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DEPENDENT FAIL PROCESSES AND CHAOTIC PROCESSES

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Abstract- The simplify and unrealistic hypothesis of fail processes independence can induce errors in identification of probabilistic distribution of function times. The paper presents a qualitative approach of fail events in systems with s-dependent components, proposing a model based on chaotic processes.

Key words: s-dependence component, variable transition rate, chaos theory.

1.Introduction

A model reproducing catastrophic fails was presented in [1]. There, a fail is called catastrophic if in well defined conditions, the value of reliability function can change by jump even the variation of some parameters (transition rates) are smooth. In the following, the "fail" denomination will used in this sense. In [2] are explained that the variation of transition rates is due to dependence of fail processes and based on a four-state fail model it proved that a catastrophic fail is produced by the simultaneous change of two parameters of reliability function:

a) The increase of an stabile state leaving probability;

b) The decrease of return probability to the stabile state from a limit stabile state;

Admitting a time-linear increasing for the probability from a) and a boundary zero value for the probability from b), a minimal slope was determined, of which the reliability function jump suddenly.

2.Chaotic processes

An elementary iterate equation given in [5] in which, for singular values of control parameter ***r***, shows a chaotic behavior.

$$p_{k+1} = r \cdot p_k \cdot (1 - p_k) \qquad (1)$$

where p_{k+1}, p_k are the population percentage from the maximum permissible *(p=1)* in a finite resource space, at the moments k, k+1. (1) describe an one- dimension unlinear application in x_k :

$$x_{k+1} = f(r, x_k) : [0,1] \rightarrow [0,1] \qquad (2)$$

A symbolic form for (2) is like (3):

$$x_k = f^{(t)}(r, x_0) \qquad (3)$$

Let the equation $x = f(r, x)$ and its roots $x^{\bullet}$ the fix points of a *f* transform. For (1):

$$x_1^{\bullet} = 0 \qquad x_1^{\bullet} = \frac{r-1}{r} \tag{4}$$

Let arbitrary choose *r=2*. A geometrical construction based on a fix point definition and the graphic of the *f* application is given in fig.1:

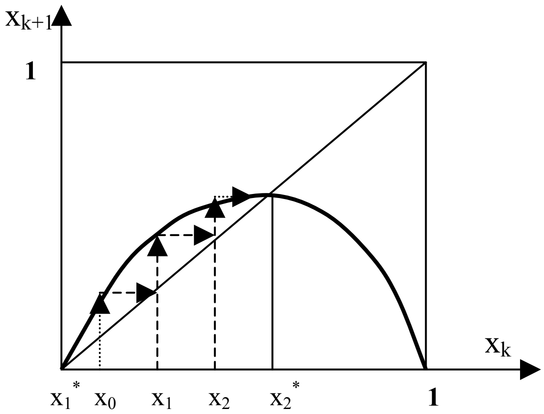


Fig.1- The fixed points of equation (1)

Regarding fig,1, is perceptible that leaving an arbitrary beginning point $x_0 \neq 0$, x verge to x_2^*, and on a x-axis the successive values of iteration x_1, x_2, …, x_2^* are found. The fixed point x_1^* is unstabile because it can't be reached from any of $x_0 \neq 0$. The fixed point x_2^* is stabile because for the process described by (1) and it is an attractor for the interval (0,1].

If the fixed point of $f(r,x)$ become unstabile for a r_1, for the process evolution near $r_1$is recommended to investigate the application:

$$x_{k+2} = f^{(2)}(r, x_t) \tag{5}$$

which, according to form (3), become:

$$x_{t+2} = r \cdot x \cdot \left[1-(r+1)x_t + 2rx_t^{\ 2} - rx_t^{\ 3}\right] \tag{6}$$

with the fixed points:

$$x_1^* = 0, \quad x_2^* = \frac{r-1}{r}, \quad x_{3,4}^* = \frac{1}{2r\left[r+1 \pm \sqrt{(r+1)(r-3)}\right]} \tag{7}$$

The two firsts are roots also for pentru *f(r,x),* but $x_3^* = f\left(r, x_4^*\right)$ şi $x_4^* = f\left(r, x_3^*\right)$.

Since, for $r \geq 3$, x_3^*, x_4^* are in [0,1], an $x_3^*, x_4^*, x_3^*, x_4^* \ldots$ array can occur and equivalent to a limit cycle with the periodicity 2 , that is to say a bifurcation a singular point (attractor) x_2^* to x_3^* and x_4^*.

For $r \geq 4{,}33$, the process yield two new bifurcations the periodicity 4, and so on. Therefore (1) describe a chaotic evolution.

3.A failure model reproducing chaotic behavior

The first section explain the way through can varied certain transition rates. Let the Markow chain for a bivalent component (fig.2) and p_{t+1}^{0}, p_{t+1}^{1} the associate probabilities for the stable sate *1* respectively to a limit stable state *2* at the moment *t+1*.

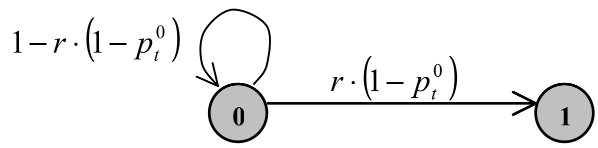


Fig.2 – A Markow chain for a bivalent component

Naturally, the transition probability from 0 to 1 is as bigger as: a) the 0 state's probability is smaller; b) the component's dependence to the system's state (given by a system's state vector) is stronger: the dependence is nominate by a term *r*, $r > 1$;
The associate equations to the Markov chain from the din fig.2 are:

$$p_{t+1}^{1} = r \cdot \left(1 - p_t^{0}\right) \cdot p_t^{0} \quad p_t^{0} + p_t^{1} = 1 \tag{8}$$

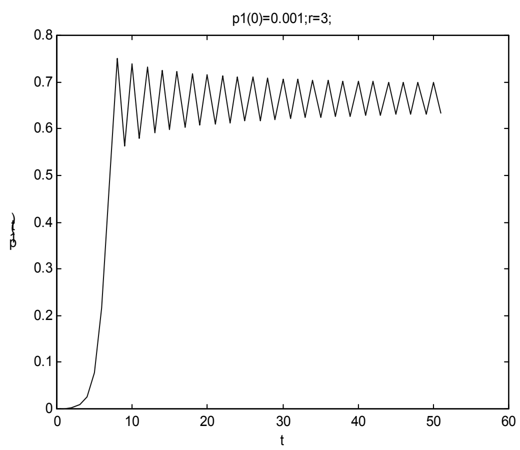
Excluding p_t^{0}, result:

$$p_{t+1}^{1} = r \cdot \left(1 - p_t^{1}\right) \cdot p_t^{1} \tag{9}$$

hence an identical form with (1).

4.Example

Let: a) $p_0^1 = 0.001; \quad r = 3;$ b) $p_0^1 = 0.001; \quad r = 3.44;$ c) $p_0^1 = 0.001; \quad r = 4;$
The evolution of p_t^1 are depicted in fig.3a, 3b,3c:



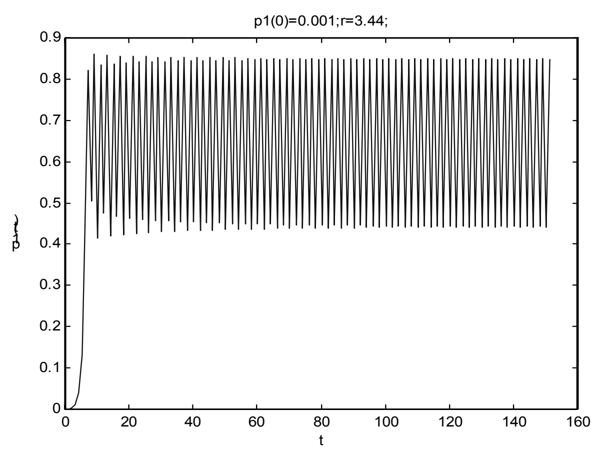


Fig.3a - $p^1(0)$=0.001; r=3

Fig.3b - $p^1(0)$=0.001; r=3.44

In the c) case (fig.3c) the histograme of frequencies of p_t^1 is required.

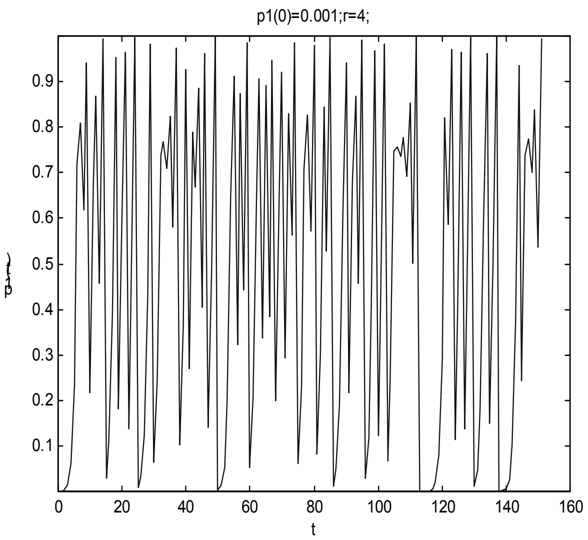


Fig.3a- $p^1(0)$=0.001; r=4

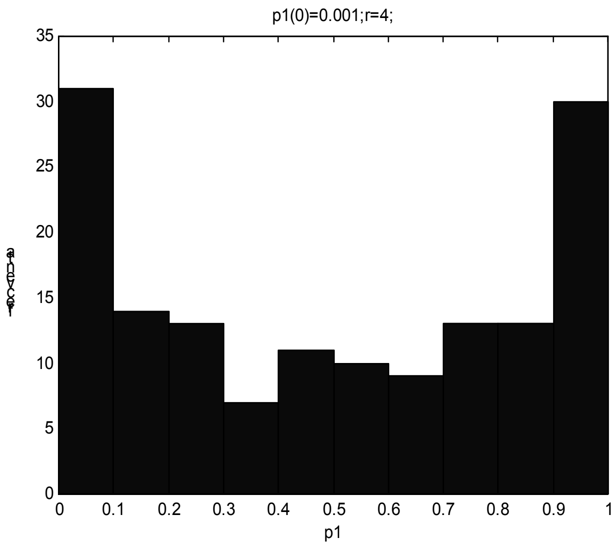


Fig.4 - Histogram for $p^1(0)$=0.001; r=3

Conclusion.

The presented model prove that it is able to reproduce chaotic behaviors of fail processes, if it is possible to evaluate an increasing rate of fails probabilities due to a system state. So it can explain the inherent limits of failure predictions in systems with s-dependent componets.

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