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NUMERICAL SIMULATION OF HYBRID CONTROL SYSTEMS – TWO APPROACHES

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ABSTRACT. This contribution treats some aspects concerning the numerical simulation of a class of hybrid control systems. The studied hybrid control system structure comprises a continuous plant that interacts, through an interface, with a pure logic discrete event controller. The controller coupled to the interface behaves like a switching control law, thus first order discontinuities appear in the derivative of the continuous state. In the case of linear plant models, two approaches for the numerical simulation of closed loop continuous evolution are compared: the first one is based on the variable step integration methods and the second one is based on the time discretisation of the plant model.

KEYWORDS: discrete event system (DES), hybrid control systems (HCS), switching control law, numerical integration, discrete-time model.

1. INTRODUCTION AND MOTIVATION

There has been a considerable interest, in recent years, in the modeling, analysis and design of hybrid systems, which typically arise from the interaction of continuous systems with discrete event dynamics or discrete planning algorithms [1]. As emphasized in [4], the consideration of both discrete and aspects in a single system is in itself not new in control theory. What is new is the aspiration to develop a theory of hybrid systems in a more unified and systematic way. The simulation of hybrid systems plays a key role within this framework. It's worth to mention that in the 60's the terms simulation and modelling were used interchangeably [3], but in the late 70's simulation was understood as one of the several possible usages of models [7].

The class of hybrid control systems (HCS) considered in this paper comprises a continuous plant that interacts, through an interface, with a pure logic controller (Figure 1); this structure was first proposed by P.J. Antsaklis and his co-workers from the ISIS-Group [9], and was studied and extended also in [5], [6]. The continuous state space is partitioned, by smooth hypersurfaces, into open cells. The plant receives a piece-wise constant control signal, according to the control policy and the discrete event controller coupled to the interface is perceived, by the plant, as a multilevel relay or as a switching control. The closed loop system can be modeled as a nonlinear differential system, with first-order discontinuities in the right-hand side. Typical applications for this approach arise from the necessity to govern autonomous systems (like mobile robots or underwater guided vehicles), as well as complex batch processes [6].

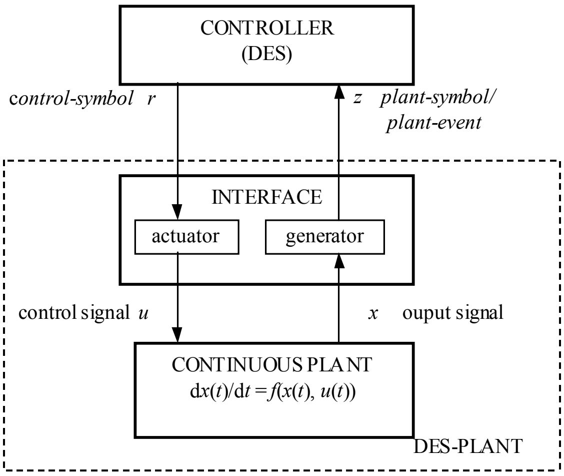

Figure 1. The architecture of the HCS with continuous-discrete interface [9].

This contribution treats some aspects concerning the numerical simulation of the above mentioned classe of hybrid dynamical systems.

The numerical simulation of the hybrid control systems with continuous-discrete interface implies the handling of the discontinuities of the right-hand term of the nonlinear differential system; this is a general simulation problem, which was intensively studied in the last decade [2]. If the numerical simulation is performed within general purpose simulation environments (like MATLAB), then attractive integration routines are the ones based on variable steps methods (e.g. Runge-Kutta methods). However, despite the simplicity of this solution, there is a main problem: the variable step integration methods are very sensitive to the first order discontinuities of the speed of the state vector, induced by the nonlinear switching control law [10]. This sensitivity generally leads to the possibility of generation of a *false simulated trajectory*, immediately after the moment of a commutation. An alternate solution, proposed in this paper, for the case of linear continuous plants, is based on the discretization of the continuous model, with a chosen constant step.

In the following, section 2 presents a brief overview of the hybrid control system framework with continuous-discrete interface. Section 3 treats the related numerical simulation problems, accompanied by a simple simulation example. Finally, concluding remarks are emphasized.

2. THE STRUCTURE OF THE HYBRID CONTROL SYSTEM WITH INTERFACE – A BRIEF REVIEW

In the HCS with continuous-discrete interface (Figure 1), the controller is modeled as a deterministic Moore machine. The plant is modeled by a set of state equations

$$dx(t)/dt = f(x(t), u(t)), \qquad (1)$$

with $t \in \Re$ the continuous time-variable, $x \in \Re^n$ the state vector and $u \in \Re^m$ the control-vector. It is assumed that the state vector is measurable.

The interface comprises the *actuator* and the *event generator* and it converts sequences of control-symbols into continuous-time, *vector-valued*, *piece-wise constant control signal* for the plant and vice-versa. Starting from a *primal control objective* (modeled as a string of inequality-like restrictions imposed to the plant's state vector), the continuous state space is partitioned, by a set of smooth hypersurfaces, $\mathrm{Ker}(h_i)$, with $h_i : \Re^n \to \Re$, $i = \overline{1, N}$, into open cells; the continuous state trajectory evolves from cell to cell, generating plant-events (labeled with plant-symbols), when crossing the adjacency hypersurfaces.

The continuous plant coupled to the interface is represented as a logic, discrete event, nondeterminstic model, with symbolic inputs and outputs, called the DES-plant. The primal control objective is converted into a subautomaton of the DES-plant, representing the desired discrete closed loop evolution. Then the DES controller can be synthesized by adapting the techniques from the DES control theory [8].

The plant perceives the DES-controller coupled to the interface as a *continuous-time switching control law*, which generally depends on the signum of the functionals of the state space partition, i.e.

$$u^*(x) = F(\mathrm{sgn}(h_1(x), \ldots, \mathrm{sgn}(h_N(x)),\ F : \Re^n \to \Re^m . \tag{2}$$

The control law may switch, in response to the occurrence of a plant-event.

Under certain hypotheses, the map F is a linear combination of $\mathrm{sgn}(h_i(x))$, $i = \overline{1, N}$ and the DES controller coupled to the interface acts like a multilevel relay without hysteresis [5]. The related particular formula of the control law is important for the implementation of the continuous-time control law, by means of relay circuits. It also permits the study of the HCS within general simulation environments, like MATLAB.

3. THE NUMERICAL SIMULATION OF THE CONTINUOUS-TIME CLOSED LOOP EVOLUTION

3.1 The general approach

Returning to the continuous model (1), with $f(x(t), u(t)) \equiv f(x(t), u^*(x(t)))$, the problem is to choose the adequate integration method. The presence of the switching logic (2) leads to the necessity of handling the discontinuities in the simulation process, which implies (a) discontinuities detection, (b) location of the discontinuities and (c) passing the discontinuities [2].

This subsection is focussed on the numerical simulation of the closed loop hybrid control system with interface, within general purpose simulation environments, like MATLAB. As shown in the introduction, the variable step integration routines (like ODE23 or ODE45) are attractive, but, there are two riks when using them : (*i*) they are sensitive to the discontinuities induced by the switching logic and (*ii*) the user cannot directly control the integration time-variable, even if the exact switching moment can be analitycally precalculated (depending on the initial conditions).

An alternate general solution for linear continuous plants is based on the discretization of the plant's model. Consider the linear continuous system

$$dx(t)/dt = Ax(t) + Bu(t), \tag{3}$$

with $A \in \Re^{n \times n}$, $B \in \Re^{n \times m}$, and x, t and u having the same significance as in (1). The system (3) has to be integrated on a given time interval $[0, t_f]$. The input signal is constant on each subinterval $[kh, (p+1)k)$, $k = 0 : Q-1$ of a uniform division of $[0, t_f]$, with $h > 0$ the discretisation step, i.e. $u(t) = u_k$, $t \in [kh, (k+1)h)$, where the Q vectors $u_k \in \Re^m$ are known. Using the formula of the solution of the differential system (3), for $t \in [kh, (k+1)h)$, and introducing $t = (k+1)h$, one obtains the discrete-time system associated to (3),

$$x((k+1)h) = A_d x(kh) + B_d u_k, \; k = 0 : Q-1, \tag{4}$$

with

$$A_d = e^{hA}, \; B_d = \int_0^h e^{\theta A} d\theta \, B. \tag{5}$$

With these considerations, the alternate integration method for continuous plant (3), with the input $u(.)$ generated by the switching logic (2), implies two main steps:

Step 1. The continuous plant is converted into a discrete-time model, $x_d(k+1) = A_d x_d(k) + B_d u_d(k)$, with A_d, B_d given by (5) and $h > 0$ the chosen discretization step, $x_d(k) = x(kh)$ and $u_d(k) = u^*(x(kh))$.

Step 2. The discrete-time model is numerically integrated, on a chosen time interval $[t_0, t_f]$, with the discrete-time variable $t_d = t_0 + kh$. **Stop.**

3.2 Example : the Simulation of a Hybrid System with Interface and the Continuous Plant Modeled as the Double Integrator

Consider a HCS in which the plant is the continuous-time double integrator, with the state equations

$$dx^1(t)/dt = x^2(t), \; dx^2(t)/dt = u(t). \tag{6}$$

The control objective is to drive the state, in the clockwise direction, in the four quadrants of the state-space, starting from the IInd quadrant (Figure 2(a)).

The set $U = \{-1, 0, 1\}$ of control values is associated to the *alphabet* $R = \{r_1, r_2, r_3\}$ *of control-symbols* by the function $\gamma : R \rightarrow U$, with $\gamma(r_1) = -1$, $\gamma(r_2) = 0$, $\gamma(r_3) = 1$. The DES-plant model (Figure 2(b)) is built based on the state space partition and on the phase-portraits of the double integrator. The control objective is modeled by the sequence of discrete states $(p_2 p_1 p_4 p_3)^*$ and a corresponding DES controller is represented in Figure 2(c).

The control law, corresponding to the DES controller coupled to the interface, is $u^*(x) = -sign(h_1(x))$, with $x = (x^1 \;\; x^2)^T$ and $h_1(x) = x^1$ the switching hypersurface. It can be analytically shown that the double integrator, controlled by $u^*(x)$, has a cyclic trajectory, according to the control objective. The simulation experiments with ODE45 (Figure 3(a),(b)) show that, depending on the chosen tolerance, for higher values of the integration time, the simulated trajectory may trend towards the origin, which is false. Figure 4(c) depicts the simulation result for the discrete time model of the closed loop HCS associated to the double integrator (7).

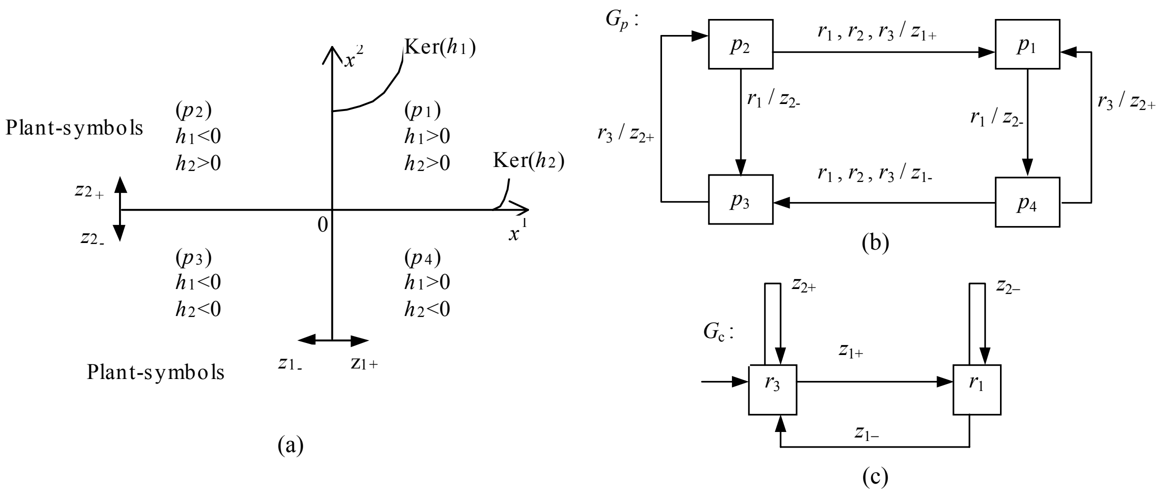

Figure 2. Example : (a) the state space partition with,; (b) the DES-plant model; (c) the DES controller with the control objective.

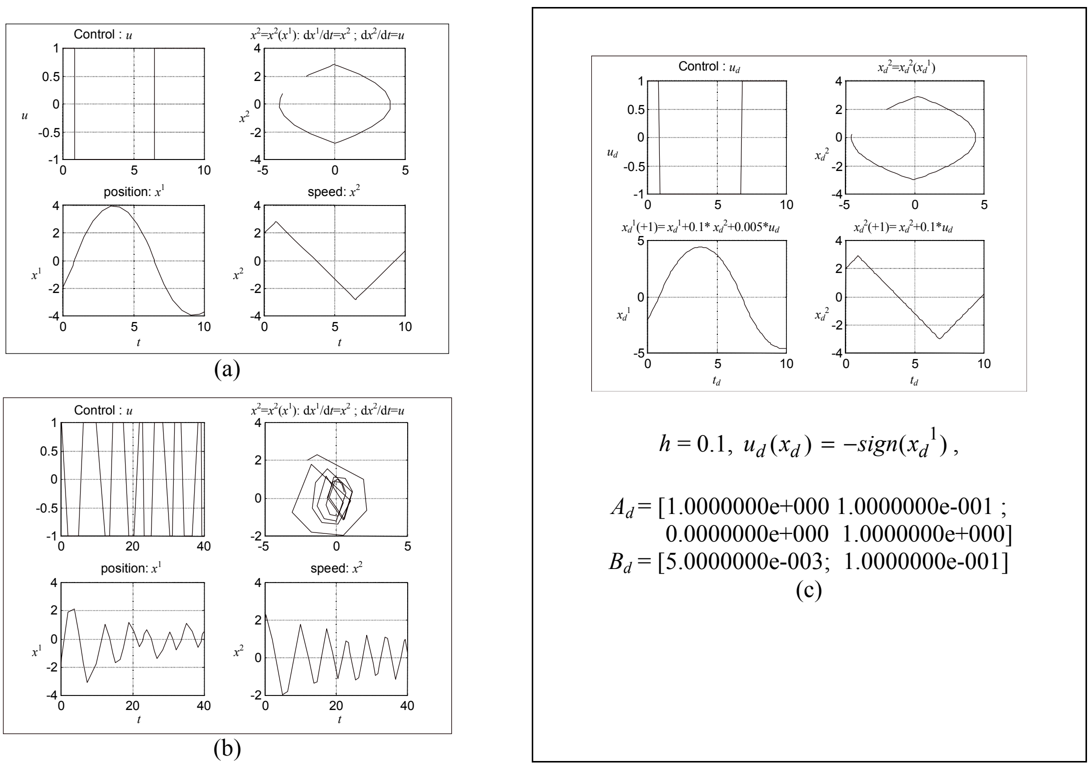

Figure 4. Example: MATLAB simulation results of the system (7) with $u^*(x) = -sign(h_1(x))$, using: (a) ODE45, tol=10^{-4}; (b) ODE45, tol=10^{-2}; (c) the discrete-time model of (7)

4 CONCLUSIONS

In the HCS structure with interface, the crucial problem of the numerical closed loop simulation, with variable step integration methods, is the proper handling of the first order discontinuities induced by the switching control law, i.e. of the derivative of the continuous state. This problem – which is common to variable structure systems - was intensively studied in the past decade and solutions based on "IF THEN ELSE" or "AS SOON AS" structures are presented in the literature [2]. When using general-purpose simulation environments, like MATLAB, the Runge-Kutta integration routines are quick, but they present the risk of generation of false simulated trajectories. This risk is avoided by the alternate discrete time proposed solution, which is slower, but well suited for the large class of linear continuous plants. The general simulation problem of the HCS with interface is still open. Note that the derivative discontinuity means a discontinuous change of models, i.e. a variable model structure [7].

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