MONTE CARLO SIMULATION USING EXCEL FOR PREDICTING
RELIABILITY OF A DANGER CONTROL SYSTEM

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Abstract: A Monte-Carlo Simulation using Excel Spreadsheet has been used for determining the reliability of a security system. This simulation technique utilizes the powerful mathematical and statistical capabilities of Excel. Simulation time is dependent on the complexity of the system, computer speed, and the accuracy desired, and may range from a few minutes to a few hours.

Key Words: Modeling. Simulation, Monte Carlo method, Reliability Estimate.

1   INTRODUCTION

A reliability prediction [1] of a complex system is done by transforming the physical and functional block model of the system into a functional RBD. Units or modules are placed in series if failure of one results in system failure. If redundant units are available, they are placed in parallel and system failure occurs only when the number of module failures exceeds a given number, say 'm' of the 'n' units present in the system. Very often, each of the blocks present in the RBD may be comprised of units placed in series, parallel or a combination of both. Moreover, the system may be so complex that it may not be possible to render its RBD as a conventional network of series / parallel modules. Under such circumstances, a formula for the system reliability may be difficult, if not impossible to derive. Use of Bayes decomposition technique may become extremely cumbersome and time consuming requiring several hours, or even days, of derivations and/or calculations. The possibility of introducing human error during derivations further aggravates the issue. There may also be a desire to perform studies of reliability performance; such as calculation of reliability over time (instead of at a point in time) or determining failure distributions. Under such circumstances, Monte Carlo simulation [3],[5] is the only time effective way of estimating reliability.

2   GENERATION OF TIME TO FAILURE

Monte Carlo simulation relies on pseudo random numbers to generate random times to failure based on a failure distribution. Excel contains a pseudo random number generator that was tested for sufficiency [3]. The function is invoked using the Excel function =RAND(). When this function is entered in a cell in an Excel spreadsheet, it

generates a uniformly distributed pseudo random number between 0 and 1. Its value can be easily updated by pressing the Calculation Key F9.

Distribution	PDF or f(t)	Excel Expression for time to failure
Exponential	$\exp(-\lambda t)$	- LN [RAND()] / λ λ = failure rate
Weibull	$\dfrac{\beta}{\alpha}\left(\dfrac{t}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\dfrac{t}{\alpha}\right)^{\beta}\right\}$	α {-LN(1- RAND()}^(1/β) α = scale parameter, β = shape parameter
Normal	$\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\dfrac{1}{2}\left(\dfrac{t-\mu}{\sigma}\right)^{2}\right\}$	NORMINV(RAND(), μ,σ) $\mu=mean,\ \sigma=\mathrm{var}iance$
Uniform	1/T	T x RAND() T = max time
Lognormal	$\dfrac{1}{\sigma t\sqrt{2\pi}}\exp\left\{-\dfrac{1}{2}\left(\dfrac{\ln t-\mu}{\sigma}\right)^{2}\right\}$	LOGINV(RAND(), μ,σ) $\mu=mean,\ \sigma=\mathrm{var}iance$

Figure 1

Using Excel's pseudo random number generator, it is possible to generate random numbers having any other distribution (Figure 1).
So, let f(x) be the probability density function (PDF) of a random variable X_i, and $F(x_0)$ its cumulative distribution function (CDF).

$$F(x_0) = \int_{0}^{x0} f(x)dx$$

While x may take any value, F(x), the cumulative distribution function (CDF), ranges from 0 to 1. If we assign the uniformly distributed random number to F(x) and solve for x, we will arrive at a random number x having PDF = f(x) [6].
Figure 1shows the Excel expressions for time to failure for various distributions of interest in reliability.

3   MONTE CARLO ESTIMATION SPREADSHEET

In a series system, two modules A and B are in series and failure of any one results in system failure. We enter the failure times T_A and T_B of module A and module B in Cell 1 and Cell 2 (in a row) respectively. Cell 3 contains the mission time T_0.   he mission is successful if the following expression yields a '1':

= IF (AND (($T_A > T_0$), ($T_B > T_0$)), 1, 0)   (1)

We enter this expression in Cell 4. The above expression ascertains that '1' is entered in the Cell 4 if the failure times of both modules are greater than T_0. If either one is less than T_0, a '0' is entered in Cell 4. Additional rows of these columns represent additional Monte Carlo runs. We estimate the reliability by generating N= 10000 rows of such data, counting the 1's in column 4, and then dividing by N.

For parallel redundancy, at least one (instead of both) of the two modules A and B connected in parallel needs to survive the mission time T_0. Hence, the result cell contains the expression

= IF (OR (($T_A > T_0$), ($T_B > T_0$)), 1, 0)   (2)

For an M of N active redundancy, additional columns are added to represent the N items. We enter expressions for failure times for each of the "N" items and then examine the number of cells that contain failure time $> T_0$. If this number is "M" or more, the mission is successful. If this number is less than "M", it is a failure. Thus the excel expression to be entered in the cell containing the result takes the following form:

$$= IF((IF((T_A > T_0),\ 1, 0) + IF((T_B > T_0),\ 1, 0) + ... + IF((T_N > T_0),\ 1, 0)) >= M),\ 1, 0)$$

(3)

Where, $T_A, T_B, ..., T_N$ are the failure times of modules A, B, & N respectively.

In a standby redundant electronic system, the standby module B is powered off and hence the failure rate associated with it is different from the failure rate associated with the active module A

The implementation of the above system (one active module & one in standby mode) in Excel spreadsheet can be done as follows (shown in Figure 2):

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
λ (fits)->	7500	750	7500		
Component Time to Failure	Mod-A Active Failure Time	Mod-B Standby Failure Time	Mod-B Active Failure Time	Mission Time T0 (Hours)	Pass/Fail
Run 1	7.49E+04	1.76E+06	3.82E+04	100,000	1
Run 2	1.42E+04	1.76E+05	7.31E+04	100,000	0

Figure 2

- C1: contains an exponentially distributed random number corresponding to failure time T_1 of active module-A having a failure rate = λ
- C2: contains failure time T_2 of standby module-B having a failure rate = $\rho\lambda$, where ρ is the ratio of standby failure rate to active failure rate.
- C3: contains failure time T_3 of standby module B having a failure rate = λ, after it is turned ON (upon failure of the active module A)
- C4: contains the value of mission time T_0, (a constant).
- C5: contains the logical expression for success. '1' stands for TRUE or mission success and '0' stands for FALSE or system failure.

The expression for success is:

$$= IF\ (OR\ ((IF\ (T_1 > T_0),\ AND\ ((T_1 < T_0), (T_2 > T_1), ((T_1 + T_3) > T_0))),\ 1, 0)$$

(4)

10000 rows reprezenting Monte Carlo runs are filled with failure times generated in the above-mentioned fashion. The sum of all successes divided by the total number of rows gives the reliability of the standby system.

Instead of duplicating rows to represent successive Monte Carlo runs, a single row that describes the system can be used and recalculated a number of times. Macros can be created for automating re-calculation, variation, of parameters, and collection of results (Figure 3).

```
Sub Macro()
For N = 1 To 10000 (*/ This statement is entered manually)
        Calculate
        Range("AA15:AH15").Select (*/ These cells contain results of simulation)
        Selection.Copy
        ActiveCell(N, 5).Select
```

Selection.PasteSpecial Paste:=x1Values,Operation:=x1None, SkipBlanks:= _
False, Transpose:=False (*/ Paste results 5 columns away, in same rowgiven by N)
Application.CutCopyMode = False
Next N (*/ Repeat, stop after 10,000 runs)
End Sub

Figure 3

After all 10,000 simulations are done, we transfer the results into another spreadsheet in the same workbook for further processing and generating of Excel charts. This is accomplished by the following Macro.

```
Sub Macro()
        Range("AA15:AL10014").Select (*/ These cells contain results of 10,000 runs)
        Selection.Copy
        Sheets("Sheet2").Select
        Range("A2").Select
        Selection.PasteSpecial                           Paste:=x1Values,
Operation:=x1None, SkipBlanks:=
        False, Transpose:=False (*/ Copy results of 10,000 runs to Sheet2)
        Application.CutCopyMode = False
End Sub
```

Figure 4

Excel's built-in functions allow simple statistical evaluation of the results and the use of Excel's plotting capabilities allows data to presented graphically for increased understanding.

4   MONTE CARLO SIMULATION PREDICTING RELIABILITY OF A DANGER CONTROL SYSTEM

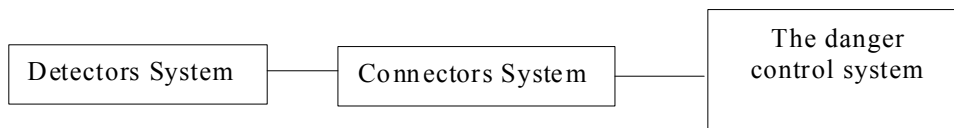In Figure.5. we have the security system RBD [2].

Figure 5.

where the detectors system is a serie-parallel conncetion, connectors system is a serie connection and danger control system is a 2 of 3 connection (or 3 of 4 connection). We take for analyze a detector system with 100 detectors and the total number of runs was 10000. The reliability, that is the percentage of successful runs recorded in the simulation was calculated by using the AVERAGE function applied to the columns where were recorded the results of individual Monte Carlo runs. In our case, the resulting reliability was: 97,435%.

Since simulation gives only an estimate and not an exact value, it is necessary to know the confidence interval of the estimate. This is easily accomplished in Excel with the following expression:

Confidence Interval = CONFIDENCE (α, σ, N)   (5)

Where:

α = 1 – confidence interval (for 95% conf. interval, α = 0.05)

σ = Standard deviation of outcomes of N trials.

N = the number of simulations for which outcomes were recorded.
We obtained a confidence value of: 1,98%

In a system where failure is defined in terms of M-of-N units, one can record the number of unit failures experienced in each run in addition to recording success or failure of the system as a whole. The distribution of failures is computed and plotted using "*Tools*", "*Data Analysis*" and "*Histogram*" menu.

Each of these features allow the binning of failure frequencies, i.e. how many runs result in 0, 1, 2, 3, 4, etc. failures. The plot of the percentage of runs experiencing each quantity of failure is the PDF. The CDF is generated by plotting the percentage of runs against the cumulative number of failures.
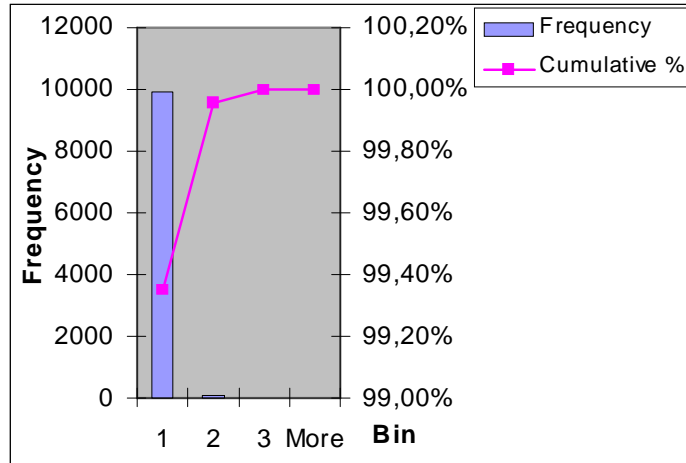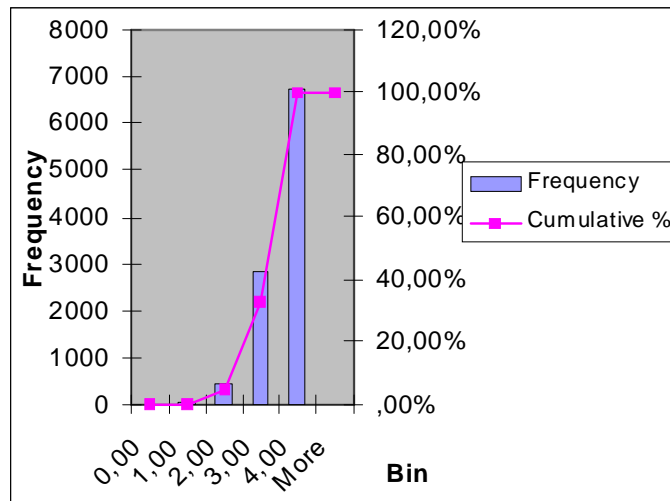

Figure 6

Figure 7

So, if the security system is a 2 out of 3 connection, the PDF and CDF we obtained is in Figure 6, else, if it is a 3 out of 4 connection, the PDF and CDF generated during the simulation are represented in Figure 7.

5   CONCLUSIONS

The techniques presented allow reliability estimation of very complex systems.The Functional Reliability Block Diagram (RBD) of the system under investigation is first

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transformed into a table in an Excel Spreadsheet. Each cell within the table corresponds to a specific block in the RBD. Formulae for failure times entered into these cells are in accordance with the failure time distribution of the corresponding block and can follow exponential, normal, lognormal or Weibull distribution. The Excel pseudo random number generator is used to simulate failure times of individual units or modules in the system. Logical expressions are then used to determine system success or failure. Excel's macro feature enables repetition of the scenario thousands of times while automatically recording the failure data. Excel's graphical capabilities are later used for plotting the failure probability density function (PDF) and cumulative distribution function (CDF) of the overall system. These plots can be used by the Reliability Engineer to understand failure performance in addition to being able to estimate reliability.

By applying for 10000 runs, this Monte Carlo technique for predicting the reliability for a complex danger control system the reliability obtained was: 97,435.

Additional reliability studies may be performed by modifying or adding Excel cells to collect the desired information. For example, if one desires to calculate the reliability as a function of time, the spreadsheet values for Mission Time (T_0) would be modified to accomplish this.

REFERENCES

1) Kececioglu, D., (1991), Reliability Engineering Handbook, *Vol. 2, Prentice Hall*, pp.66-80
2) Corneliu Popescu, (2001), Contributii privind restera testabilitatii si fiabilitatii sistemelor de Securitate, *teza de doctorat* pp.101-103
3) Law, A. M., Kelton, D.W., (1991), Simulation Modeling and Analysis, *McGraw Hill Series in Industrial Engineering & Management*, pp.78-80
4) Popescu, C., Popescu, D., (2001), Fiabilitatea i testabilitatea sistemelor, *Editura Matrix,* pp.70-75
5) Sobol, I. M., (1994), A Primer for the Monte Carlo Method, *CRC Press* , pp.30-33
6) Meyer, S. L., (1975), Data Analysis for scientists and engineers, *John Wiley & Sons, Inc.* pp.123-125