A&QT-R (THETA 13)
International Conference on Automation, Quality and Testing, Robotics
May 23-25, 2002, Cluj-Napoca, Romania

OPTIMAL SYNTHESIS OF A 3-TTSR PARALLEL MANIPULATOR

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ABSTRACT

The paper presents a method for dimensional optimization of the arhitecture of a 3-TTSR parallel manipulator by minimizing a purpose function defined in a four dimensional space so that the end-effector reaches a point in the working space with a given orientation without overrunning the joints' limits.

Keywords: Parallel Robots, Close-loop mechanisms, Synthesis, Kinematics

1. INTRODUCTION

In the last two decades, the technological progress and especially the industrial robotics development has practically led to the appearance of a new robots generation based on the closed-loop kinematic chains, namely **parallel robots**. These robots can provide the complementarity of the serial classical robots owing to high positioning accuracy, high speeds of displacement and very good dynamic behaviour. While the serial robots are mainly used for simple industrial loads of a low precision in a large working space, the parallel robots can fulfil high precision tasks, being able to manipulate heavy loads of high speeds and accelerations.

Efforts for the research in the parallel robot domain are also made in Romania. Within the Department of Mechanics and Computer Programming at the Technical University of Cluj-Napoca there are researches in the parallel mechanism domain from the years of 70s materialized in different scientific papers of Prof. Nicolae Plitea who, for the first time, has practically created this research direction at the Technical University of Cluj-Napoca. Many members of this department are strongly involved in this scientific research by developing research projects and participating directly in some of scientific manifestation in this domain.

In April 2001 at the Technical University of Cluj-Napoca was opened the first "Centre for Simulation and Testing of Industrial Robots" within are developed research and teaching activities on the parallel robots field.

On the practical and theoretical required experience there are results scientifically proved, concerning the mathematical description of the parallel robots. Before the parallel structures replace the classical ones in the industrial production, it is necessary to develop an intense research activity referring to the implementation of these structures in manufacturing some functional robots.

One of the important aspects in the research of parallel robots is their optimization.

In this paper we propose to present a method for dimensional optimisation of the architecture of the 3-TTSR parallel manipulator. The dimensional synthesis of the kinematic chain should be of primary importance in the mechanical design of parallel manipulators with the aim to achieve an optimal use of the workspace possibilities.

2. DESCRIPTION OF THE 3-TTSR PARALLEL MANIPULATOR

The mechanism of the 3-TTSR manipulator is a parallel manipulator of type B guided in three points [8]. The structural scheme of these parallel mechanisms of type B is illustrated in the figure 1 and the theoretical model in figure 2.

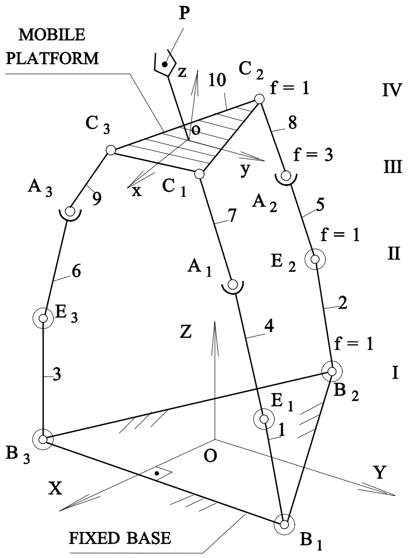

Figure 1

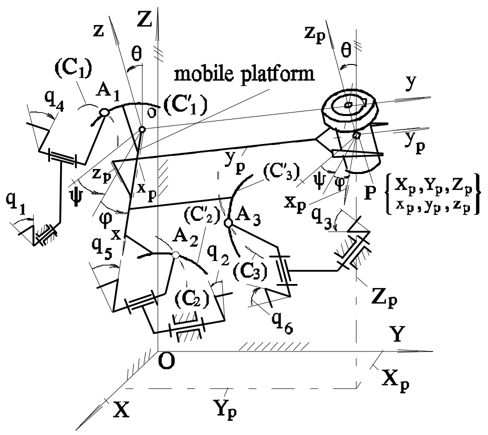

Figure 2

In figure 3 the kinematic scheme of the 3-TTSR parallel manipulator is presented. The points B_i (i=1,2,3) and E_i (i=1,2,3) represent active prismatic joints whereas ball joints and the revolute joints are located in A_i (i=1,2,3) respectively in C_i (i=1,2,3). The six prismatic joints are actuated, the rest of them are passive.

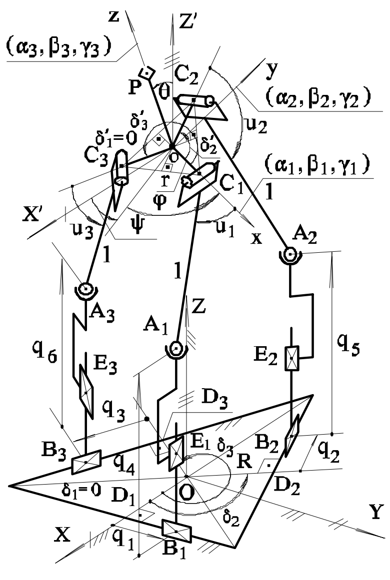

Figure 3

Figures 4 and 5 show the shape of the mobile platform and the base platform.

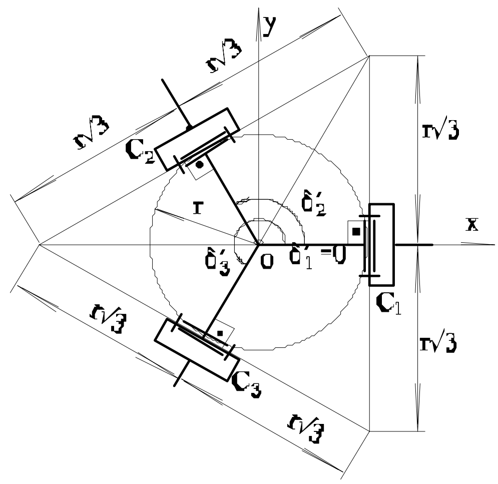

Figure 4

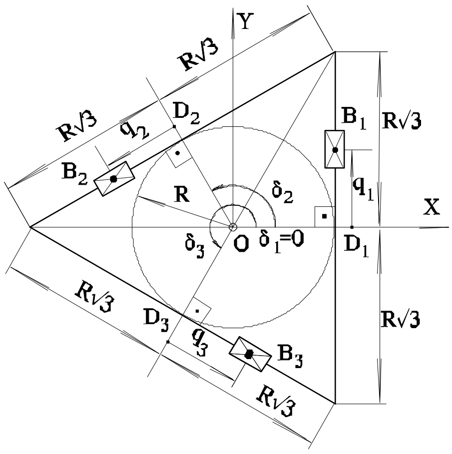

Figure 5

3. INVERSE GEOMETRIC MODEL

The inverse geometrical model consists of the calculus of the manipulator joint coordinates q_i and q_{i+3} (i=1,2,3) when the position X_p, Y_P, Z_p and the orientation ψ, θ, ν (Euler angles) of the end-effector are known.

The closure equations for the inverse geometrical model are the following [5]:

$$a_i cu_i + b_i su_i + c_i = 0 \qquad (1)$$

where

$$a_i = l[\alpha_1 c^2\delta_i + \frac{\alpha_2+\beta_1}{2} s(2\delta_i) + \beta_2 s^2\delta_i];$$
$$b_i = -l(\alpha_3 c\delta_i + \beta_3 s\delta_i); \qquad (2)$$
$$c_i = X_P c\delta_i + Y_P s\delta_i - R + \frac{r}{l} a_i - h(\alpha_3 c\delta_i + \beta_3 s\delta_i)$$

The sinus and cosine functions where abbreviated through “s” and “c”.

After the calculation of the relative rotation angles u_i from the passive revolute joints C_i, it follows the calculation of the absolute coordinates of the guiding points A_i (spherical joints):

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \cdot \begin{bmatrix} (r+l\,cu_i)c\delta_i \\ (r+l\,cu_i)s\delta_i \\ -l\,su_i - h \end{bmatrix} \qquad (3)$$

The joint coordinates are derived as follows:

$$q_i = Y_i c\delta_i - X_i s\delta_i \ ; \ q_{i+3} = Z_i, \quad i=1,2,3. \qquad (4)$$

The nutation angle of the bar A_iC_i in the spherical joint is derived from the following equation:

$$\begin{aligned}\cos\theta_i = \{[(&\alpha_1\, c\delta_i\, cu_i + \alpha_2\, s\delta_i\, cu_i - \alpha_3\, su_i)\, c\delta_i + \\ &(\beta_1\, c\delta_i\, cu_i + \beta_2\, s\delta_i\, cu_i - \beta_3\, su_i)\, s\delta_i]\,(R - r) + \\ &(-\gamma_1\, c\delta_i\, cu_i - \gamma_2\, s\gamma_i\, cu_i + \gamma_3\, su_i)\sqrt{l^2 - (R - r)^2}\ \}/l\end{aligned} \quad (5)$$

4. KINEMATIC SYNTHESIS OF THE 3-TTSR MECHANISM

The synthesis problem of the mechanisms consists of the evaluation of the constant parameters for certain given or imposed conditions [3]. In [4] the mechanism synthesis is defined as the calculus of the structural and geometrical parameters of the mechanisms in order to be able to perform some functional constructive demands, which were established through the design phase. The stages in the basis mechanism synthesis of the manipulators are the followings: structural synthesis, kinematic synthesis and dynamic synthesis. In the kinematic synthesis stage the constant parameters of the mechanism kinematic scheme are found out.

This paper solves the kinematic synthesis depending on the position and the orientation of the end effector in the workspace. It means that the geometric parameters (figures 4 and 5)

R – radius of the basis
r – radius of the mobile platform
l – length of the guiding bars of the mobile platform;
h – length of the gripper;

should be calculated so that the manipulator is able to move the object into a position and an orientation given by the operational coordinates: $X_{P,}\ Y_{P,}\ Z_{P,}\ \psi,\ \theta,\ \nu$.

The dimensional-kinematic synthesis consists of a non-linear programming problem.

We minimize the purpose function: $F(R, r, l, h, q_1, \dots, q_6)$ under the following constraints:

$R_{min} \# R \# R_{max}$;
$r_{min} \# r \# r_{max}$;
$l_{min} \# l \# l_{max}$;
$q_{i,min} \# q_i \# q_{i,max}$; $q_{i,\,min,\,max} = f(R)$
$q_{i+3,min} \# q_{i+3} \# q_{i+3,max}$;
$\cos\theta_i \geq \cos\theta_{i,ma}$.

The optimisation methods are divided into three big categories: null order methods, first order methods (they use the purpose function gradient) and second order methods (they use the gradient and the Hesse matrix [1]).

In the paper the null order method is proposed to be used. The algorithm is as follows:

1. The position and the orientation of the object $X_p, Y_p, Z_p, \psi, \theta, \nu$ are given;
2. The rotational matrix elements $\alpha_1, \dots, \gamma_3$ are evaluated;
3. The existence intervals for the design parameters R, r, l, h are discretizated that leads to a four dimensional hypermatrix;
4. For each parameter vector { R, r, l, h, }, using the inverse geometric mode we verify the constraints

   $a_i^2 + b_i^2 - c_i^2 \exists 0$
   $q_{i,min} \# q_i \# q_{i,max}$; $q_{i,\,min,\,max} = f(R)$
   $q_{i+3,min} \# q_{i+3} \# q_{i+3,max}$
   $\cos\theta_i \geq \cos\theta_{i,ma}$

5. If all the constraints are fulfilled we evaluate the scope function:

$$F = 3(R + l + r) + h + \sum_{i=1}^{3} |q_i| + \sum_{i=1}^{3} q_{i+3}$$

If at least one of these constraints is not fulfilled, we give the function a greater value;

6. Back to the step 4. The iteration follows until all the possible combinations of {R, r, l, h} are considered;
7. From the obtained functions, the smallest function is chosen with the appropriate optimal design parameters R^{opt}, r^{opt} , l^{opt}, h^{opt} and the displacements from the actuated joints: q_i^{opt}, q_{i+3}^{opt}.

This algorithm was implemented in a simulation program. Some simulation results are presented in the table 1.

Table 1

Position [cm]	Orientation [°]	Dimensions				F [cm]
		R^{opt}	r^{opt}	l^{opt}	h^{opt}	
$X_p = 0$ $Y_p = 20$ $Z_p = 55$	$\psi = 0$ $\theta = 0$ $\varphi = 0$	15	5	32,5	12,5	257,3096
$X_p = 0$ $Y_p = 10$ $Z_p = 50$	$\psi = 0$ $\theta = 30$ $\varphi = 0$	17,5	5	27,5	5	277,0324
$X_p = -5$ $Y_p = -15$ $Z_p = 55$	$\psi = 10$ $\theta = 10$ $\varphi = 10$	20	5	32,5	5	278,55
$X_p = 15$ $Y_p = 15$ $Z_p = 50$	$\psi = 0$ $\theta = 0$ $\varphi = 0$	15	12,5	32,5	7,5	270,6471
$X_p = 15$ $Y_p = 15$ $Z_p = 50$	$\psi = 0$ $\theta = 0$ $\varphi = 10$	15	15	32,5	7,5	282,4007
$X_p = -15$ $Y_p = -15$ $Z_p = 50$	$\psi = 0$ $\theta = 0$ $\varphi = 0$	15	7,5	30	12,5	249,5544

For the design variables we considered the following steps: step R = 2.5 cm, step r = 2.5 cm, step l = 2.5 cm, step h = 2.5 cm and the following extreme values:

R_{min} = 5 cm; R_{max} = 30 cm;
r_{min} = 5 cm; r_{max} = 30 cm;
l_{min} = 10 cm; l_{max} = 40 cm;
h_{min} = 5 cm; h_{max} = 15 cm;
$q_{i,min}$ = -(%3 R - 5) cm; $q_{i,max}$ = (%3 R - 5) cm;
$q_{i+3,min}$ = 10 cm; $q_{i+3,max}$ = 30 cm;

The maximum allowed rotational angle in the ball joints was chosen so that $\cos \theta_{i,max}$=0.75.

5. CONCLUSIONS

In this paper a null order optimizing method for the dimensional synthesis of a 3-TTSR manipulator is proposed, considering as optimizing criteria the capacity of the manipulator with a given orientation to attain a certain point in the working space. The particularity of the proposed method is that the limits of the variables q_i are not constant, being depend of the base platform radius "R", another variable during the optimizing process. Also other expressions of purpose functions can be conceived, to illustrate ass good as possible correlation between the manipulator cost and the required energy.

The proposed optimizing method is of medium complexity, less time consumer and can be easily extended to other parallel manipulator types.

It is considered necessary the kinematic optimization of the parallel manipulator mechanisms before a real prototype is build.

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