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BOOST DC-DC CONVERTER MODELS FOR CONTROL (GENERAL FRAME FOR CONTINUOUS CONDUCTION MODE)

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Abstract –DC-DC power converters are a particular class of time variant nonlinear discrete systems. In order to control the converters, a model is a stringent necessity but exact models are complicated and cannot be easily applied.

The paper presents frames in which a variety of models can be derivate, as a starting point for controller design.

Key – words: DC-DC converters, Models, Sliding- mode, passivity – based control

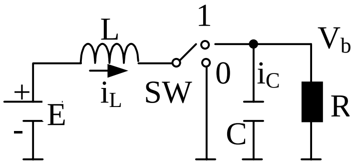
1. INTRODUCTION

The models of the DC-DC power converters are chosen in accord to the aim of modeling. These can be:

a) converter design (power semiconductor, components, etc.);
b) converter analysis (in open loop);
c) converter control design (in closed loop) in order to exhibit some desired performance.

Because the power converters require invariably feedback, the paper focuses on the model proper to the control system design.

The boost (step-up) converters operate through cyclic energy transfer, regulated by the state (ON/OFF) of the power semiconductor switches. During the period $t_{ON} = \mu \cdot T_S$ (μ = duty ratio, T_S = switching period), the switches allow that energy of the external power supply be stored in “input inductive branches. The stored energy is than transferred to the output energy reservoirs (capacitors) and to the load.



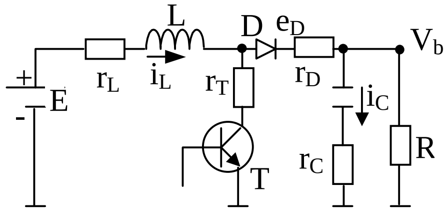


Figure 1. The ideal a) and a real circuit b) of a boost converter

By boost converters, the output voltage (V_B) is higher than the supply voltage and it is the same polarity. The input current is practically continuous (or, more exactly **non-pulsating**) because this current is the same as the inductor current. The continuous mode of operation (inductor current do not decay to zero during the switching period).

The idealized diagram and a circuit for the boost converter are given in figure 1.

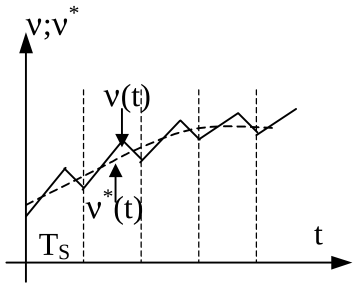


Figure 2. Actual waveform ν(t) with ripple and averaged waveform $\nu^*(t)$ with neglected ripple

2. PRINCIPLES OF MODELING

Due to the switching mode of operation, the variables of the converters contain a DC component – the averaged value – and a residual high-frequency component (ripple). The models (for control) are based on hypothesis of **small-ripple approximation** (more exactly small-ripple neglect). Around a state point of operation, whose coordinates give the DC components, the small deviations of the variables describe the **small – signal AC model**. If the ripple is small, the deviation are replaced with their **low-frequency averaged values**, figure 2 (averaged over each switching period T_S).

It is usual to denote:

ν_0: steady – state value;

$\nu^*(t)$: averaged value;

$\nu(t)$: "whole" value with

$$\nu(t) = \nu_0 + \nu^*(t) \tag{1}$$

For the boost converter:

$$V_b^* = \left(\frac{1}{1-\mu^*}\right) \cdot E_0 \tag{2}$$

2.1. **Conventional linear models**

To obtain the "conventional" models, linear circuit models are identified for each switch position ("state") during the switching period. These equation are the **averaged** over the switching period, than linearized in order to lead to a low frequency Ac equivalent model, proper for the **linear** control theory in frequency domain.

In the **preliminaries** of the modeling, it is necessary to determine the **averaged** evolution of the voltages and currents appearing in the boost converter circuit. For instance, if the inductor current (with $r_L = 0$, $r_C = 0$, e = 0, $r_D = 0$) is described by the equation:

$$\begin{aligned} L\frac{di}{dt} &= \mu(t)\cdot E(t) + [1-\mu(t)]\cdot V_b(t) = \\ &= [\mu_0 + \mu^*(t)][E_0 + E^*(t)] + [1-\mu_0 - \mu^*(t)][V_{b0} + V_b^*(t)] = \\ &= \mu_0 E_0 + [1-\mu_0]\cdot[V_{b0} - E_0] + \mu_0 E^*(t) + E_0\mu^*(t) + (1-\mu_0)[V_b^*(t) - E^*(t)] - \\ &\quad - (V_{b0} - E_0)\cdot\mu^*(t) + \mu^*(t)E^*(t) - \mu^*(t)[V_b^*(t) - E^*(t)] \end{aligned} \tag{3}$$

AC – nonlinear terms (of second order).

Provided that the deviations are small, the second – order nonlinear terms may be neglected and the **AC – linear small – signal model** is:

$$L\frac{di_L^*(t)}{dt} = A_{i1}\cdot\mu^*(t) + A_{i2}\cdot E^*(t) + A_{i3}\cdot V_b^*(t) \qquad (4)$$

In this manner it is possible to compute the equation of ode variables.

2.2. MODELS FOR SLIDE MODE CONTROL

2.2.a. Preliminaries

For switched-mode power converter, whose the control variable (base current, gate-voltage, etc.) assumes only a discrete set of values, it is natural to consider sliding mode strategies as a proper switching policy.

Sliding mode techniques belong to the category of **true domain** design and characterize the system both under **small-signal** and **large-signal** conditions. The most important feature of sliding mod approach is the low sensitivity to system parameter variation (R, E, etc.).

The general behaviors of the DC/DC converters can be given by a **bilinear** form of the state-space formulation:

$$\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} + u\cdot\mathbf{B}\cdot\mathbf{x} \qquad (5)$$

where (u) is a scalar control with discrete value {0, 1} given by:

$$u = \frac{1}{2}[1 - \mathrm{sign}(s)].$$

s(**x**) is a scalar switching function for sliding mode:

$$s(\mathbf{x}) = \mathbf{C}^T\cdot\mathbf{x}$$

with vector $\mathbf{C} = \left(\partial s / \partial \mathbf{x}\right)$ (gradient of the scalar function (s)).

The evolution of the converter is given by the equations:

$$\begin{aligned}\dot{s} &= \mathbf{C}^T\cdot\dot{\mathbf{x}} = \mathbf{C}^T\cdot\mathbf{A}\cdot\mathbf{x} + u\cdot\mathbf{C}^T\cdot\mathbf{B}\cdot\mathbf{x} = \\ &= \mathbf{C}^T\cdot\mathbf{A}\cdot\mathbf{x} + \frac{1}{2}\mathbf{C}^T\cdot\mathbf{B}\cdot\mathbf{x} - \frac{1}{2}\mathrm{sign}(s) - \mathbf{C}^T\cdot\mathbf{B}\cdot\mathbf{x}\end{aligned} \qquad (6)$$

2.2.b. Dynamic mode of the boost converter

With $x_1 = i_L$; $x_2 = V_b$ $(r_L = r_C = r_D = r_T = 0; e = 0)$ results the model:

$$\begin{cases}\dot{x}_1 = -(1-\mu)\cdot x_2 + \frac{1}{L}E \\ \dot{x}_2 = \frac{(1-\mu)}{C}\cdot x_1 - \frac{1}{R_C}\cdot x_2\end{cases} \qquad (7)$$

With this model it is possible to control the converter in two loop: an internal current – control loop and an external voltage – control loop.

2.3. MODELS FOR PASSIVITY – BASED CONTROL

2.3.a. Preliminaries

The models are based on the energy balance of the converter over a switching period, operate with generalized coordinates (q) and ignore the parasitic ripple.

To define the models, a Lagrangian and a Hamiltonian dynamics approach [??] are used. The Lagrangian consist in establishing the Euler – Lagrange "parameters" of the circuit in accord to each switch – states.

If (q_L) and (q_C) are the general coordinates:

1 – the general kinetic energy Euler – Lagrange "parameter" is:

$$\tau_u = \frac{1}{2}(\dot{\mathbf{q}}_L)^T \cdot \mathbf{L} \cdot (\dot{\mathbf{q}}_L)$$

where (**L**) is a diagonal positive definite matrix and $(\dot{\mathbf{q}}_L)$ is a vector of inductor currents;

2 – the generalized potential energy has two components:

$$\nu_u = \frac{1}{2}(\mathbf{q}_{CI})^T \cdot (\mathbf{C}_I)^{-1} \cdot (\mathbf{q}_{CI}) + \frac{1}{2}(\mathbf{q}_{CV})^T \cdot (\mathbf{C}_V)^{-1} \cdot (\mathbf{q}_{CV})$$

$$\mathbf{q}_{CV} = \mathbf{V}(u) \cdot \mathbf{q}_L$$

where ($\mathbf{q}_{CI}$) and ($\mathbf{q}_{CV}$) are the capacitor charge vectors and V(u) is a matrix parameterized terms of switch position (u).

3 – the Rayleigh dissipation Euler – Lagrange "parameter" is:

$$F_u = \frac{1}{2}[\mathbf{Z}(u) \cdot \dot{\mathbf{q}}_L - \dot{\mathbf{q}}_{CI}]^T \cdot \mathbf{R} \cdot [\mathbf{Z}(u) \cdot \dot{\mathbf{q}}_L - \mathbf{q}_{CI}]$$

where **R** is a diagonal "regular" matrix and **Z**(u) is a matrix of "inductive loop mode insertion".

4 – the generalized forcing function is:

$$Q_u = \left[(\mathbf{Q}^*(u) \cdot \mathbf{E})^T; \mathbf{0}\right]^T$$

where $\mathbf{Q}^*$(u) is the "input loop external source matrix" and (**E**) is a external sources subvector.

Applying the Euler – Lagrange equation to the previous form Euler – Lagrange "parameters", with: $\mathbf{x}_L = \dot{\mathbf{q}}_L$; $\mathbf{x}_{CV} = \mathbf{C}_v^{-1} \cdot \mathbf{q}_{CV} = \mathbf{C}_v^{-1} \cdot \mathbf{V}(u) \cdot \mathbf{q}_{CV}$; $\mathbf{x}_{CI} = \mathbf{q}_{CI}$, the **general state – space model** is:

$$\begin{vmatrix} \mathbf{L} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_V & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_I \end{vmatrix} \cdot \begin{vmatrix} \dot{\mathbf{x}}_L \\ \dot{\mathbf{x}}_{CV} \\ \dot{\mathbf{x}}_{CI} \end{vmatrix} = \begin{vmatrix} \mathbf{0} & -\mathbf{Z}^T(u) & -\mathbf{V}^T(u) \\ \mathbf{Z}(u) & \mathbf{0} & \mathbf{0} \\ \mathbf{V}(u) & \mathbf{0} & \mathbf{0} \end{vmatrix} \cdot \begin{vmatrix} \mathbf{x}_L \\ \mathbf{x}_{CV} \\ \mathbf{x}_{CI} \end{vmatrix} + \tag{8}$$

$$+ \begin{vmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}^{-1} \end{vmatrix} \cdot \begin{vmatrix} \mathbf{x}_L \\ \mathbf{x}_{CV} \\ \mathbf{x}_{CI} \end{vmatrix} = \begin{vmatrix} \mathbf{Q}^*(u) \cdot \mathbf{E} \\ 0 \\ 0 \end{vmatrix}$$

In an equivalent general form:

$$\mathbf{D}^* \cdot \dot{\mathbf{x}} + \mathbf{G}^*(u) \cdot \mathbf{x} + \mathbf{R}^* \cdot \mathbf{x} = \boldsymbol{\varepsilon}^*$$

with $\mathbf{x} = \left[\mathbf{x}_L^T; \mathbf{x}_{CV}^T; \mathbf{x}_{CI}^T\right]$ and $\mathbf{D}^*$ a diagonal matrix.

In the particular case of the boost converter:

$$\mathbf{D}^* = \begin{vmatrix} L & 0 \\ 0 & C \end{vmatrix};\ \mathbf{G}^*(u) = u\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix};\ \mathbf{R}^* = \begin{vmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{vmatrix};\ \boldsymbol{\varepsilon}^* = \begin{vmatrix} E \\ 0 \end{vmatrix}.$$

The differential equations of the switched system based on the Euler – Lagrange "parameters" is:

$$L \cdot \ddot{q}_L = -(1-\mu) \cdot R \cdot \left[(1-\mu) \cdot \dot{q}_L - \dot{q}_C\right] + E$$
$$\frac{1}{C} \cdot q_C = R \cdot \left[(1-\mu) \cdot \dot{q}_L - \dot{q}_C\right] \qquad (9)$$

An equivalent model is:

$$\begin{cases} \ddot{q}_L = -(1-\mu)\dfrac{q_C}{LC} + \dfrac{E}{L} \\ \dot{q}_C = -\dfrac{1}{RC} \cdot q_C + (1-\mu) \cdot \dot{q}_L \end{cases} \qquad (10)$$

which is equivalent to the model (9) for sliding mode.

If the "parasitic" elements $(r_L, r_C, r_D, r_T, e_D)$ are considered, the equations are now:

$$\begin{cases} \ddot{q}_L = -\dfrac{1}{L} \cdot \tau^* \cdot \dot{q}_L - (1-\mu) \cdot \dfrac{R}{r_C + R} \cdot \dfrac{1}{LC} \cdot q_C + \dfrac{E}{L} - (1-\mu) \cdot \dfrac{e_D}{L} \\ \dot{q}_C = (1-\mu) \cdot \dfrac{R}{r_C + R} \cdot \dot{q}_L - \dfrac{1}{(r_C + R)C} \cdot q_C \end{cases} \qquad (10^*)$$

with $\tau^* = r_L + \mu \cdot r_T + (1-\mu) \cdot r_D + (1-\mu^2)\dfrac{r_C R}{r_C + R}$.

Due to the parasitic elements, the output voltage (V_b) over the load resistor is:

$$V_b = x_0 = \frac{R}{r_C + R} \cdot \frac{q_C}{C} + (1-\mu) \cdot \frac{r_C \cdot R}{r_C + R} \cdot \dot{q}_C \qquad (11)$$

3. CONCLUSIONS

The linear control remains the most usual and frequent solution in order to ensure same steady state and transient performance specifications of the DC-DC converters.

Even for different control strategies: linear control; sliding mode control or passivity based control, the simplified models have many elements in common and can be derived, with some considerations, one from other.

Although the achieved performance is satisfactory, the main drawback is the narrow domain of validity. For large perturbations, these models cannot point out the danger of instability.

LITERATURE

1. Ortega, R a.o.(1998): Passivity – based Control of Euler – Lagrange Systems, Springer
2. Utkin, V. a.o. (1999): Sliding Mode Control in Electromechanical Systems, Taylor – Francis
3. Erickson, R. W. (1997): Fundamentals of Power Electronics, Chapman and Hall
4. Guinjoan, F. a.o. (1997): Large – Signal Modeling and Simulation of Switching DC-DC Converters, in *IEEE Trans. Pow*. Electronics, p.485-493