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DYNAMIC NETWORKS AND SWITCHED LINEAR STATE FEEDBACK CONTROL

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ABSTRACT

The purpose of this paper is to study the opportunity of a specific decentralized control system for multi-agent systems. The control law is given as a switched linear state feedback control. The multi-agent system is stabilized into a given formation by stating the problem of stabilization as a regulation / tracking problem of a dynamic network constructed into an error space.

KEYWORKDS: multi-agent system, sliding-mode control, dynamic network

1. INTRODUCTION

The field of multi-agent systems control is still under on-going investigation by both the control scientific and computer science communities. Our paper contributes to this field from the control theory point of view. We consider a collection of agents, where an agent is "just something that perceives and act", [1], and obeys a linear dynamics. We assume that the agent "perceives" just the neighboring agents. In [2], [3] a control-oriented modeling for agents formations was given. The control developed in a top-down manner stabilizes the multi-agent system into a target formation. A specific control law was proposed in [3] without giving a full explanation, from a mathematical point of view, of its opportunity in the respective context. The present paper intends to give more introspection into the mechanism of formation stabilization when the proposed control law is used. We called the control law given in [3] as sliding-mode control even if it is not in the traditional sliding-mode (SM) control form. Recently, we became aware of the work revealed by [7] and decided that the name "switched linear state feedback" (SLSF) control law is more appropriate. The paper is not intended to show simulations or give numerical results. There are many papers in the literature, (see [15] and the references given in [3]), merging formation stabilization, hybrid systems theory, graph theory and automata and all of them show successful simulations for the approached case studies. Our vision is though a little bit different in the fact that we try to state an as general as possible methodology for developing a multi-agent systems control, without using tools from graph theory and automata, if they prove to be unnecessary. Instead of "giving numbers" for examples that might be judged as trivial, in this paper we explain the opportunity of the chosen control law into the given context. In our approach the "dynamic network", [6], is a collection of non-linear systems that results as a projection of the multi-agent system dynamics into an error space. The composite non-linear systems may change their dimensionality while agents interact with each other. In the following we give an integrated analysis of the problems the emerged dynamic network has to cope with. The same specifications as in [3] stands.

2. A CONTROL-ORIENTED MODELING FOR FORMATIONS

We denote by N, N>3, the number of agents the multi-agent system consist of. We call formation, *F*, the set of relations among agents which describe the configuration of the multi-agent system. In this paper we assume that the relations among agents refer only to the distance among them. We denote the distance between two agents i and j that perceive each other by $\rho_{kj} = \left\| \tilde{x}_{kj} \right\|$ where $\tilde{x}_{ij}(t) = x_k(t) - x_j(t)$ and $x_k, x_j \in R^n$ denote the state vectors assigned to agents k and j. If $\left\| \tilde{x}_{kj} \right\| > \delta$, δ>0, the agents i and j cannot perceive each other. The δ-neighborhood, N_δ, of the agent Ag_k, $N_\delta\{Ag_k\}$, is the set of agents that are at a distance of at most δ from Ag_k: $N_\delta\{Ag_k\} = \{Ag_j \middle| \rho_{kj} \le \delta\}$. We denote the number of neighboring agents by N_k. The target (desired) formation, F_d, specifies the distances between agents, at equilibrium, as follows: $F_d = \{\rho_{kj} \middle| \rho_{kj} = w_{kj}, \dot{\rho}_{kj} = 0\}$, $\forall Ag_k$, Ag_j, and $w_{kj}>0$ is given.

The dynamic network evolves in the space of errors e_{ij}, where $e_{kj} = w_{kj} - \rho_{kj}$. For each agent Ag_k a sliding manifold is defined in the space of errors, as follows: $s_k = \sum_{j, Ag_j \in N_\delta\{Ag_k\}} c_{kj} e_{kj} = c_k^T e_k = 0$, where $c_k, e_k \in R^{N_k}$.

The collective dynamics of a multi-agent system is reflected into the dynamics of the error vector $e \in R^m$ constructed by stacking all variables e_{kj}. Thus, the collective dynamics is the continuous-time dynamics of the dynamic network. It deals with the sliding manifolds $s_\infty = \tilde{C}e = 0$, where the sliding-mode coefficients are grouped into the matrix $\tilde{C} = [\tilde{c}_1 \ \ ... \ \ \tilde{c}_N]^T$ and $\tilde{c}_k^T$ is constructed by padding with zeros the vector c_k^T so that its multiplication by e is proper.

Constraining the agents to see each other inside a limited horizon the dynamic network emerges as an autonomous switching system. Each non-linear system of the dynamical network describes the dynamics of an agent neighborhood projected into the error space. Since the number of the neighboring agents for each agent may change, the dynamic network emerges as a large-scale system undergoing autonomous switching because the connections between components appear and disappear. The switching dynamics can be captured through a matrix that we call scarcity matrix. The scarcity matrix W_p deletes from the matrix $\tilde{C}$ the agents situated at a distance higher than δ from each other. W_p is a diagonal matrix containing only 1 and 0 elements. The target sliding manifold for an agent Ag_k is estimated for $N_\delta^d\{Ag_k\}$ where the superscript *d* indicates that the respective neighborhood is extracted from the target formation, F_d. The scarcity matrix describing the target sliding manifold is denoted by W_d. A feasible formation, *F,* has the scarcity matrix W_p containing at least *N-1* unity elements and also, the matrix $\tilde{C}W_p$ has no all-zero-element line. The decentralized control implemented for the multi-agent system uses coordination schemes based on the sliding manifolds $s = \widetilde{C}W_p e = 0$. Since the dynamic network exhibit switching among different sets of neighborhoods the scarcity matrix changes in time. Consequently, the formation dynamics is as follows:

$$F(0) \to ... \to F(t) \to ... \to F_d(T) \quad \Leftrightarrow \quad f.o.[\widetilde{C}W_p(0)] \to f.o.[\widetilde{C}W_p(t)] \to f.o.[\widetilde{C}W_d] \qquad (1)$$

where *f.o.* stands for the elimination of the all-zero-element lines or/and all-zero-element columns from the matrix it acts on. We drop the argument of the scarcity matrix W_p when this is not important.

5. THE DYNAMIC NETWORK

Since for many systems the feedback linearization is possible we assume here that the agents obey a linear dynamics $\dot{x}_k = A_k x_k + b_k u_k$, A_k-Hurwitz and $u_k \in R$. The $N_\delta\{Ag_k\}$-dynamics is as follows: $e_{kj} = -(\widetilde{x}_{kj}^T \dot{\widetilde{x}}_{kj}) / \rho_{kj}$, $j=1,..,N_k$, $j \neq k$. For each neighborhood $N_\delta\{Ag_k\}$ the reference frame is joint to the agent k, therefore $x_k=0$ and $\widetilde{x}_{kj} = -x_j$. After some algebra, the error dynamics turns out to be on the following form: $\dot{e}_{kj} = -(\lambda_{kj} + d_{kj})\rho_{kj} + \gamma_{kj} u_k + \zeta_{kj} u_j$, $\forall \; j=1,..N$, $d_{kj} = \max\left((\widetilde{x}_{kj}^T / \rho_{kj})(A_j - A_j^T)(\widetilde{x}_{kj} / \rho_{kj})/2\right)$, $\gamma_{kj} = -(\widetilde{x}_{kj}^T b_k)/\rho_{kj}$, $\zeta_{kj} = -(\widetilde{x}_{kj}^T b_j)/\rho_{kj}$, $|\zeta_{kj}| < \|b_j\|$, $|\gamma_{kj}| < \|b_k\|$, $\lambda_{kj} = \widetilde{x}_{kj}^T (A_j + A_j^T)\widetilde{x}_{kj} / 2\rho_{kj}^2 \in [\lambda_m((A_j + A_j^T)/2), \lambda_M((A_j + A_j^T)/2)] < 0$. If $A_j=A_j^T$ then $d_{kj}=0$. In order to keep the notations simple we will consider $d_{kj}=0$ in this paper.

We refer to the case when the control does not require information flow. Obviously, by including information flow into the control law the control quality may be improved. Since the on-line operation implements a decentralized control the inter-connection terms u_j in the error dynamics e_{kj} enter as disturbances. Since the functions λ_{kj}, γ_{kj}, ζ_{kj} are bounded we assume that the non-linear systems the dynamic network consist of are linear systems with time-varying parametric uncertainties. A decentralized control without information flow can only take advantage of the local parameters measurements. Therefore, we are looking for a control solution where the parameters are assumed to be non-measurable.

The network dynamics may be captured by the following system with overlapping decompositions:

$$\dot{e}_k = \Lambda_k(t)e_k + C_k(u_1,...,u_N) - \Lambda_k(t)w_k; \quad C_k(u_1,...,u_N) = \gamma_k(t)u_k + \Xi_k(t)v_k = \Theta(t)u \qquad (2)$$

where $v_k^T = [u_j]_{j \neq k, Ag_j \in N\{Ag_k\}}$, $\gamma_k^T = [\gamma_{kj}]_{j=1,..,N_k}$, $w_k^T = [w_{kj}]_{j=1,..,N_k}$, $\Xi_k = diag([\zeta_{kj}]_{j=1,..,N_k})$, $\Lambda_k = diag([\lambda_{kj}]_{j=1,..,N_k})$.

The network dynamics may be also captured by the following system:

$$\dot{e} = \Lambda(t)e + \Theta(t)u - \Lambda(t)w \qquad (3)$$

where the matrices and vectors are properly written using the equation (2).

The off-line design may deal with the inter-connection terms differently. It may either try to reject them as in the methodology based on large-scale systems, [4], [5], [9], [10], or, may benefit of them as in the chaos control [14], [13], [12]. Despite the chosen alternative the collective dynamics stability is critically linked to the inter-connection terms. In the following, we refer to the network dynamics as being an uncertain system with the following structural uncertainties: $\Lambda(t) = \Lambda_0 + F_1(t)E_1$ and $\Theta(t) = F_2(t)E_2$ if $B_k = B_j$ $\forall$ Ag$_k$, Ag$_j$ or $\Theta(t) = F_2(t)E_2 + F_3(t)E_3$ if $B_k \neq B_j$, for $F_i^T(t)F_i(t) < 1$, i=1,2,3. In the manner introduced in [11] the system may be also written as a parameter time-varying linear system, as follows:

$$\dot{e} = \Lambda_0 e + p - \Lambda_0 w; \quad p = F_1 q + F_2 q_u; \quad q = E_1 e - E_1 w; \quad q_u = E_2 u \tag{4}$$

6. THE CONTROL LAW ANALYSIS

The main issue of the control design is how to choose the control law for each agent so that the stabilization of the network into one of the equilibrium states is possible. We are interested to coordinate agents through sliding manifolds by stating the problem of formation stabilization as a regulator/ tracking problem. In this section we try to argue two things. First, that the SLSF control law is able to implement a SM regime if properly designed and secondly, that the switching which the network undergoes might require a SLSF control law instead of a SM control law. As a general idea the SM control law is used in relation to (2) and the SLSF control law is used in relation to (3).

The equilibrium of the network dynamics is as follows:

$$\dot{e} = 0 \quad \rightarrow \quad \left(\Lambda(t)\big|_{\rho=w}\right)w = \left(\Theta(t)\big|_{\rho=w}\right)u_w \tag{5}$$

where $u_w^T = [u_{1_w} \quad \dots \quad u_{N_w}]$ is the control responsible for choosing one of the equilibrium states. Since $\Lambda(t)\big|_{\rho=w}$ and $\Theta(t)\big|_{\rho=w}$ depend on the relative position between agents, $\tilde{x}_{kj}$, it turns out from (5) that the equilibrium state depends on $\tilde{x}_{kj}$.

The control law may be chosen as a SM control law ($M_k, \tilde{c}_k^T$), [5], as follows: $u_k = M_k \|x_k\| sign(s_k) sign(\tilde{c}_k^T W_p \gamma_k) + u_{k_w}$. For this case the control vector is $u_{sm} = T_p T_\gamma M \psi + u_w$, where $T_p = diag([sign(s_k)]_{k=1,..,N})$, $T_\gamma = diag([sign(\tilde{c}_k^T W_p \gamma_k)]_{k=1,..,N})$, $M = diag([M_k]_{k=1,..,N})$, $\psi^T = [\|x_k\|]_{k=1,..,N}$. Another possible control law is the one given in [3] in the form of a switched linear state feedback (SLSF) ($\tilde{k}_k^T, \tilde{c}_k^T$), as follows: $u_k = \left(\tilde{k}_k^T W_p e\right) sign(s_k) + u_{k_w}$ where $\tilde{k}_k^T$ has the same structure as $\tilde{c}_k^T$. The control vector is $u_{slsf} = T_p \tilde{K} W_p e + u_w$ where $\tilde{K}$ has the same structure as $\tilde{C}$.

In the following we justify that the SLSF control law can implement a SM regime. A SM regime for the non-linear system j of the network can be imposed if $\exists$ ($\tilde{k}_j^T, \tilde{c}_j^T$) so that $s_j \dot{s}_j < 0$. We suppose that $\tilde{c}_j^T \Theta_j > 0$, $\Theta = [\Theta_j]_{j=1,..,N}$. Let e$_1$ and e$_2$ be two error vectors so that: $s_j(e_1) = \tilde{c}_j^T e_1 > 0$ and $s_j(e_2) = \tilde{c}_j^T e_2 < 0$ and also $\dot{s}_j(e_1) < 0$, $\dot{s}_j(e_2) > 0$. From $s_j(e_1)\dot{s}_j(e_1) < 0$ and $s_j(e_2)\dot{s}_j(e_2) < 0$, after some algebra we get:

$$e_1^T \tilde{c}_j^T \tilde{k}_j^T e_1 < \varsigma(e_1, \tilde{C}, \tilde{K}, F_1, F_2, w, \Lambda, \Theta), \quad e_2^T \tilde{c}_j^T \tilde{k}_j^T e_2 < \varsigma(e_2, \tilde{C}, \tilde{K}, F_1, F_2, w, \Lambda, \Theta) \tag{6}$$

Since the inequalities (6) are non-conflicting, we conclude that the SLSF control law could implement a SM regime through the full rank matrices ($\tilde{K}$, $\tilde{C}$). The same reasoning stands also for $\tilde{c}_j^T \Theta_j < 0$.

Since the system we are dealing with is time varying and the matrix $\widetilde{K}$ from the SLSF control law is a constant matrix we have to show that the SLSF control law is the solution of the optimization problem with the Hamiltonian: $H = s^T s + p^T \dot{s}$. By elimination, the control coefficient in H is $p^T Z = s^T H^{-1} Z$, where $Z = \widetilde{C}\Theta$. If we choose $u = Z^{-1} H v$ the Hamiltonian $H = s^T s + s^T H^{-1} C \Lambda e + s^T v$ can be minimized by $v_k = (c_k^T e + l_k^T e)\,\mathrm{sgn}(s_k)$. The linear dependence of the gain of v_k on error is due to the fact that H does not depend explicitly on e. To prove this fact we write $\dot{s} = \widetilde{C}\Lambda e + \widetilde{C}\Theta T_p k e = Y + Zu$ and notice that H does not depend on the error e explicitly because $\dot{s}$, Y, $\dot{u}$, $\dot{Y}$ are linear functions of e. Consequently, the SLSF control law is able to minimize the proposed Hamiltonian if the worst case dynamics of the structural uncertainties are properly captured by l_k.

Since the multi-agent system is stable if the collective dynamics is stable the network stability is investigated using the common Lyapunov function $V = e^T P e$, $P = \widetilde{C}^T \widetilde{C} + \widetilde{K}^T \widetilde{K}$. The switching dynamics through different formations (modes) means switching through energetic levels $V_p = e^T W_p P W_p e$. Since the inequalities:

$$V_p < \frac{\lambda_M(P)}{\lambda_m(P)} \left(\frac{\|W_p e\|}{\|e\|} \right)^2 V; \quad \lambda_m(W_p P W_p)\|e\|^2 \le V_p \le \lambda_M(W_p P W_p)\|e\|^2 \tag{7}$$

have to be fulfilled the matrix $\widetilde{C}$ should be estimated so that $V_p<V$ for as many feasible formations as possible.

In order to solve the trade-off between the high gain required by the stability and the low gain that restrict the disruption of the loosely-connected formations, see [3], the control law spectrum should be as diverse as possible. Therefore, it is worth comparing the following values: $\|u_{sm}\| = \|T_\gamma M\psi\| = \|M\psi\|$ and $\|u_{slsf}\| = \|KW_p e\| \le \|M_p \psi\| \le \|M\psi\|$ where $M_p = diag([\|k_i^T W_p\|]_{i=1,..N})$. We can notice that for the SLSF control law the spectrum of the control values is more diverse than for the SM control law. Therefore, if the SLSF control law is implemented, given the Lyapunov function for the collective dynamics, many more modes of the switching system may be estimated so that $V_p<V$ than if the SM control law is implemented. In order to check this assumption, we compare the implications of both the SM control law and SLSF control law on the control system design. Both control laws provide a piece-wise continuous input. We assume w=0. Since the following inequalities are true:

$$s_\infty^T \widetilde{C}\Theta u_{sm} \le \|s_\infty^T \widetilde{C}\Theta T_p\| \|M\psi\|; \quad s_\infty^T \widetilde{C}\Theta u_{slsf} \le \|s_\infty^T \widetilde{C}\Theta T_p\| \|M_p \psi\| \le \|s_\infty^T \widetilde{C}\Theta T_p\| \|M\psi\| \tag{8}$$

from the passivity theory, [8], for the following dynamic system: $\dot{s}_\infty = \widetilde{C}^T \Lambda e + \widetilde{C}\Theta u$, $y = \widetilde{C}\Theta s$ with the supply rate $w_s(\tau) = y^T(\tau) u(\tau) < 0$, the following is true: $0 \le V_{a_{slsf}} \le V_{a_{sm}} \le V$, where $V_{a_{sm}} = -\int_0^T w_{sm}(\tau) d\tau$ and $V_{a_{slsf}} = -\int_0^T w_{slsf}(\tau) d\tau$.

A good control solution requires as many as possible feasible formations to be distributed between $V_{a_{slsf}} \le V_p \le V$ and $V_{a_{sm}} \le V_p \le V$, according to the control law used. Note that this is easier to be accomplished by the SLSL control law.

7. CONCLUSIONS

In this paper we have shown that the SLSF control law does implement a sliding-mode regime. However, this control law does not necessarily guarantee that a sliding-mode is always implemented on the intersection of all the imposed sliding manifolds. In the process of converging towards an equilibrium the sliding manifolds can be either just hit or followed for some time only. Since the control is designed so that $\sum_{i=1}^{N} s_k \dot{s}_k < 0$ at each moment at least one manifold is either attractive or followed. The SLSF-control law does not aim at forcing the system trajectory towards the intersection of all sliding manifolds if the natural dynamics is stressed by such a maneuver. However, the trajectory orientation is mainly guided by the sliding manifolds.

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