# CONTRIBUTIONS TO THE DYNAMIC MODELING OF TRTTT SERIAL MODULAR PORTAL ROBOTS

**Ioan Ovidiu Costin*, Iulia Costin****

**Technical University of Cluj-Napoca, Machine Tools and Industrial Robots Department, B-dul Muncii 103-105, Cluj-Napoca, ROMANIA, iocostin@yahoo.com*
***University of Agricultural Sciences and Veterinary Medicine, Mathematics and Computer Science Department, Cluj-Napoca, ROMANIA, icostin@personal.ro*

**Abstract.** In this paper the authors consider a category of portal industrial robot with five degrees of freedom and TRTTT structure. Each module has a single degree of freedom. For this robot, the geometrical-constructive dimensions, the modules' masses respectively the inertial moments are given. Using the Lagrange's dynamic equation of the second form, we establish the five differential equations system that characterizes the robot's dynamic behavior. For this, we found the robot's kinetic energy based on the component modules' kinetic energy and the generalized forces corresponding to the exterior forces and couples. The dynamic equations system allows the solving of the direct and inverse problem for the robot system's dynamics.

**Keywords:** portal robot, dynamics, serial, modular, modeling.

## 1. INTRODUCTION

The portal robots represent the class of the hanged robots, for which the robots' weight force will act oppositely to that of the classical serial robots, built from the base module to the final module. The modular robots allow to achieve separately the modules, and to put them together in order to obtain different configurations. The modularity increases the flexibility level in building various robot structures. The robots that are analyzed in the paper have five degrees of freedom, with TRTTT structure, according to figure 1.

We use the following notations:

$l_{i,i+1}$, $i=\overline{0,5}$ - constructive parameters of the robot, representing the relative distances between the $i$-th coordinate system and the ($i$+1)-th coordinate system (the intersection between the $\overrightarrow{O_3O_4}$ and $\overrightarrow{O_5O_6}$ directions gives the constructive parameters $l_{4c}$ and $l_{c5}$).

$q_i$, $i=\overline{1,5}$ - the generalized coordinates of the robot system

$\dot{q}_i$, $i=\overline{1,5}$ - the generalized velocities of the robot's modules

$\ddot{q}_i$, $i=\overline{1,5}$ - the generalized accelerations of the robot's modules

$m_i$, $i=\overline{1,6}$ - the masses for the robot's modules and for the gripper together with the manipulated object.

$\vec{F}_k$ - the motor forces, that include also the resistant forces corresponding to each translation module $k$

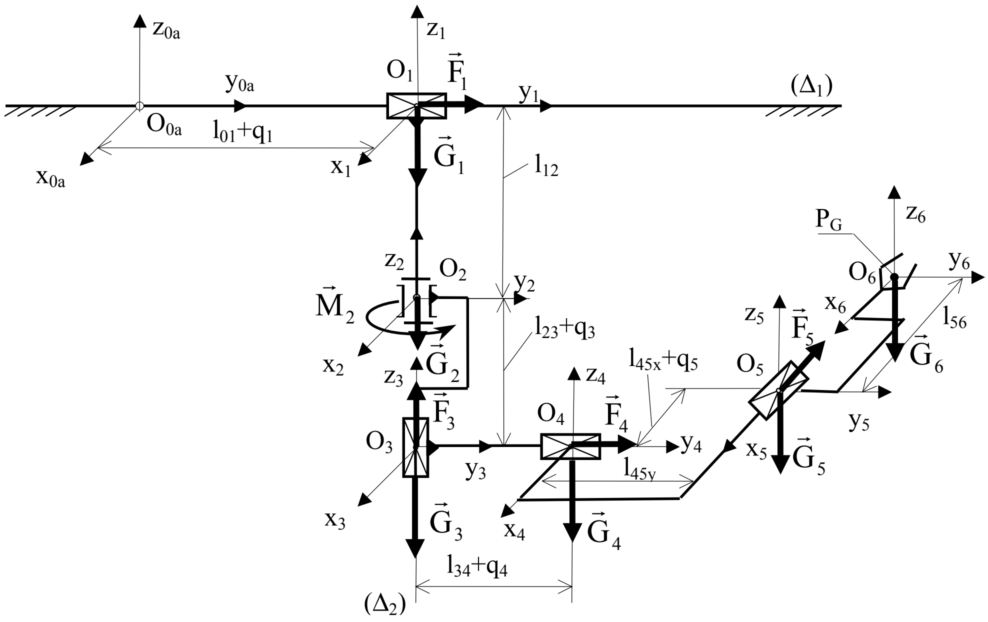


Fig.1.

$\vec{M}_j$ - the motor couples, that include also the resistant couples corresponding to each rotational module $j$

$J_{\Delta_2}^{(i)}$ - the inertial couples of the $i$-th translation module with respect to the rotation axis $\Delta_2$

$J_{z_k}^{(j)},\ j = 4,5$ - the inertial couples of the $j$-th with respect to the axes $z_k$ that pass through $O_k$ and are parallel to $\Delta_2$

$\vec{G}_i,\ i = \overline{1,5}$ - the gravitational forces

In the calculus we made, we considered that the mass of the 5$^{th}$ module, the gripper's mass and that of the manipulated object are concentrated in the origin of the 5$^{th}$ coordinate system ($O_5$).

## 2. THE DYNAMIC MODEL

For obtaining the dynamic model of the two robot structures, the Lagrange equations of the 2$^{nd}$ form are used:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial E}{\partial q_i} = Q_i,\ i = \overline{1,5} \qquad (1)$$

where $E$ represents the kinetic energy, and $Q_i$ represents the generalized forces.

The modules are considered as rigid bodies, and the kinetic energy is determined according to [4], [5], [2].

We consider the following simplifying hypotheses:

- each module of the robot has the reference system's origin in the mass center (which means that $x_c = y_c = z_c = 0$);
- the reference systems' axes are coincident to the principal directions of inertia ($J_{xy} = J_{yz} = J_{xz} = 0$);
- the inertial moments do not modify their values with respect to the gravitational center.

Consequently, the kinetic energy will have the form:

$$E = \tfrac{1}{2} m \cdot \left(v_x^2 + v_y^2 + v_z^2\right) + \tfrac{1}{2}\left(J_x \omega_x^2 + J_y \omega_y^2 + J_z \omega_z^2\right) \qquad (2)$$

For the TRTTT structure, if we apply the relation (2) to each module, we obtain the relations:

$$E_1 = \tfrac{1}{2} m_1 \cdot \dot{q}_1^2$$

$$E_2 = \tfrac{1}{2} m_2 \cdot \dot{q}_1^2 + \tfrac{1}{2} J_{\Delta_2}^{(2)} \dot{q}_2^2$$

$$E_3 = \tfrac{1}{2} m_3 \cdot \left(\dot{q}_1^2 + \dot{q}_3^2\right) + \tfrac{1}{2} J_{(\Delta_2)}^{(3)} \cdot \dot{q}_2^2$$

$$E_4 = \tfrac{1}{2}\left[J_{z_4}^{(4)} + m_4 \cdot (l_{34} + q_4)^2\right] \cdot \dot{q}_2^2 + \tfrac{1}{2} m_4 \cdot \left[\dot{q}_1^2 + \dot{q}_3^2 + \dot{q}_4^2 + \dot{q}_2^2 \cdot (l_{34} + q_4)^2 + 2\dot{q}_1\dot{q}_2(l_{34} + q_4)\sin q_2 + \right.$$
$$\left. + 2\dot{q}_1\dot{q}_4 \cos q_2\right]$$

$$E_5 = \tfrac{1}{2}\left[J_{z_5}^{(5)} + m_5 \cdot \left[(l_{34} + q_4 + l_{4c})^2 + (l_{c5} + q_5)^2\right]\right] \cdot \dot{q}_2^2 + \tfrac{1}{2} m_5 \cdot \left\{\dot{q}_1^2 + \dot{q}_3^2 + \dot{q}_4^2 + \dot{q}_5^2 + \left[(l_{34} + l_{4c} + q_4)^2 + \right.\right.$$
$$\left. + (l_{c5} + q_5)^2\right] \cdot \dot{q}_2^2 + \left[(l_{34} + l_{4c} + q_4)\sin q_2 + (l_{c5} + q_5)\cos q_2\right] \cdot 2\dot{q}_1\dot{q}_2 + 2\dot{q}_1(\dot{q}_4 \cos q_2 + \dot{q}_5 \sin q_2) +$$
$$\left. + 2\dot{q}_2\left[\dot{q}_4 \cdot (l_{c5} + q_5) - \dot{q}_5 \cdot (l_{34} + q_4 + l_{4c})\right]\right\}$$

The total energy of the TRTTT robot contains the kinetic energy for all of the five component modules:

$$E_{TRTTT} = E_1 + E_2 + E_3 + E_4 + E_5$$

Using the notation:

$$c_{11} = m_1 + m_2 + m_3 + m_4 + m_5$$

$$c_{22} = J_{\Delta_2}^{(2)} + J_{\Delta_2}^{(3)} + J_{z_4}^{(4)} + J_{z_5}^{(5)} + 2m_4 \cdot (l_{34} + q_4)^2 + 2m_5 \cdot \left[(l_{34} + q_4 + l_{45y})^2 + (l_{45x} + q_5)^2\right]$$

$$c_{33} = m_3 + m_4 + m_5$$

$$c_{44} = m_4 + m_5$$

$$c_{55} = m_5$$

$$c_{12} = 2m_4(l_{34} + q_4)\sin q_2 + 2m_5\left[(l_{34} + l_{4c} + q_4)\sin q_2 + (l_{c5} + q_5)\cos q_2\right]$$

$$c_{14} = 2(m_4 + m_5)\cos q_2$$

$$c_{15} = 2m_5 \sin q_2$$

$$c_{24} = 2m_5(l_{c5} + q_5)$$

$$c_{25} = -2m_5(l_{34} + l_{4c} + q_4)$$

we obtain

$$E_{TRTTT} = \tfrac{1}{2}\left(c_{11}\dot{q}_1^2 + c_{22}\dot{q}_2^2 + c_{33}\dot{q}_3^2 + c_{44}\dot{q}_4^2 + c_{55}\dot{q}_5^2 + c_{12}\dot{q}_1\dot{q}_2 + c_{14}\dot{q}_1\dot{q}_4 + c_{15}\dot{q}_1\dot{q}_5 + c_{24}\dot{q}_2\dot{q}_4 + c_{25}\dot{q}_2\dot{q}_5\right)$$

The generalized forces and couples $\vec{F}_1, \vec{M}_2, \vec{F}_3, \vec{F}_4, \vec{F}_5$ are obtained by giving virtual displacements such that generalized coordinates $q_i$ should vary with $\delta q_i$ and correspond to the five degrees of freedom.

Through the linking elements, the driving forces and moments act based on the action and reaction principle. For example, in the following way:

Corresponding to the virtual displacements $\delta q_i$ and to the external forces and moments, the expression of the virtual work is:

$$\delta L = \left(\vec{F}_1 + \vec{G}_1\right) \cdot \delta\vec{r}_1 + \vec{G}_2 \cdot \delta\vec{r}_2 + \vec{M}_2 \cdot \delta\vec{q}_2 + \left(\vec{F}_3 + \vec{G}_3\right) \cdot \delta\vec{r}_3 + \left(\vec{F}_4 + \vec{G}_4\right) \cdot \delta\vec{r}_4 + \left(\vec{F}_5 + \vec{G}_5\right) \cdot \delta\vec{r}_5$$

where the $\delta\vec{r}_i$ represents the position vector of the coordinate system's origin (for each $i$ module), with respect to the absolute reference system. In calculus, we apply the principles of the vectorial composition.

After calculations, we obtain:

$$\delta L = (F_1 + F_4 \cdot \cos q_2 + F_5 \sin q_2) \cdot \delta q_1 + M_2 \cdot \delta q_2 + (F_3 - G_3 - G_4 - G_5) \cdot \delta q_3 +$$
$$+ F_4 \cdot \delta q_4 + F_5 \cdot \delta q_5$$

According to (1) and $Q_i = \dfrac{\partial L}{\partial q_i}, i = \overline{1,5}$, we obtain the dynamics equations for the analyzed robot:

$$c_{11}\cdot\ddot{q}_1+\tfrac{1}{2}\frac{d}{dt}\left(c_{12}\dot{q}_2+c_{14}\dot{q}_4+c_{15}\dot{q}_5\right)=F_1+F_4\cos q_2+F_5\sin q_2$$

$$c_{22}\cdot\ddot{q}_2+\tfrac{1}{2}\left(\frac{d}{dt}\left(c_{12}\dot{q}_1+c_{24}\dot{q}_4+c_{25}\dot{q}_5\right)-\frac{\partial c_{12}}{\partial q_2}\dot{q}_1\dot{q}_2-\frac{\partial c_{14}}{\partial q_2}\dot{q}_1\dot{q}_4-\frac{\partial c_{15}}{\partial q_2}\dot{q}_1\dot{q}_5\right)=M_2$$

$$c_{33}\cdot\ddot{q}_3=F_3-\sum_{i=3}^{5}G_i$$

$$c_{44}\cdot\ddot{q}_4+\tfrac{1}{2}\left(\frac{d}{dt}\left(c_{14}\dot{q}_1+c_{24}\dot{q}_2\right)-\frac{\partial c_{22}}{\partial q_4}\dot{q}_2^2-\frac{\partial c_{12}}{\partial q_4}\dot{q}_1\dot{q}_2-\frac{\partial c_{25}}{\partial q_4}\dot{q}_2\dot{q}_5\right)=F_4$$

$$c_{55}\cdot\ddot{q}_5+\tfrac{1}{2}\left(\frac{d}{dt}\left(c_{15}\dot{q}_1+c_{25}\dot{q}_2\right)-\frac{\partial c_{22}}{\partial q_5}\dot{q}_2^2-\frac{\partial c_{12}}{\partial q_5}\dot{q}_1\dot{q}_2-\frac{\partial c_{24}}{\partial q_5}\dot{q}_2\dot{q}_4\right)=F_5$$

Using the dynamics equations, the direct and inverse problems of the dynamics can be analyzed. Therefore, knowing the differential equations for the robot motion and imposing the geometrical-constructive parameters as well as the motion laws on every axis, we can determine the time-function variation laws of the motor forces and couples. Conversely, if we know the geometrical-constructive parameters, the initial conditions for the motion, and the variation laws of the motor forces and couples, we can determine the motion laws.

## 3. CONCLUSIONS

The dynamic equations for the TRTTT robot systems are so complex because they involve many variable components. Knowing the expression of the kinetic energy, as determined in the paper, for component modules as well as for the whole robot, it can be viewed the variation curve of these energies for various laws of gripper motion. This implies to be known the time variation of the generalized coordinates, respectively the generalized velocities and accelerations variation. For the motion along the $q_1$ degree of freedom, a contribution have not only the $\vec{F}_1$ force, but also the $\vec{F}_4$ and $\vec{F}_5$ forces' components. The last two equations from the dynamics equations have similar forms, because they refer to the $q_4$, respectively $q_5$ translations in the same horizontal plane, perpendicular on the vertical motion of the arm, the differences appearing due to the different positions of the two translations with respect to the rotation axis of the robot.

## 4. REFERENCES

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