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ON THE FUZZY CONTROL OF A CLASS OF LINEAR TIME-VARYING SYSTEMS

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Abstract: The paper deals with Takagi-Sugeno fuzzy models dedicated to a class of plants characterized by Two Input-Single Output linear time-varying systems. It is offered a stability test algorithm of the closed-loop systems fuzzy control systems involving Takagi-Sugeno fuzzy controllers to control the accepted class of plants.

Key words: linear time-varying systems, Takagi-Sugeno fuzzy models, stability test algorithm.

1. INTRODUCTION

Fuzzy logic is used nowadays in modeling and control of complex systems. As function of the structure of the inference rules, fuzzy systems can be divided in several classes characterized by the following rule-based fuzzy models [1]: linguistic fuzzy models, fuzzy relational models, and Takagi-Sugeno (TS) fuzzy models.

The linguistic fuzzy models and the fuzzy relational ones are called Mamdani fuzzy models. A linguistic fuzzy model represents a special case of a fuzzy relational model, with the fuzzy relation particularized by a binary relation having only one nonzero element in each row of the matrix corresponding to the relation.

The main features of TS fuzzy models are [2]: firstly, the input space is decomposed into subspaces; then, within each subspace representing fuzzy regions in the input space, the system model can be approximated by simpler models, in particular linear ones; it is possible to use conventional controller development techniques for controlling these relatively simple local models; finally, the global fuzzy model in the state-space is derived by blending the subsystems' models in terms of the weighted average of the rule contributions.

These features determine the wide application area of TS fuzzy models in spite of their drawbacks such as: the behavior of the global TS fuzzy model can significantly divert from the expected behavior obtained by the merge of the local models [1]; the stability analysis and testing of fuzzy control systems based on TS fuzzy models is relatively difficult because of the complex aggregation of the local models in the inference engine.

The presented drawbacks become stronger when there are developed fuzzy controllers to cope with complex plants including the linear time-varying (LTV) ones. LTV systems are used in practice because most real-world systems are time-varying as a result of system parameters modification as function of time. LTV systems may also be a result of linearizing nonlinear systems in the vicinity of a set of operating points or of a trajectory. Several techniques are employed in the analysis and development of control systems meant for LTV systems. These techniques mainly deal with the eigenstructure assignment [3] … [6].

The first objective of the paper is to express a class of TS models for Two Input-Single Output (TISO) LTV plants. Based on these models, the second aim of the paper is to propose a stability test algorithm for a class of fuzzy systems with TS fuzzy controllers controlling the TISO LTV plants.

The paper structure is the following. The section 2 is dedicated to expressing the TS fuzzy models for TISO LTV systems. Then, in section 3 there are defined the TS fuzzy controllers meant for controlling the TS fuzzy models. In the next section it is discussed the stability test algorithm based on Lyapunov stability theory. The section 5 presents the final concluding part of the paper.

2. A CLASS OF TAKAGI-SUGENO FUZZY MODELS

There will be used in the sequel the following Takagi-Sugeno fuzzy model to represent a TISO LTV system that models the controlled plant:

$$\begin{aligned} &R^l: \text{IF } z_1(t) \text{ is } F_1^l \text{ AND } z_2(t) \text{ is } F_2^l \text{ AND } \ldots \text{ AND } z_n(t) \text{ is } F_n^l \\ &\qquad \text{THEN } y(s) = H_{P,l}^u(s)u(s) + H_{P,l}^v(s)v(s),\ l = 1 \ldots m, \end{aligned} \tag{1}$$

where: $u(s)$ – the Laplace transform of the plant input (the control signal) $u(t)$; $v(s)$ – the Laplace transform of the disturbance input $v(t)$; $y(s)$ – the Laplace transform of the controlled output $y(t)$; m – the number of inference rules; R^l – the lth inference rule, l = 1 … m; n – the number of measurable plant (system) variables pointing out the time-variation of the plant; $z^i(t)$ – the measurable plant variables, i = 1 … n, in particular these ones can be state variables, and:

$$\mathbf{z}(t) = [z_1(t)\ z_2(t)\ \ldots\ z_n(t)]^T; \tag{2}$$

F^l – the linguistic terms associated to the measurable variable $z^i(t)$ and to the rule R^l; $H_{P,l}^u(s)$ and $H_{P,l}^v(s)$ – the local transfer function of the plant with respect to the control signal and the disturbance input, respectively.

It can be observed that the TS fuzzy model (1) represents a continuous-time model including both the inference rules as part of the rule base and the local analytic models of the TISO LTV system. The controlled output is inferred by taking the weighted average of all local models appearing in (1).

Since the model (1) characterizes the properties of the controlled plant in a local region of the input space, it is referred to as fuzzy dynamic local model [7].

The following notation is introduced:

$$\mu_l(t) = \mu_l(\mathbf{z}(t)),\ l = 1 \ldots m, \tag{3}$$

for the membership degrees of the normalized membership functions μ_l of the inferred fuzzy set F^l, where:

$$F^l = \bigcap_{i=1}^{n} F_i^l,\ l = 1 \ldots m, \tag{4}$$

$$\sum_{l=1}^{m} \mu_l(t) = 1. \tag{5}$$

By using the product inference method in (4) and the weighted average method for defuzzification, the TS fuzzy model (1) can be expressed in terms of the following fuzzy dynamic global model that can be considered as TS fuzzy model of the plant:

$$y(s) = H_P^u(s)u(s) + H_P^v(s)v(s),$$
$$H_P^u(s) = \sum_{l=1}^{m} \mu_l(t) H_{P,l}^u(s),\ H_P^v(s) = \sum_{l=1}^{m} \mu_l(t) H_{P,l}^v(s). \qquad (6)$$

The model (6) represents the model of a TISO LTV system because the inferred transfer functions, $H_P^u(s)$ and $H_P^v(s)$, have time-varying coefficients.

3. TAKAGI-SUGENO FUZZY CONTROLLERS. CLOSED-LOOP SYSTEM MODELS

The TS fuzzy models (1) or (6) could be very useful in comparison with other conventional techniques in nonlinear control. This is the case of piecewise linearization [8], where the plant is linearized around a nominal operating point, and there are applied linear control techniques to the controller development. But this approach divides the input space into crisp subspaces, and the result is in a non-smooth connection of the linear subsystems to build the closed-loop system model.

This is not the case with the TS fuzzy models (1) and (6). These models are based on the division of the input space into fuzzy subspaces and use linear local models in each subspace. Furthermore, the fuzzy sets F_i^l and the inference method permit the smooth connection of the local models to build the fuzzy dynamic global model of the closed-loop system.

For controlling the TISO LTV plant (6) there is proposed a TS fuzzy controller with the following model:

$$R^l: \text{IF } z_1(t) \text{ is } F_1^l \text{ AND } z_2(t) \text{ is } F_2^l \text{ AND } \ldots \text{ AND } z_n(t) \text{ is } F_n^l$$
$$\text{THEN } u(s) = H_{C,l}(s)e(s),\ l = 1 \ldots m, \qquad (7)$$

where: $e(s)$ – the Laplace transform of the control error $e(t)$:

$$e(t) = r(t) - y(t), \qquad (8)$$

with $r(t)$ – the reference input; $H_{C,l}(s)$ – the transfer functions of the local controllers, l = 1 … m.

The local controllers in (7) are developed for the local analytic models in (1) by parallel distributed compensation [9].

By the feedback connection of the plant (1) and of the fuzzy controller (7) in terms of the conventional control structure presented in Figure 1, the closed-loop system can be described by the following fuzzy dynamic local model:

$$R^l: \text{IF } z_1(t) \text{ is } F_1^l \text{ AND } z_2(t) \text{ is } F_2^l \text{ AND } \ldots \text{ AND } z_n(t) \text{ is } F_n^l$$
$$\text{THEN } y(s) = H_{r,l}(s)r(s) + H_{v,l}(s)v(s),\ l = 1 \ldots m, \qquad (9)$$

where: $H_{r,l}(s)$ and $H_{v,l}(s)$ – the local transfer functions of the closed-loop system with respect to the reference input and to the disturbance input, respectively, l = 1 … m.

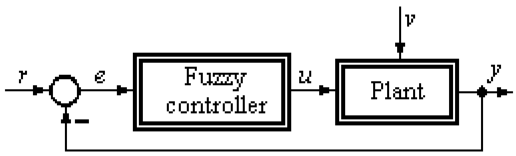

Figure 1 – Control system structure.

In the conditions (3) … (5), by accepting the same inference method and defuzzification method as in the previous section, the fuzzy dynamic global model of the closed-loop system can be expressed in terms of (10):

$$y(s) = H_r(s)r(s) + H_v(s)v(s),$$
$$H_r(s) = \sum_{l=1}^{m} \mu_l(t) H_{r,l}(s),\ H_v(s) = \sum_{l=1}^{m} \mu_l(t) H_{v,l}(s)\ , \tag{10}$$

where the inferred transfer functions, $H_r(s)$ and $H_v(s)$, have time-varying coefficients.

It is justified to consider the TS fuzzy model (10) as TISO LTV system. Therefore, for its analysis there can be applied methods specific to LTV systems [3] … [6] which require numerical techniques for the calculation of $H_r(s)$ and $H_v(s)$.

For the development of the fuzzy controllers it is necessary to perform the stability analysis and testing. A stability analysis test algorithm for the closed-loop system (10) will be discussed in the next section.

4. STABILITY TEST ALGORITHM

Although in the development of the fuzzy controllers there are used the fuzzy dynamic local models (1), (7) and (9), for the stability analysis of the fuzzy control systems two approaches can be employed. The first one is base on the use of the fuzzy dynamic global model (10). The second one can be developed by starting with the definition of a piecewise smooth quadratic Lyapunov function [10]. But, this is based on the fuzzy dynamic local model (9).

In the case of the second approach for the system (10) there can be used several approaches based on either transferring the ideas from hybrid systems [8] or by using, since this system can be considered as a variable structure one with possible discontinuous right-hand side, the stability analysis methods dedicated to variable structure systems [11].

For the stability analysis and testing of the fuzzy control system modeled by the fuzzy dynamic global model (10) it will be presented as follows the first approach, based on the Lyapunov stability theory in terms of the definition of a piecewise smooth quadratic Lyapunov function V:

$$V = \sum_{l=1}^{m} q_l V_l\ ,\ \mathrm{V}_l = \mathbf{x}^T \mathbf{P}_l \mathbf{x}\ , \tag{11}$$

where: $\mathbf{x}$ – the state vector, dim $\mathbf{x}$ = (1, n_S), $\mathbf{P}_l$ – positive definite symmetric matrices, dim $\mathbf{P}_l$ = (n_S, n_S), q_l – weighting coefficients ensuring the smoothness of the function V, l = 1 … m, n_S – the system order.

The matrices $\mathbf{P}_l$ are obtained by ensuring the negative definiteness of the derivative of the Lyapunov function. This can be ensured by solving the algebraic Riccati equations (12):

$$\mathbf{A}_l^T \mathbf{P}_l + \mathbf{P}_l \mathbf{A}_l = -\mathbf{Q}_l\ ,\ l = 1 \ldots m\ , \tag{12}$$

with $\mathbf{Q}_l$ – positive definite symmetric matrices, dim $\mathbf{Q}_l$ = (n_S, n_S), and $\mathbf{A}_l$ – the system matrices in the systemic realizations corresponding to the closed-loop transfer functions $H_{r,l}(s)$ and $H_{v,l}(s)$, dim $\mathbf{A}_l$ = (n_S, n_S),

The stability analysis test algorithm consists of the following steps:

- step 1: based on the knowledge and experience concerning the controlled plant operation, determine the number of inference rules m for controlling the plant, the partition of the input space in fuzzy regions, assign the linguistic terms F_i^l to the

measurable plant variables $z^i(t)$, $i = 1 \ldots n$, and define the membership functions corresponding to F_i^l, $l = 1 \ldots m$;
- step 2: for each inference rule R^l, $l = 1 \ldots m$, derive the linear local models of the plant, characterized by the transfer functions $H_{P,l}^{u}(s)$ and $H_{P,l}^{v}(s)$;
- step 3: develop a conventional controller with the transfer function $H_{C.l}(s)$ for each of the local models of the plant by a linear control development technique such that the m closed-loop local systems, with the transfer functions $H_{r,l}(s)$ and $H_{v,l}(s)$, $l = 1 \ldots m$, have the required control system performance;
- step 4: set the values of the positive definite symmetric matrices $\mathbf{Q}_l$ and solve the algebraic Riccati equations (12); if the solutions $\mathbf{P}_l$ of (12) prove to be not positive definite, then jump to the step 3; otherwise, the system is stable.

It must be highlighted that the step 4 requires the largest computational effort.

5. CONCLUSIONS

The paper derives continuous-time TS fuzzy models dedicated to TISO LTV systems together with a stability test algorithm for the fuzzy control systems modeled by TS fuzzy models based on Lyapunov stability theory.

The models and the stability analysis algorithm can be used in the development of TS fuzzy controllers based on the parallel distributed compensation with several applications.

One real-world application can be in the area of electrical drives with variable inertia [12], [13], where the development of the local controllers can be performed in terms of the ESO method [14]. This real-world application is necessary for the validation of the proposed stability analysis test algorithm.

The main limitation of the stability analysis algorithm concerns its computational complexity.

Further research must be concentrated firstly on the development of computer-aided implementation of the stability analysis algorithm. This is a relatively difficult to be done because the closed-loop system poles are not, as expected, the union of the local systems poles. Secondly, another research direction is in the development of discrete-time and of hybrid TS fuzzy models for the considered class of plants.

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