A&QT-R 2002 (THETA 13)
2002 IEEE-TTTC International Conference on Automation, Quality and Testing Robotics
May 23 – 25, 2002, Cluj-Napoca, Romania

MOVEMENT ANALYSIS OF THE RAPID AUTONOMOUS UNDERWATER VEHICLE

Paul Burlacu, *UM 02003 Constanța, Romania*
Cosmin Bucur, *Ovidius University, Romania*
Mihai Alexa, *CCSFN, Constanta, Romania*
Email: burlacupaul@hotmail.com

Abstract

This paper presents a comprehensive analysis of the dynamics of combined guidance and control laws for autonomous vehicles. The control law is based on vehicle dynamics while the guidance law is based solely on geometry and kinematics.

This paper contents the following chapters: problem formulation, motion equation, linearized system and linear model, state-space synthesis, guidance problem and stability analysis of linear model.

1.Introduction

This article presents a comprehensive analysis of the dynamics of the combined guidance and control laws for autonomous vehicle. The control law is based solely on geometry and kinematics. It is shown that, unless proper conditions are met, the combined scheme may suffer loss of stability.

The need arises to maintain an accurate path keeping in confined spaces and shallow waters under the influence of steady and time varying external forces. In order to deal with the complexity of such unstructured environments, a three layer software organization is utilized, consisting of an organizational layer, a coordination layer, and an execution layer.

At the heart of the execution level there exist the functions of navigation, guidance, and control. The fundamental breakdown of the motion control functions between guidance and control relies on the notion that an autopilot is responsible for stabilizing the motion dynamics of the vehicle in terms of its speed, heading, and depth. The guidance law combines commands for the path and position to be followed and other attitude requirements with navigational estimates of true position and orientation, to generate the speed, heading and depth commands for the autopilot. Such a distinction between guidance and control is necessary in the present aplications since it allows for the required flexibility in selecting the appropriate scheme for the task at hand.

2.Equations of Motion

Restricting our attention to the horizontal plane, the mathematical model consists of the nonlinear sway (translational motion with respect to the vehicle longitudinal axis) and yaw (rotational motion with respect to the vertical axis) equations of motion. In a local (moving) coordinate frame fixed at the vehicle's geometrical center, Newton's equations of motion are

$$m\left(\dot{v}+ur+x_G\dot{r}-y_Gr^2\right)=Y(1)$$

$$I_z\dot{r}+mx_G\left(\dot{v}+ur\right)-my_Gvr=N(2)$$

where v and r are relative sway and yaw velocities of the moving vehicle, Y and N represent the total excitation sway force and yaw moment, respectively, x_G and y_G are the coordinates of the vehicle center of gravity in the body fixed local frame; and m and I_z are the vehicle mass and mass moment of inertia. The nonlinear equations of motion in the horizontal plane become

$$m\left(\dot{v}+ur+x_G\dot{r}-y_Gr^2\right)=Y_{\dot{r}}\dot{r}+Y_{\dot{v}}\dot{v}+Y_rur+Y_vuv-\frac{\rho}{2}\int_{tail}^{nose}C_{D_y}h(\xi)\frac{(v+\xi r)^3}{|v+\xi r|}d\xi+Y_\delta u^2\delta(3)$$

$$I_z\dot{r}+mx_G\left(\dot{v}+ur\right)-my_Gvr=N_{\dot{r}}\dot{r}+N_{\dot{v}}\dot{v}+N_rur+N_vuv-\frac{\rho}{2}\int_{tail}^{nose}C_{D_y}h(\xi)\frac{(v+\xi r)^3}{|v+\xi r|}\xi d\xi+N_\delta u^2\delta(4)$$

where only the coefficients that have nonzero values in the present model have been kept. Y_a, N_a represent partial derivatives of Y and N with respect to the a, C_{Dy} is the drag coefficient, ρ is the water density and δ is the rudder angle. Equations (3) and (4) can be nondimensionalized with respect to the constant forward speed u, and the vehicle length l; the dimensionless time variable being tu/l.

To complete the model, the expressions for the vehicle yaw rate are

$$\dot{\Psi}=r\,(5)$$

and the inertial positions rates are

$$\dot{x}=u\cos\Psi-v\sin\Psi(6)$$

$$\dot{y}=u\sin\Psi+v\cos\Psi(7)$$

The symbols are defined in figure 1. The variable y in (7) represents the cross track error, or the deviation of the vehicle geometrical center off the desired straight line path.

3.Control

A heading angle control law is based on equation (3), (4), (5) which can be written as a set of three nonlinear coupled differential equations in the form

$$\dot{\Psi}=r \qquad (8)$$

$$\dot{v}=a_{11}uv+a_{12}ur+b_1u^2\delta+d_v(v,r) \qquad (9)$$

$$\dot{r}=a_{21}uv+a_{22}ur+ur+b_2u^2\delta+d_r(v,r)\,(10)$$

The coefficients a_{ij} and b_i are functions of the vehicle hydrodynamic and geometric properties . Because $d_v(v,r)=0$ and $d_r(v,r)=0$, effective steering control can be maintained by using the linearized versions of equations (8), (9), (10)

$$\dot{x}=Ax+B\delta\,(11)$$

where the state vector $x=[\Psi,v,r]^T$ has been introduced. Linear full state feedback is introduced in the form

$$\delta=k_1(\Psi-\Psi_c)+k_2v+k_3r(13)$$

where Ψ_c is the commanded heading angle.

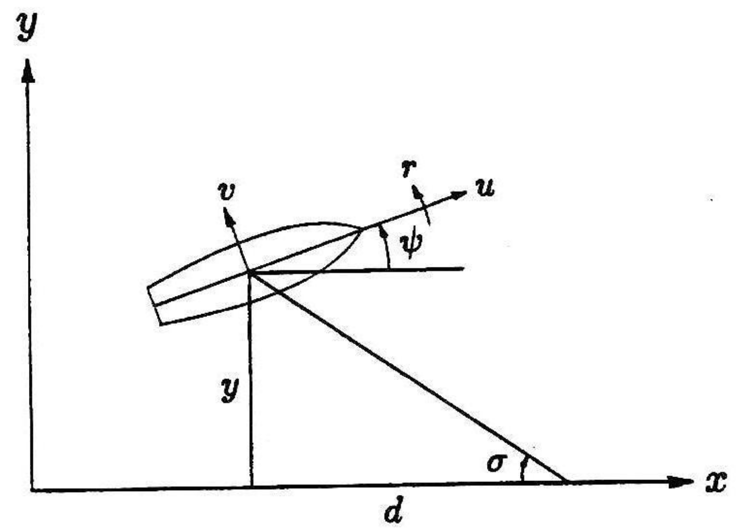

Figure 1. Vehicle geometry and definitions of symbols.

The general form of the desired characteristic equations is,

$$\lambda^3 + \alpha_1 \lambda^2 + \alpha_2 \lambda + \alpha_3 = 0 \quad (13)$$

and the controller gains are computed from

$$k_1 = \frac{a_3}{(b_2 a_{11} - b_1 a_{21}) u^3}$$

$$(b_1 a_{22} - b_2 a_{21}) u^3 k_2 + (b_2 a_{11} - b_1 a_{21}) u^3 k_3 = \alpha_2 - (a_{11} a_{22} - a_{12} a_{21}) u^2 + b_2 u^2 k_1$$

$$b_1 u^2 k_2 + b_2 u^2 k_3 = -\alpha_1 - (a_{11} + a_{22}) u$$

Due to the explicit dependence on u, the gains are continuously updated for different nominal forward speeds.

A turning rate linear rudder feedback control law based on the linearized set of equations (9) and (10),

$$\dot{v} = a_{11} u v + a_{12} u r + b_1 u^2 \delta \quad (14)$$

$$\dot{r} = a_{21} u v + a_{22} u r + b_2 u^2 \delta \quad (15)$$

is

$$\delta = k_v v + k_r r \quad (16)$$

where k_v, k_r are feedback gains. By substituting (16) into (14) and (15) we can find the closed loop characteristic equation

$$\lambda^2 + A_1 \lambda + A_2 = 0 \quad (17)$$

where

$$A_1 = -[a_{11} a_{22} + (b_1 k_v + b_2 k_r) u] u$$

$$A_2 = [a_{11} a_{22} - a_{12} a_{21} + (b_1 a_{22} - b_2 a_{12}) u k_v + (b_2 a_{11} - b_1 a_{21}) u k_r] u^3$$

If the desired characteristic equation is

$$\lambda^2 + \alpha_1 \lambda + \alpha_2 = 0 \quad (18)$$

we can equate the coefficients of (17) and (18) and get the following system of linear equations

$$k_v b_1 u^2 + k_r b_2 u^2 = -\alpha_1 - (a_{11} + a_{22})u$$

$$k_v (a_{22} b_1 - a_{12} b_2)u^3 + k_r (a_{11} b_2 - a_{21} b_1)u^3 = \alpha_2 - (a_{11} a_{22} - a_{12} a_{21})u^2$$

to be solved for the gains k_v and k_r.

The control law (16) guarantees stability of $v=r=0$ of (14) and (15)(straight line motion at an arbitrary heading). When the commanded angular velocity r_c is nonzero the control law is slightly modified to

$$\delta = k_v v + k_r r(r - r_c + k_c r_c) \quad (19)$$

where k_c is the feedforward gain. The feedback gains k_v, k_r remain the same since $d_v(v,r)=0,\ d_r(v,r)=0$ and, therefore, the liniarized dynamics of (9) and (10) around r_c do not differ significantly from (14) and (15). The feedforward gain k_c is computed based on steady state accuracy requirements. At steady state, equations (14) and (15) yield

$$v = \frac{b_1 a_{22} - b_2 a_{12}}{b_2 a_{11} - b_1 a_{21}} r_c, \quad \delta = \frac{a_{21} a_{12} - a_{11} a_{22}}{(b_2 a_{11} - b_1 a_{21})u} r_c \quad (20)$$

Substituting (20) into (19) and requiring that $r=r_c$ at steady state we can solve for k_c and finally write the control law (19) in the form

$$\delta = k_v v + k_r r - k_0 \alpha_2 r_c \quad (21)$$ where

$$k_0 = \frac{1}{(b_2 a_{11} - b_1 a_{21})u^3} \quad (22)$$

With the above feedforward gain, the control law is complete. Again, as before, all gains k_v, k_r, k_0 are continuously updated every time a different forward speed is commanded. The feedforward gain k_0 computed from (22) ensures that the steady state turning rate r equals the commanded value r_c for the linear system (14) and (15). In general, we can see from (9) and (10) that at steady state $r \neq r_c$ unless $d_v=d_r=0$. As the control law becomes tighter, the steady state error $|r-r_c|$ will be smaller.

4.Guidance

The previous control laws must be accompanied by a suitable guidance law. The guidance law must be compatible with the control law selection; for example, heading angle guidance law must be coupled with a heading control law. The simplest heading guidance law is a pure pursuit navigation. In figure 1 we can see :

$$\tan\sigma = -\frac{y}{d} \quad (23)$$

Pure pursuit navigation then corresponds to taking

$$\Psi_c = \sigma \quad (24)$$

as the commanded heading angle in the control law (12).

Instead of equating the command heading angle Ψ_c to the line of sight angle σ as in the pure pursuit navigation case, equation (24), a first order lag can be introduced as in

$$\dot{\Psi}_c = k_{\dot{\sigma}} \dot{\sigma} + k_\sigma (\sigma - \Psi) \quad (25)$$

In order to compute the coefficients $k_{\dot{\sigma}}$ and k_σ based on kinematics only while keeping the vehicle dynamics in the background, we take $\Psi_c = \Psi$ and $v=0$ and then equation (7), (23), and (25) yield

$$\dot{\Psi}_c = k_{\dot{\sigma}} \frac{d}{d^2 + y^2} u \sin \Psi_c - k_\sigma \left(\tan^{-1} \frac{y}{d} + \Psi_c \right) (26)$$

$$\dot{y} = u \sin \Psi_c (27)$$

The characteristic equation of (26) and (27) is

$$d\lambda^2 + \left(k_\sigma d + k_{\dot{\sigma}} u \right) \lambda + k_\sigma u = 0 (28)$$

If the desired guidance characteristic equation is

$$\lambda^2 + \beta_1 \lambda + \beta_2 = 0 (29)$$

the proportional navigation gains are computed from

$$k_\sigma = \frac{\beta_2 d}{u} (30)$$

$$k_{\dot{\sigma}} = \frac{\beta_2 du - \beta_2 d^2}{u^2} (31)$$

A guidance law to accompany turning rate control is based solely on kinematics, whereas vehicle dynamics are handled by the rudder control law, as before. Guidance law development is therefore based on

$$\dot{\Psi} = r_c (32)$$

$$\dot{y} = u \sin \Psi (33)$$

where r_c is the commanded turning rate and the lateral velocity v is assumed to be zero in (33). Cross track error guidance is achieved by

$$r_c = k_\Psi \Psi + k_y y (34)$$

The closed loop characteristic equation is of the same form as (29), the guidance law gains k_Ψ, k_y are obtained by equating the coefficients of (29), and (35)

$$k_\Psi = -\beta_1, \quad k_y = -\frac{\beta_2}{u} (36)$$

5.Stability

The complete system is given by the heading equation (8), (9), (10), the kinematic relation (7), the control law (12), and the navigational equations (23), (24). It can be easily verified that the trivial equilibrium state which corresponds to a straight line motion is characterized by

$$\Psi = v = r = y = 0 (37)$$

Linearization of the state equation in the vicinity of (37) produces the linear system

$$\dot{x} = Ax (38)$$

where the complete state vector is

$$x = [\Psi, v, r, y]^T (39)$$

Local stability properties of (38) are established by the eigenvalues of A. We get

$$\lambda^4 + (-B_1 - C_2)\lambda^3 + (-D_1 + B_1 C_2 - C_1 B_2 - C_1 B_2 - A_2)\lambda^2 + (-C_1 D_2 + D_1 C_2 - uD_2 - A_1 B_2 + A_2 B_1)\lambda$$
$$+ (uB_1 D_2 - uD_1 B_2 - A_1 D_2 + A_2 D_1) = 0 (40)$$

where

$$A_1 = b_1 u^2 k_1, B_1 = a_{11}u + b_1 u^2 k_2, C_1 = a_{12}u + b_1 u^2 k_3, D_1 = b_1 u^2 k_1 \frac{1}{xd}$$

$$A_2 = b_2 u^2 k_1, B_2 = a_{21}u + b_2 u^2 k_2, C_2 = a_{22}u + b_2 u^2 k_3, D_2 = b_2 u^2 k_1 \frac{1}{xd}$$

Loss of stability of equation (40) occurs when

$$BCD - B^2E - AD^2 = 0(41)$$

where

$$A = 1$$

$$B = -B_1 - C_2$$

$$C = -D_1 + B_1C_2 - C_1B_2 - A_2$$

$$D = -C_1D_2 + D_1C_2 - uD_2 - A_1B_2 + A_2B_1$$

$$E = uB_1D_2 - uD_1B_2 - A_1D_2 + A_2D_1$$

Next step, we obtain after some algebra

$$a_1d^2 + a_2d + a_3 = 0(42) \text{ where}$$

$$a_1 = \alpha_1\alpha_2 - \alpha_3$$

$$a_2 = \frac{(\alpha_1\alpha_2 - 2\alpha_3)(b_1a_{22} - b_2a_{12} - b_2)}{b_2a_{11} - b_1a_{21}} - \frac{b_1\alpha_1\alpha_3}{(b_2a_{11} - b_1a_{21})u} - \alpha_1^2 u$$

$$a_3 = \frac{-(b_1a_{22} - b_2a_{12} - b_2)[b_1\alpha_1(b_1a_{22} - b_2a_{12} - b_2)u]\alpha_3}{(b_2a_{11} - b_1a_{21})^2 u}$$

The positive root of the quadratic equation (42) determines the critical visibility d_{crit} for stability. For $d > d_{crit}$ the equilibrium state (37) is stable which means that the control law will drive and keep the vehicle onto the straight line path. For $d < d_{crit}$ the equilibrium state loses its stability and the vehicle response become oscillatory as a result of the pair of complex conjugate eigenvalues with positive real parts.

6.Conclusions

An analytic investigation of nonlinear dynamic response characteristic of pursuit guidance coupled with orientation control law of rapid marine vehicles has been presented. There exists a critical preview distance d_{crit} for stability of straight-line motion. For $d < d_{crit}$ the nominal equilibrium state loses its stability and the response admits oscillatory characteristics.

References

Papoulias Fotis, [1995], Nonlinear Dynamics and Bifurcations in Autonomous Vehicle Guidance and Control, *Underwater Robotic Vehicles: Design and Control*

Fotis Papoulias, Venne D.V., Cumrnings G.P., [1994], Effects of positional information time lags on autonomous vehicle guidance and control, *International Shipbuilding Progress.*