ANALOGICAL MODELING VARIANT OF MASTICATION PROCESSES USING DISTRIBUTED PARAMETERS

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ABSTRACT

The paper presents a less conventional intoductory approach on analogical modeling of mastication processes. A reasonable compromise is attempted, between a statistical interpretation and a mathematical formalism which is specific for distributed parameter processes.

In this context, the simplified balance of mastication forces is expressed on two abscisses, respectively the time (t) and the active length (p) of the dental arches which contributes in the mastication process.

The use of partial derivative equations proved to be an advantage, while the fenomenological interpretation and some interesting conclusions represent a possible simplified variant of analogical modeling of mastication processes.

These results were used in a further study regarding numeric simulation of mastication processes.

KEYWORDS: mastication forces, analogical modeling, partial derivative equations

1. INTRODUCTION

For modeling purposes and as a new approach to previous studies [1], [2], [3], [4], [5], [6], on modeling of mastication dynamics, a food bolus is considered having average statistical dimensions in a spherical shape of a diameter (w).

The initial diameter (w_{00}) will reduce progressively, by means of mastication, to a final diameter (w_{ff}), as shown in figure 1.

The occlusal plane of the mandibular dental arch is approximated to be in a horizontal and parallel position to the maxillary occlusal plane at all times during mastication.

The dotted wrapping line which limits the margins of the foood bolus diameters (w_{00},…, w, …, w_{ff}) is approximated as an exponential function, growing from its initial value, y_{00}, to its final value, y_{ff}, with an intermediate inflexion point.

Mastication is performed between incision, at p_0=0, and the last molar region, at a distance (p_f) from the incisal point, where the diameter (w_{ff}) of the food bolus reaches small enough dimensions in order for the bolus to be collected by the tongue and swallowed.

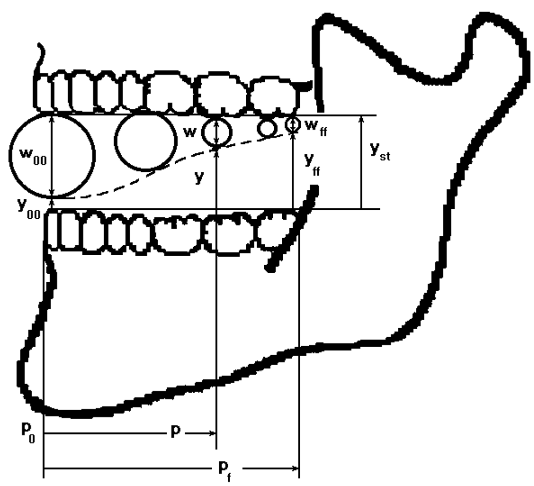

Figure 1. Symplified mastication process

The exponential function of the wrapping line in figure 1 has a theoretic tendency (when $p \to \infty$) towards its stationary value y_{st} and :

$$y_{st} = y_{00}+w_{00} = y+w = y_{ff}+w_{ff} \qquad (1),$$

in which (y_{00}) is a static component (for t_0=0 and p_0=0).

As a result, the deformation of the food bolus can be expressed in the following three variants :

$$y = y_{st}-w = y_{00}+(w_{00}-w) = y_{ff}-(w-w_{ff}) \qquad (2).$$

As mastication progresses, the position of the food bolus becomes more posterior between the dental arches, increasing the value of (p). Meanwhile, the initial food bolus is divided into a number (N) of smaller diameter food boluses, meaning that (w) decreases progressively.

N, the number of smaller size food boluses emerging by means of mastication, respects the initial mass conservation law, as follows:

$$N = \frac{\gamma_{00}}{\gamma} \cdot \left(\frac{w_{00}}{w} \right)^3 \qquad (3)$$

in which (γ_{00}) and (γ) represent the specific weights of the spheres of diameters (w_{00}), respectively (w).

If, for example, the diameter decreases by one half, meaning that $w = \frac{1}{2} w_{00}$, but $\gamma \approx \gamma_{00}$, mastication to that point will result in N=8 boluses of half the initial diameter.

Because the progression of the mastication process is time-dependant, the dotted wrapping curve in figure 1 can be represented in its time dependance, as seen in figure 2.

The points between (t_0, p_0) and (t_f, p_0) mark the intake of food into the oral cavity, for which the deformation of the food bolus maintains an initial arbitrary value (y_{00}).

The points between (t_0, p_0) and (t_0, p_f) correspond to an hypothetic initial moment of mastication (t_0=0) in which the whole oral cavity contains food boluses of diameter (w_{00}) and deformation (y_{00}).

The points between (t_f, p_0) and (t_f, p_f) frame the evolution of the bolus deformation $y(t_f, p)$ between incision, at (p_0), and the last molar region, at a distance (p_f). One can note the exponential aspect of the curve, showing an intermediate point of inflexion.

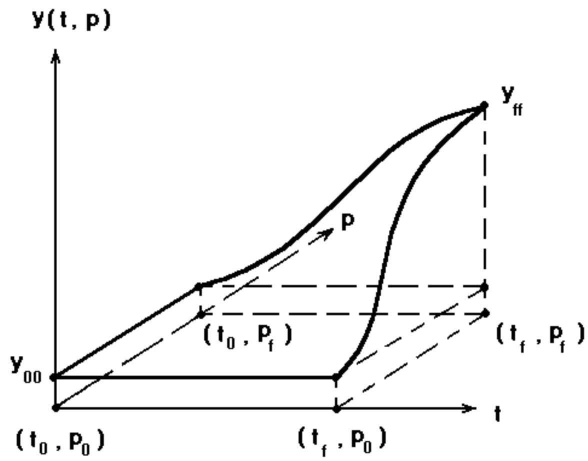

Figure 2. Time and space dependancy of the wrapping curve

The points between (t_0, p_f) and (t_f, p_f) show the evolution of mastication at the most distal points of the dental arches (p_f). In this case as well, one can note the exponential aspect of the curve, showing an intermediate point of inflexion.

The maximum deformation of the food bolus corresponds to $y_{ff} = y(t_f, p_f)$, at the end of its mastication, right before swallowing, when the bolus reaches its most distal position between the dental arches.

From statement (2) we are able to express the average statistic diameter (w) of the food bolus as follows :

$$w = y_{st}-y = y_{00}+w_{00} = y_{ff}+w_{ff} \qquad (4),$$

which allows altering the curve surface in figure 2 to the one seen in figure 3.

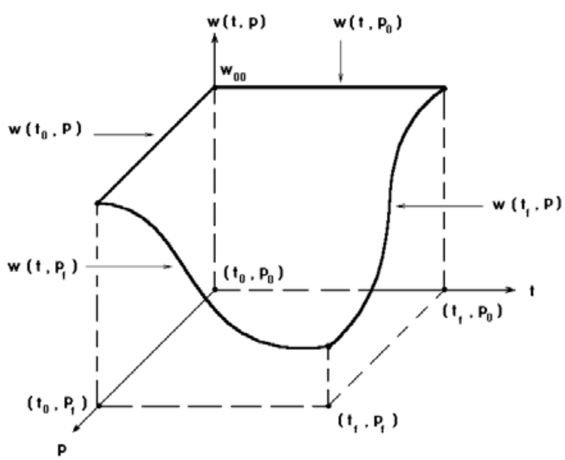

Figure 3. Altered representation of the wrapping curve

2. ANALOGICAL MODELING VARIANT

Every food bolus, having a diameter (w) and beeing deformed by (y), develops the following three categories of forces (figure 4):

- an *acceleration-deceleration* force of the deformation y=y(t, p), proportional with the mass M of the food bolus.
- a force of *elastic* deformation, graphically symbolized as the spring R.
- a force of *plastic* deformation, symbolized as the absorber F.

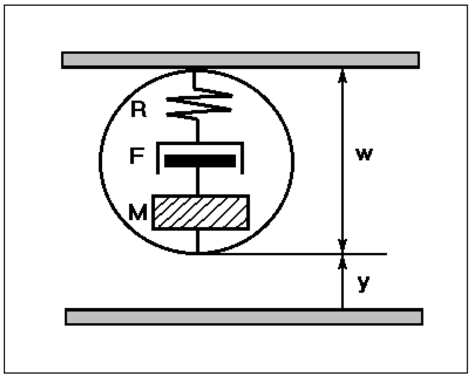

Figure 4. Force categories developed by the deformation of a food bolus

These three categories of forces are generated both by direct vertical compression of the bolus and by the progressive horizontal condensing of the raising number (N) of smaller emerging boluses (see statement 3).

The sum of these six categories of forces can be approximated by the following partial derivative equation (pde) :

$$\tilde{a}_{00}y + \tilde{a}_{10}\frac{dy}{dt} + \tilde{a}_{01}\frac{dy}{dp} + \tilde{a}_{20}\frac{d^2y}{dt^2} + \tilde{a}_{11}\frac{d^2y}{dtdp} + \tilde{a}_{02}\frac{d^2y}{dp^2} = u \qquad (5)$$

in which all coefficients (a_{00}, a_{10}, ...) as well as (b_0), in a further statement, are considered to be constant.

The right side of the equation (u) represents the sum of the six force components.

If (5) is divided by ($\tilde{a}_{00}$), we have :

$$a_{00}y + a_{10}\frac{dy}{dt} + a_{01}\frac{dy}{dp} + a_{20}\frac{d^2y}{dt^2} + a_{11}\frac{d^2y}{dtdp} + a_{02}\frac{d^2y}{dp^2} = b_0 u \qquad (6)$$

in which $a_{00}=1$; $a_{10}=\frac{\tilde{a}_{10}}{\tilde{a}_{00}}$; $a_{01}=\frac{\tilde{a}_{01}}{\tilde{a}_{00}}$; $a_{11}=\frac{\tilde{a}_{11}}{\tilde{a}_{00}}$; $a_{02}=\frac{\tilde{a}_{02}}{\tilde{a}_{00}}$; $b_0=\frac{1}{\tilde{a}_{00}}$.

A general expression for the solution y(t, p) of equation (6) can be written as follows :

$$y(t, p) = y_{00} + y_T(t) * y_P(p) * b_0 * u_0 \qquad (7)$$

in which $y_T(t)$ and $y_P(p)$ are usually exponential or polynomic functions. Exponential functions are the ones preferred for most scientific applications, so we will also consider :

$$y_T(t) = \left(1 - \frac{T_1}{T_1 - T_2}\cdot \varepsilon^{-t/T_1} - \frac{T_2}{T_2 - T_1}\cdot \varepsilon^{-t/T_2}\right) \qquad (8)$$

$$y_P(t) = \left(1 - \frac{P_1}{P_1 - P_2}\cdot \varepsilon^{-p/P_1} - \frac{P_2}{P_2 - P_1}\cdot \varepsilon^{-p/P_2}\right) \qquad (9)$$

T_1 and T_2 in (8) stand for the time constants (measured in seconds) and P_1, P_2 in (9) stand for length constants (measured in meter).

Solution (7) can be represented similar to figure 2, while te existence limits for t, p and y have to be chosen by the physician, depending on the particularities of the mastication process to be simulated. An example of choosing these limits for an average mastication process could be : y_{00} = 0,00005 m ; w_{00} = 0,02 m ; u_0 = 1 ; t_0 = 0 sec. ; t_f = 20 sec. ; p_0 = 0 m ; p_f = 0,06 m.

To conclude, the partial derivative equation (6) can represent a possible analogical model variant of mastication, with a high degree of generalization, on both a time and a space axis.

3. CALCULATION OF THE COEFFICIENTS IN PDE (6)

In order to calculate the coefficients (a_{00}, a_{10}, ...) in pde (6) the program COEFA has been developed and run. For its initialization the following data have been declared : y_{00} ; y_{st} ; u_0 ; t_0 ; t_f ; p_0 ; p_f ; $\lambda_T = T_2/T_1 \geq 1{,}01$; $\lambda_P = P_2/P_1 \geq 1{,}01$; $a_{00} = 1$.

The six unknowns (b_0, a_{10}, a_{01}, a_{20}, a_{11}, a_{02}) have been calculated by means of a six liniar algaebric equations system, representing the pde considered for the six points represented in figure 2 : $y_{st} = y(\infty,\infty)$; $y(t_0,p_f)$; $y(t_f,p_0)$; $y(t_f,\infty)$; $y(\infty,p_f)$; $y(t_f,p_f)$.

The COEFA program has been initialized with the following data (time in seconds, length in meter) : y_{00} ; w_{00}; y_{st} ; b_0 ; t_0 ; t_f ; p_0 ; p_f ; λ_T and λ_P. The results are saved in : b_0, a_{10}, a_{01}, a_{20}, a_{11}, a_{02}, T_1, T_2, t_i, P_1, P_2 and p_i.

Table 1 shows two examples of initialization and the results obtained after running the COEFA program :

Nr.	Initialization data	a_{00}	a_{10}	a_{01}	a_{20}	a_{11}	a_{02}	T_1	T_2	t_i	P_1	P_2	p_i
1	$y_{00} = 5 * 10^{-4}$ m $w_{00} = 2 * 10^{-2}$ m $w_{ff} = 8 * 10^{-4}$ m $y_{st} = 2{,}05 * 10^{-2}$ m $t_f = 20$ sec. $p_f = 6 * 10^{-2}$ m $\lambda_T = 2$ $\lambda_P = 1{,}5$	1	1,596349	$3{,}808809 * 10^{-3}$	9,817215	3,19112	$9{,}571476 * 10^{-5}$	2,2222	4,4444	3,080654	0,008	0,012	$0{,}9731161 * 10^{-3}$
2	$y_{00} = 0$ $w_{00} = 3 * 10^{-2}$ m $w_{ff} = 1{,}2 * 10^{-3}$ m $y_{st} = 3 * 10^{-2}$ m $t_f = 25$ sec. $p_f = 5 * 10^{-2}$ m $\lambda_T = 1{,}5$ $\lambda_P = 2$	1	8,405286	0,0167737	17,04323	$5{,}752954 * 10^{-2}$	$6{,}293086 * 10^{-5}$	3,3333	5	4,054651	$5{,}5555 * 10^{-3}$	$1{,}1111 * 10^{-2}$	$7{,}701635 * 10^{-3}$

Table 1. Examples of initialization and the results obtained after running the COEFA program

In these examples : $b_0 = \dfrac{y_{st} - y_{00}}{u_0}$; $t_0=0$; $p_0=0$ and $u_0=1$. The inflexion abscisses t_i [seconds] and p_i [meter] are in the range of the time constants (T_1 and T_2), respectively in the range of the length constants (P_1 and P_2). All above calculated coefficients (a_{10} , a_{01} , ...) of pde (6) are positive, which confirms the fact that all six force components (associated with these coefficients) correspond to a mechanical work needed during mastication.

Once these results calculated, the complete partial derivative equation (6) which models a simplified mastication process has been defined.

4. CONCLUSIONS

1. Analogical modeling using partial derivative equations based on statistical interpretation of mastication processes represents a less conventional approach in the study of mastication physiology.
2. The use of two abscisses, respectively a temporal and a spatial one contributes to a more complete view, allowing a better identification of food granulation, at different moments during mastication and in different locations between dental arches.
3. Mastication forces can be determined and interpreted with a higher accuracy in dependance of the two abscisses (t) and (p).
4. Numeric integration of the pde (6), in a further study, will also highlight other advantages of using and perfecting the orientation presented in this paper regarding the study of mastication processes.

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