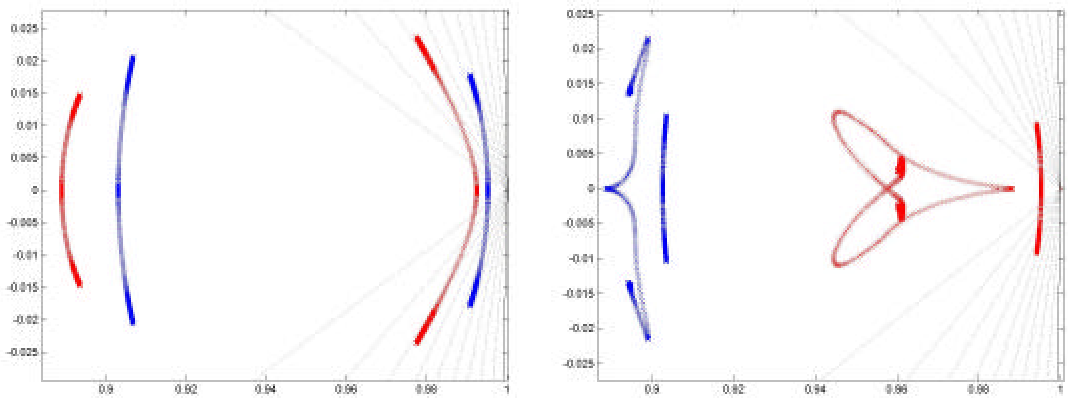
has been considered. In order to obtain the effects of the use of the Kalman filter in feed-back with an induction motor of the type mentioned, the mathematical model of the motor and also the one of the Kalman filter have been implemented in Matlab-Simulink, using the S_Function blocks. The sampling time used is $300[\mu s]$. In this simulation it has been considered that the $K(k)$ matrix is variable and it actualizes at every step. Through simulation the fallowing graphics are obtained:



f=50[Hz]



f=100[Hz]



a) f=20[Hz] b)

Fig.1 Proper values of the robust Kalman filter (a) and of the extended Kalman fiter (b) that function at the constant couple

3. CONCLUSIONS

Luenberger has studied the problem of the estimators included in the continual linear control systems. He showed that the estimator adds poles to the feed-back system without affecting the others poles. Due to the simulation of the three cases presented we showed that this is true only when the estimator is made to give a model to the motor (the last case presented), even if this is not an optimal project (in stochastic sense). After the analysis, it has been noticed that at speeds close to the nominal speed and also at very small speeds, proper values of the estimator are very close of the ones of the motor, and we can conclude that the control system works fine. Bigger differences appear only at high speed. After the analysis it also has been noticed that part of the proper values of the estimator fall behind the proper values of the motor, that practically means a decrease of the control system performances. The speed limit when this phenomenon appears depends of the sampling step, diminishing in the same time with the increase of the latter. While using the Kalman filters in vectorial control systems for speed, the fact that they become unstable when using a big sampling step, which means a decrease in the control system performances must be take in account.

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