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ADJOINT SENSITIVITY AND UNCERTAINTY ANALYSIS FOR RELIABILTY/AVAILABILITY MODELS, WITH APPLICATION TO THE INTERNATIONAL FUSION MATERIALS IRRADIATION FACILITY

D. G. Cacuci and M. Ionescu-Bujor

Forschungszentrum Karlsruhe
Institute for Reactor Safety
Hermann-von-Helmholtz-Platz 1
76344 Eggenstein-Leopoldshafen, Germany

ABSTRACT: This work illustrates the use of the Adjoint Sensitivity Analysis Procedure (ASAP) for performing sensitivity and uncertainty analysis of the preliminary design of the projected International Fusion Materials Irradiation Facility (IFMIF) facility. The preliminary results obtained here indicates that considerable work is still needed to attain the level of performance envisaged for IFMIF.

Keywords: IFMIF, Adjoint Sensitivity/Uncertainty Analysis

1 INTRODUCTION

There is a wide-based international consensus that the qualification of materials in a test environment close to the conditions predicted in fusion reactors is indispensable not only for the construction and operation of such a device, but also for the calibration of data generated from material irradiation and damage in fission reactors and accelerators. Therefore, an international design team with members from the European Union, the United States of America, Japan and the Russian Federation have been working since 1994, under the auspices of the International Energy Agency, on the reference design for the International Fusion Materials Irradiation Facility (IFMIF), based on an accelerator-driven Deuterium-Lithium stripping source.

The three-dimensional artist's view of IFMIF is presented in Figure 1, below. The IFMIF project [1] is organized into five subsystems: (1) accelerator facilities to produce and transport accelerated deuterons; (2) target facilities, which provide a flowing Lithium-jet to convert deuterons into neutrons; (3) test facilities for irradiating, handling and examining specimens; (4) conventional facilities; and (5) a central control and common instrumentation facility; the estimated costs for building the IFMIF Plant are of the order of 700 million US Dollars.

As is well known, measuring devices have a finite accuracy; hence, an actual system cannot be identified exactly. Furthermore, theoretical concepts cannot be implemented exactly because of manufacturing tolerances. The time-behavior of a real system may change unpredictably because of environmental, material property, or operational influences. Even if a mathematical model were an exact representation of physical reality, the computational methods needed to solve mathematical models introduce themselves a variety of numerical approximations. The issues enumerated in the foregoing illustrate clearly the major sources of uncertainties in models and experiments, and also underscore the need for systematic sensitivity and uncertainty analyses. The objective of **uncertainty analysis** is to calculate the PDF (or its moments) of the system's response (e.g., reliability or failure probability) when the PDF's (or moments) of the system parameters (e.g., input data) and modeling uncertainties are considered known. On the other hand, the main objectives of **sensitivity analysis** are to: (i) understand the system by highlighting important data; (ii) eliminate unimportant data; (iii) determine effects of parameter variations on system behavior; (iv) design and optimize the system (e.g., maximize availability/minimize maintenance); (v) reduce over-design; (vi) prioritize the improvements effected in the respective system; (vii) prioritize introduction of data uncertainties.

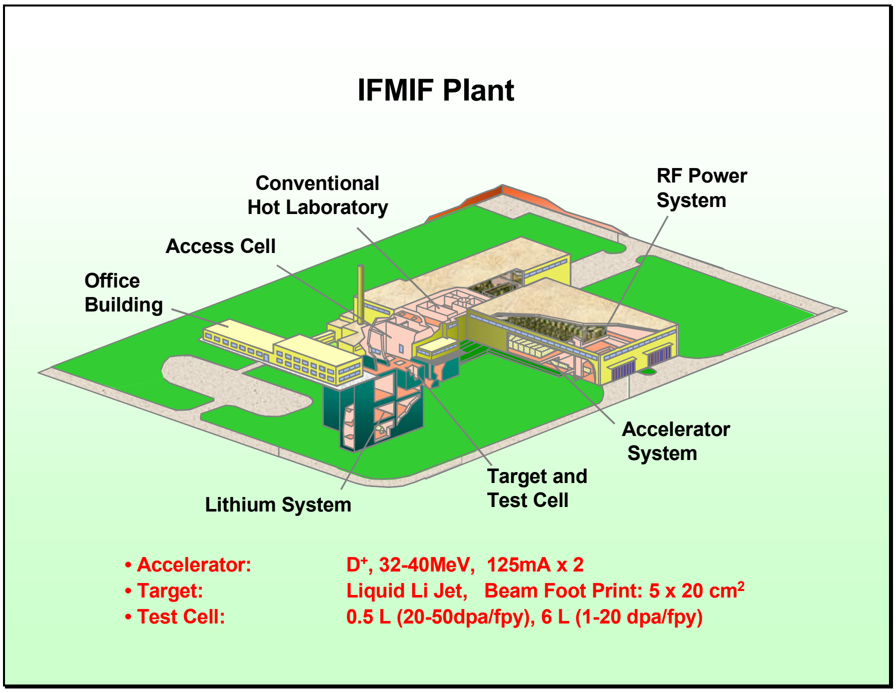

Fig. 1: Artist's view of IFMIF

2 RELIABILITY, AVAILABILITY, MAINTAINABILTY AND INSPECTABILITY (RAMI) OF IFMIF

The Reliability/Availability/Maintainability/Inspectability (RAMI) procedure is employed to optimize the design of IFMIF. For the RAMI procedure, the parameters x_i, $i=1,\ldots,m$, that enter a reliability and/or risk model are considered random variables distributed according to given probability distribution functions (PDF's). Hence, the reliability or availability of a system will itself be a random variable (since it is a function of random variables), and an uncertainty band must surround every point estimate of a demand failure or of any of the parameters that specify a time-dependent failure probability of a component. RAMI is employed for: (a) establishing reliability and maintainability requirements at the subsystem and component levels; (b) identifying system sensitivities to RAMI uncertainties; (c) influencing the level of design redundancy; (d) estimating the contribution of maintenance to the life cycle cost (spares and replacements); and (e) identifying the areas for potential technology development.

There are two major RAMI modeling steps; the first RAMI step involves a "Top-Down Analysis", which comprises (i) identification of the major subsystems, (ii) breaking up each major subsystem in its constitutive assemblies, (iii) breaking up each assembly in sub-assemblies, and, finally (iv) breaking up each sub-assembly in its components. The second RAMI step involves a "Bottom-Up Synthesis", which comprises: (i) determination of RAMI values for individual components, (ii) calculation of RAMI values for the sub-assembly, using the values for each of its components, (iii) continuation of RAMI calculations at successively higher-level structures (by considering the structures at each level as the components for the next-higher-level structure), and, finally (iv) calculation of the TOP-level RAMI values, for the highest-level system. Note that, at each level, Markov-type models are usually used to calculate RAMI values for the respective components, sub-structures and successively higher-level structures.

The Fault Tree diagram and the estimated parameters for the five (5) subsystems comprising the IFMIF accelerator systems are provided in Figure 2 and Table 1, respectively. (Note: MTTF = mean time to failure; MTTR = mean time to repair; A = availability; R = reliability; HEBT = High Energy Beam Tubes)

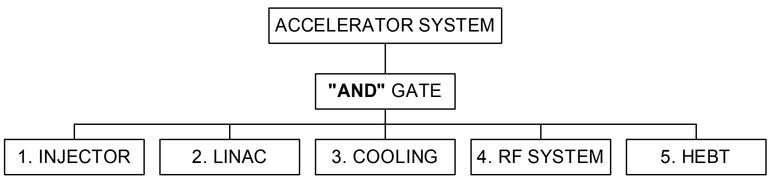

Figure 2. IFMIF Accelerator Fault Tree

	1. INJECTOR	2. LINAC	3. COOLING	4. RF SYSTEM	5. HEBT
MTTF [hrs.]	156.7	465.0	500 000	225.5	224.5
MTTR [hrs.]	2.2	19.0	4.0	9.0	7.7
A	0.9866	0.9623	1.0000	0.9630	0.9640
R	0.3479	0.7885	0.9997	0.4942	0.4936

Table 1. Estimated RAMI Data for IFMIF Component Systems

3 ILLUSTRATIVE EXAMPLE: ADJOINT SENSITIVITY AND UNCERTAINTY ANALYSIS OF IFMIF ACCELERATOR AVAILABILITY

For large-scale systems such as IFMIF, it is **not** possible to find an explicit, analytical solution for the reliability PDF(R), so the "Moment Matching" Method is used to approximate it, as follows: (i) given the PDF's or the (central) moments $m_j(x_i)$, i=1,…,m, of the input data, calculate the first **four** (central) moments $m_1(R)=E(R)$, $m_2(R)$, $m_3(R)$, $m_4(R)$ of the response; and (ii) approximate the unknown PDF(R) by $P(R;\beta;\gamma)$, a two-parameter PDF, that has the same four moments, $m_i(R)$, i=1,...,4, as calculated above. Note that $m_1(R)$ and $m_2(R)$ determine the two parameters β & γ of $P(R;\beta;\gamma)$, while $m_3(R)$ and $m_4(R)$ determine the shape (skewness and kurtosis) of $P(R;\beta;\gamma)$

The current state-of-the-art methods for obtaining the moments $m_i(R)$ of PDF(R) are Monte Carlo Method and/or the Propagation of Moments (Taylor-Series) Method.

Within the Monte Carlo Method, simulations are employed to: (i) generate randomly a sample of N m-tuples $\{z_{ij}\}$ for i=1,2,...,m, and j=1,2,...,N, where z_{ij} denotes the j^{th} random value of the i'th input variable, x_i; (ii) solve the reliability model N-times for each m-tuple $\{z_{ij}\}$ to obtain a sample of N values of the "reliability"; (iii) from this sample, **estimate** the moments $m_i(R)$, i=1,...,4, confidence limits, and the approximate PDF, P(R;x;y), of the reliability. R. Clearly, the main advantages of using the Monte Carlo Method are: (ii) it is conceptually easy to use and (ii) it requires little additional modeling. However, the Monte Carlo Method brings with it at least two major, inherent, disadvantages, as follows: (i) since many thousands of Monte Carlo simulations are needed, Monte Carlo simulations are at best expensive (for small systems), or, at worst, impracticable (e.g., for large **time-dependent** systems); and (ii) since the response sensitivities and parameter uncertainties are amalgamated, improvements in parameter uncertainties **cannot** be directly propagated to improve response uncertainties; rather, the entire set of simulations must be repeated anew !

In the Propagation of Moments (Taylor-Series) Method, the multivariate response (e.g., reliability) $R(x_1,\ldots,x_m)$ is expanded in a Taylor series around $\bar{x} \equiv (\bar{x}_1,\ldots,\bar{x}_m)$; then, the mean, [E(R)], variance, $[m_2(R)]$, skewness, $[m_3(R)]$, and kurtosis, $[m_4(R)]$, of the response R are calculated by integrating the Taylor series over the respective multivariate PDF's. For example, if the parameters x_i's are uncorrelated, then, to first order, the Propagation of Moments Method yields:

$$E(R)=R(\bar{x}_1,\ldots,\bar{x}_m);\ m_2(R)=\sum_{i=1}^{m}\left\{\left(\frac{\partial R}{\partial x_i}\right)^2\right\}_{\bar{x}} m_2^i;\ m_3(R)=\sum_{i=1}^{m}\left\{\left(\frac{\partial R}{\partial x_i}\right)^3\right\}_{\bar{x}} m_3^i$$
$$m_4(R)=\sum_{i=1}^{m}\left\{\left(\frac{\partial R}{\partial x_i}\right)^4\right\}_{\bar{x}}\left[m_4^i-3\left(m_2^i\right)^2\right]+3\left[m_2(R)\right]^2 \qquad (1)$$

Major *advantages* of using the "Propagation of Moments" Method are: (i) if all sensitivities are available, then *all of the objectives of sensitivity analysis* (enumerated above) can be pursued efficiently and exhaustively; and (ii) since the *response sensitivities and parameter uncertainties* are obtained *separately from each other*, improvements in parameter uncertainties can immediately be propagated to improve the uncertainty in the response, without the need for expensive model recalculations. The *major disadvantage of the "Propagation of Moments" Method is that the sensitivities need to be calculated a priori; such calculations are* ***extremely expensive****, particularly for large (and/or time-dependent) systems, unless they are performed by using the Adjoint Sensitivity Analysis Procedure (ASAP), as originally developed by Cacuci [2,3] for nonlinear systems.*

In the following, we will illustrate the efficient calculation of sensitivities using ASAP, for the IFMIF Accelerator Fault Tree depicted in Figure 2. Thus, consider that each primary fault events, i=1,...,5, for the five systems listed in Table 1 (i=1=Injector, i=2=Linac = Linear Accelerator, i=3= Cooling, i=4=RF System, i=5=HEBT) consists of a single unit that can be repaired. For each unit i, i=1,...,5, we introduce the following notation: μ_i = instantaneous repair rate for unit i; λ_i = instantaneous failure rate for unit i; $U_i(t)\equiv$ time-dependent reliability (System is "Up"); $D_i(t)\equiv$ time-dependent failure (System is "Down") probability: Then, the Markov-Model for the state transition diagram is given by the following system of ten (10) coupled differential equations:

$$\begin{cases}\dfrac{dU_i(t)}{dt}=-\lambda_i U_i(t)+\mu_i D_i(t); \quad \dfrac{dD_i(t)}{dt}=\lambda_i U_i(t)-\mu_i D_i(t) \\ U_i(0)=\alpha_i \equiv \text{probab. that unit } \boldsymbol{i} \text{ is in operation at t = 0}; \quad D_i(0)=1-\alpha_i\end{cases}. \qquad (2)$$

The response, $R_{sys}(t)$, of interest is the overall time-dependent availability of the IFMIF accelerator system. This response is represented mathematically by the output of the "AND" gate, namely:

$$R_{sys}(\tau) = A_{IFMIF}(\tau) = \prod_{i=1}^{5}\left[1 - D_i(\tau)\right]. \quad (3)$$

Note that there are 15 parameters to be considered for sensitivity/uncertainty analysis, namely: $\alpha_i, \lambda_i, \mu_i$. The sensitivities $\partial R_{sys}/\partial\alpha_i$, $\partial R_{sys}/\partial\lambda_i$, $\partial R_{sys}/\partial\mu_i$, (i=1, ...,5), are needed in order to rank the importance of each parameter α_i, λ_i, μ_i, in affecting $R_{sys}(\tau)$, and to compute the variance $V[R_{sys}(t)] \equiv m_2[R_{sys}(t)]$, and higher-order moments, at any time $t = \tau$. In principle, these sensitivities are obtained by differentiating Eq.(3), to obtain

$$\frac{\partial R_{sys}}{\partial x_i} = \sum_{k=1}^{10}\frac{\partial R_{sys}}{\partial p_k}\frac{\partial p_k}{\partial x_i},\ i = 1,\ldots,15;\ \text{where}\ \frac{\partial R_{sys}}{\partial p_k} = \begin{cases} 0 \ \text{if}\ p_k = U_j;\ j = 1,\ldots,5 \\ \partial R_{sys}/\partial D_j,\ \text{if}\ p_k = D_j;\ j = 1,\ldots,5 \end{cases} \quad (4)$$

Applying now the ASAP (see, Cacuci, op.cit.), we obtain the Adjoint Sensitivity Model in the form:

$$\begin{cases} -\frac{d\Phi_i^U}{dt} + \lambda_i\Phi_i^U(t) - \lambda_i\Phi_i^D(t) = 0;\ \mathit{for}\ i = 1,\ldots,5 \\ -\frac{d\Phi_i^D}{dt} - \mu_i\Phi_i^U(t) + \mu_i\Phi_i^D(t) = \frac{\partial R_{sys}(t)}{\partial D_i}\delta(t-\tau)\ ; \\ \Phi_i^U(t_f) = \Phi_i^D(t_f) = 0;\ \text{at}\ t = t_f \end{cases} \quad (5)$$

Finally, in terms of the adjoint functions $\Phi_i^U(t;\tau)$ and $\Phi_i^D(t;\tau)$, the sensitivities become:

$$\frac{\partial R_{sys}(\tau)}{\partial x_j} = \begin{cases} \int_0^{t_f} U_i(t)\left[\Phi_i^D(t;\tau) - \Phi_i^U(t;\tau)\right]dt,\ \text{for}\ x_i \equiv \lambda_i \\ \int_0^{t_f} D_i(t)\left[\Phi_i^U(t;\tau) - \Phi_i^D(t;\tau)\right]dt,\ \text{for}\ x_i \equiv \mu_i; \\ \Phi_i^U(0;\tau) - \Phi_i^D(0;\tau),\ \text{for}\ x_i \equiv \alpha_i \end{cases} \quad (6)$$

Note that the Adjoint Sensitivity System, i.e., Eq.(5), depends on $R_{sys}(t)$, but does not depend on parameter variations. Hence, *one adjoint calculation* per response suffices to obtain, very inexpensively, *all* of the sensitivities, exactly. In the example above, conventional sensitivity calculations are a factor of at least 15 more expensive than the ASAP; this is because there would be at least 5x15 (since there are 15 parameters) conventional calculations for solving the respective coupled systems of differential equations, as opposed to 5x1 (since there is 1 response) calculations for the ASAP. This example demonstrates that, *for large-scale systems, the ASAP is the most effective, if not the only, method to calculate exactly, all of the sensitivities*. A *reliability* calculation, for example, involves usually *one response* only, namely the probability of the Top Event, but thousands of parameters. Hence, *one adjoint calculation* (as opposed to thousands of MC simulations or conventional sensitivity calculations) suffices to obtain *all* of the (thousands of) sensitivities.

It is informative to consider the effects of uncertainties in the five subsystems listed in Table 1 on the availability uncertainty for the accelerator system. Thus, considering variances $\sigma_i^2 = 10\% \ x_i$ in the systems i = 1, ..., 5, and using them together with Eq. (6) in Eq. (1), yields uncertainty bands for $R_{sys}(t)$ = IFMIF Availability (t), as depicted in Figure 3, below. Since the target value for IFMIF-availability is at least 88%, our calculation shows that this target value is barely reached, and the uncertainties in the accelerator subsystems must still be reduced considerably in order to attain continuous operation goals at >88% availability.

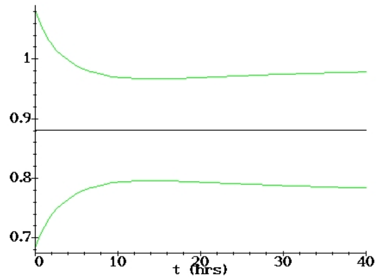

Figure 3: Uncertainty bands for IFMIF Availability, $R_{sys}(t) \pm \sqrt{m_2(t)}$; (68.27%)

4 SUMMARY AND CONCLUSIONS

Using the preliminary design of the projected IFMIF facility, this work has illustrated the advantages of using the Adjoint Sensitivity Analysis Procedure (ASAP) for performing an exhaustive and systematic sensitivity and uncertainty analysis of large-scale nonlinear systems, with many uncertain parameters. The preliminary results obtained for the IFMIF design indicates that considerable additional research is needed to attain the level of performance envisaged for this facility. For this purpose, ASAP will be used to perform a systematic sensitivity analysis of every IFMIF subsystem, in order to identify the weak links where the respective reliability performance needs to be improved. Current research is focusing on implementing ASAP to obtain general adjoint Markov Models for both for Monte Carlo and deterministic calculations for reliability analyses of IFMIF and ITER. Both IFMIF and ITER are large-scale, time-dependent, expensive (of the order of many billion US$) systems, with very many parameters, eminently suitable for the application of ASAP. Improving the availability and reliability of IFMIF and ITER are major international technological and economical design goals within the EURATOM Nuclear Fusion Program.

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