IDENTIFICATION AND CONTROL OF THE ISOTOPIC SEPARATION PROCESS

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Abstract - In this paper the mathematic model and the boundary conditions of the isotopic separation process are presented in the context of process identification and control. The presented model for the isotopic separation column is a general one, which accounts for the spatially varying structure of the process and which is derived from data obtained by means of simple input/output experiments.

Key-words: mathematic model, boundary conditions, isotopic separation column control

1. INTRODUCTION

The problem of maintaining certain specified variables at set values is common to any planned operation. The control problem for isotope-separation plants differs from that of other manufacturing operations in the degree of complexity. These systems are dynamic processes whose outputs, states, controls and parameters are spatially varying. If the equations, parameters, and boundary conditions, which make up the conventional partial differential equation models of these processes, are known, then there are a variety of theoretical methods available for distributed parameters system control design.

In practice, the spatially distributed nature of such processes is generally overlooked or ignored, and conventional control system design techniques are applied using approximate lumped models identified from input/output testing. However, because these simple lumped models ignore the spatially varying nature of the distributed parameters system, they will often suffer from strong interactions and apparent time delays due to the underlying diffusion and convection phenomena inherent in such processes.

Therefore there exists a need for an identification procedure that recovers a general model for the isotopic separation column, which accounts for the spatially varying structure of the process and which is derived from data obtained by means of simple input/output experiments. The present paper works with the isotopic separation column for N^{15}.

2. PRELIMINARY NOTIONS REGARDING THE ISOTOPIC SEPARATION COLUMN (ISC)

A simplified scheme of the ISC is depicted in figure 2.1 and consists of:
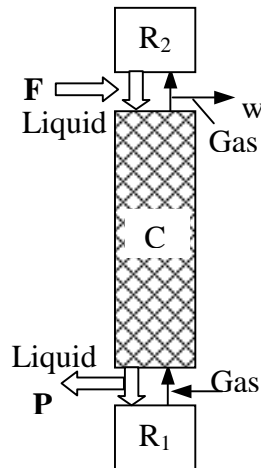

Fig 2.1. Scheme of the ISC

- separation column (C);
- reflux bottom (R_1) vessel and reflux top (R_2) vessel
- supplying system with nitric acid input flow (F) and with a small concentration in N^{15} ($N_{in} \cong 0.365\%$);
- final product of liquid flow (P) with the higher N^{15} concentration ($N_p \gg N_{in}$);
- residual product of gas flow (w), with very small concentration (N_w) in (N^{15}).

Because of the repeated isotope exchange in the column between liquid and gas, in liquid (HNO_3) is accumulated more and more concentration of (N^{15}) while in gas, this concentration decays. The phase in which enrichment of desired isotope take place will be denoted by small letters and the other phase by capitals. Thus H and h are the holdups, L and l are the flows, in liquid and gas, respectively. The direction of flow of the enriched phase will be taken as positive, and length along the column will be measured in this direction.

With this notes, the fundamental equation of the isotopic separation is:

$$(H+h)\frac{\partial N}{\partial t} = -\frac{\partial}{\partial z}\left[\frac{Ll}{k}\cdot\frac{\partial N}{\partial z} - L(\alpha-1)N(1-N) + PN\right] \qquad (2.1)$$

where N - mole fraction of the N^{15} in liquid phase, k - global gain coefficient, α - the elementary separation factor, t – time [days], z – spatial coordinate (z_f – the length of column) [m]; and with the correspondences: $c_6 \rightarrow H+h$; $c_5 \rightarrow -Ll/k$; $c_1 \rightarrow -L(\alpha-1)$

or in more general form:

$$c_1(z,t)N(z,t)[1-N(z,t)] - c_5(z,t)\frac{\partial N(z,t)}{\partial z} = \tau(z,t) - N(z,t)T(z,t) \qquad (2.2)$$

with

$$\frac{\partial T(z,t)}{\partial z} = -\frac{\partial c_6(z,t)}{\partial t} \text{ and } \frac{\partial \tau(z,t)}{\partial z} = -\frac{\partial}{\partial t}[c_6(z,t)N(z,t)] \qquad (2.3)$$

3. THE IDENTIFICATION AND CONTROL PROBLEM

The effect of perturbations in the fundamental quantities c_1, c_5 and c_6 on the separation is found by solving the basic equation with these three quantities preassigned functions of time. Of primary interest are the small fluctuations, which if imposed on a system already in the steady state will give small changes in concentration. Gross upsets and phenomena associated with the approach to equilibrium are excluded from consideration. The desired solution may thus be obtained by ordinary perturbation theory, with the steady state as the unperturbed system.

Suppose that there is a primary disturbance of some nature from an outside source. It will be written as:

$$\sigma f(z, t) \tag{3.1}$$

The parameter σ is not necessarily small – its function is to distinguish between the various orders of terms resulting from the primary disturbance.

As a consequence of σf there will be included disturbances in c_1, c_5 and c_6. Thus c_1, which before the disturbance is a function of z only, becomes a function of z, t and σ. Expanding in powers of σ:

$$c_1(z, t; \sigma) = c_{10}(z) + \sigma c_{11}(z, t) + \sigma^2 c_{12}(z, t) + \ldots \tag{3.2}$$

where $c_{10}(z)$ is the undisturbed value of c_1. Generally the symbol $c_1(z,t;\sigma)$ will be shortened to $c_1(z,t)$. Likewise:

$$c_5(z, t; \sigma) = c_{50}(z) + \sigma c_{51}(z, t) + \sigma^2 c_{52}(z, t) + \ldots \tag{3.3}$$

$$c_6(z, t; \sigma) = c_{60}(z) + \sigma c_{61}(z, t) + \sigma^2 c_{62}(z, t) + \ldots \tag{3.4}$$

As a result of the previous equation:

$$T(z, t) = P_0 + \sigma T_1(z, t) + \sigma^2 T_2(z, t) + \ldots \tag{3.5}$$

The functions $c_1(z,t)$, …, $T(z,t)$ are considered known, as well as the coefficient of their expansions. They are in fact the solutions of the hydrodynamic part of the control problem.

Now is easy to find $N(z,t)$ and $\tau(z,t)$. These will also be functions of σ, which will be written as:

$$N(z, t) = N_0(z) + \sigma N_1(z, t) + \sigma^2 N_2(z, t) + \ldots \tag{3.6}$$

$$\tau(z, t) = \tau_0 + \sigma \tau_1(z, t) + \sigma^2 \tau_2(z, t) + \ldots \tag{3.7}$$

Of course

$$\tau_0 = P_0 N_{P_0} \tag{3.8}$$

where N_{P_0} is the undisturbed mole fraction of the product.

The convergence of equation (3.6) and (3.7) is now of concern. The natural period of a cascade with respect to concentration disturbances is the equilibrium time.

From equations (3.2), (3.6), (3.7), (2.2), and (2.3) and separating the coefficients of σ^0, σ^1, σ^2, …, the following systems of equation are obtained:

$$P_0 N_{P_0} = P_0 N_0(z) - c_{50} \frac{\partial N_0}{\partial z} + c_{10} N_0 (1 - N_0) \tag{3.9}$$

$$\tau_1 = P_0 N_1 - c_{50} \frac{\partial N_1}{\partial z} + c_{10}(1 - 2N_0)N_1 + T_1 N_0 - c_{51}\frac{\partial N_0}{\partial z} + c_{11}(1 - N_0)N_0 \tag{3.10}$$

$$\frac{\partial \tau_1}{\partial z} = -c_{60}\frac{\partial N_1}{\partial t} + N_0 \frac{\partial T_1}{\partial z} \tag{3.11}$$

$$\tau_2 = P_0 N_2 - c_{50}\frac{\partial N_2}{\partial z} + c_{10}(1 - 2N_0)N_2 + T_1 N_1 - c_{51}\frac{\partial N_1}{\partial z} + c_{11}(1 - 2N_0)N_1 -$$
$$\tag{3.12}$$
$$- c_{10}N_1^2 + T_2 N_0 - c_{52}\frac{\partial N_0}{\partial z} + c_{12}(1 - N_0)N_0$$

$$\frac{\partial \tau_2}{\partial z} = -c_{60}\frac{\partial N_2}{\partial t} + N_0\frac{\partial T_2}{\partial z} - c_{61}\frac{\partial N_1}{\partial t} + N_1\frac{\partial T_1}{\partial z} \tag{3.13}$$

and so on for higher orders.

The boundary conditions for each order equation can likewise be found by expansion. When the boundary conditions are known the equations may be solved successively.

Equation (3.9) gives the steady state of the unperturbed system. The expression for τ_1 has the form:

$$\tau_1 = DN_1 + \Omega_1(z,t) \tag{3.14}$$

where D is the linear operator, independent of t,

$$D(z) = P_0 - c_{50}\frac{\partial}{\partial z} + c_{10}(1 - 2N_0) \tag{3.15}$$

and

$$\Omega_1(z,t) = T_1 N_0 - c_{51}\frac{\partial N_0}{\partial z} + c_{11}N_0(1 - N_0) \tag{3.16}$$

is, after substituting $N_0(z)$, an explicit function of z and t. Combining equation (3.14) with equation (3.11) gives:

$$c_{60}\frac{\partial N_1}{\partial t} + \frac{\partial}{\partial z}DN_1 = -\frac{\partial \Omega_1}{\partial z} + N_0\frac{\partial T_1}{\partial z} \tag{3.17}$$

The left-hand side of equation (3.17) is a linear homogeneous expression in N_1, with coefficients independent of t, the right-hand side is an explicit function of z and t. It is therefore easily solvable by standard methods. In particular the Laplace transform of equation (3.17),

$$c_{60}pN_1^* + \frac{\partial}{\partial z}DN_1^* = -\frac{\partial \Omega_1^*}{\partial z} + N_0\frac{\partial T_1^*}{\partial z} \tag{3.18}$$

is an ordinary linear second-order differential equation. The asterisk is introduced to denote the Laplace transform of any function. Thus if x(z,t) is a function of z and t,

$$x^* = x^*(z,p) = \int_0^\infty e^{-pt}x(z,t)dt \tag{3.19}$$

The chief advantage of using the Laplace transform of equation (3.17) – besides the fact that it solves the equation without trouble in spite of complicated boundary conditions – is that the transforms T_1^*, c_{51}^*, and c_{11}^* are used on the right-hand side. Since the hydrodynamic part of the control problem will also be solved by using the Laplace transform, these quantities are immediately available.

The solution of (3.17) or (3.18) may be facilitated by transforming the dependent and independent variables. The expression DN_1 becomes, using the integrating factor,

$$J(z) = \exp\left[-\int_0^z \frac{c_{10}(1 - 2N_0) + P_0}{c_{50}}ds\right] \tag{3.20}$$

$$DN_1 = -c_{50}J^{-1}\frac{\partial}{\partial z}(JN_1) \tag{3.21}$$

The dependent variable is then chosen as JN_1. The quantities c_{10}, c_{50}, etc. are most conveniently expressed in terms of the N_0. This suggests changing the independent variable from z to N_0.

The expression for τ_2 has the form

$$\tau_2 = DN_2 + \Omega_2(z,t) \tag{3.22}$$

where D is the linear operator of equation (3.15) and

$$\Omega_2(z,t) = T_1 N_1 - c_{51}\frac{\partial N_1}{\partial z} + c_{11}N_1(1-2N_0) - c_{10}N_1^2 + T_2 N_0 -$$
$$- c_{52}\frac{\partial N_0}{\partial z} + c_{12}N_0(1-N_0) \tag{3.23}$$

Combining equation (3.22) with the equation (3.13)

$$c_{60}\frac{\partial N_2}{\partial t} + \frac{\partial}{\partial z}DN_2 = -\frac{\partial\Omega_2}{\partial z} + N_0\frac{\partial T_2}{\partial z} - c_{61}\frac{\partial N_1}{\partial t} + N_1\frac{\partial T_1}{\partial z} \tag{3.24}$$

The Laplace transform of this equation is

$$c_{60}pN_2^* + \frac{\partial}{\partial z}DN_2^* = -\frac{\partial\Omega_2^*}{\partial z} + N_0\frac{\partial T_2^*}{\partial z} - c_{61}\frac{\partial N_1^*}{\partial t} + N_1\frac{\partial T_1^*}{\partial z} \tag{3.25}$$

The process can, at least in principle, be continued indefinitely for N_3, N_4, ….

The boundary conditions for the differential equations will now be obtained. First of all the initial condition $N(z,0)=N_0(z)$ leads to

$$N_1(z,0) = N_2(z,0) = \ldots = 0 \tag{3.26}$$

Assume that there are reservoirs at each end of the plant, of holdup H (product end) and H' (waste end). The product and waste are withdrawn from the reservoirs. The mole fractions in the reservoirs are therefore called $N_P(t)$ and $N_w(t)$, respectively. The waste stream is called $W(t)$. The flow from the cascade into the top reservoir is called $c_5(P,t)$; the flow from the cascade into the bottom reservoir is $c_5(W,t)$. These quantities also have expansions as powers of σ.

$$W(t) = W_0 + \sigma W_1(t) + \sigma^2 W_2(t) + \ldots$$
$$H = H_0 + \sigma H_1(t) + \sigma^2 H_2(t) + \ldots$$
$$H' = H'_0 + \sigma H'_1(t) + \sigma^2 H'_2(t) + \ldots$$
$$N_P(t) = N_{P_0} + \sigma N_{P_1}(t) + \sigma^2 N_{P_2}(t) + \ldots \tag{3.27}$$
$$N_W(t) = N_{W_0} + \sigma N_{W_1}(t) + \sigma^2 N_{W_2}(t) + \ldots$$
$$c_5(P,t) = c_{50}(P) + \sigma c_{51}(P,t) + \sigma^2 c_{52}(P,t) + \ldots$$

The mole fractions, $N(z,t)$, and the transports, $T(z,t)$ and $\tau(z,t)$, will refer to the stripper if z has negative values.

The discontinuity in τ and T at s=0 implied in the change in the equations is accounted for by the introduction of feed (F, N_F).

$$T(0_+,t) - T(0_-,t) = F; \quad \tau(0_+,t) - \tau(0_-,t) = FN_F; \quad N(0_+,t) = N(0_-,t) \tag{3.28}$$

Note, however, that N_F does not have to be the same as $N(0,t)$, or even N_{F_0} the same as $N_0(0)$. F and N_F may both vary with time (the cascade under consideration might be the upper cascade of a combination of plants). Expanding equation (3.28) in powers of σ,

$$P_0 + W_0 = F_0; \quad P_0 N_{P_0} + W_0 N_{W_0} = F_0 N_{F_0}; \quad N_0(0_+) = N_0(0_-) \tag{3.29}$$

$$T_1(0_+,t) - T_1(0_-,t) = F_1; \quad \tau_1(0_+,t) - \tau_1(0_-,t) = F_0 N_{F_1} + F_1 N_{F_0}; \quad N_1(0_+,t) = N_1(0_-,t) \tag{3.30}$$

$$T_2(0_+,t) - T_2(0_-,t) = F_2; \quad \tau_2(0_+,t) - \tau_2(0_-,t) = F_0 N_{F_2} + F_1 N_{F_1} + F_2 N_{F_0};$$
$$N_2(0_+,t) = N_2(0_-,t) \tag{3.31}$$

The boundary conditions at the top of the rectifier are obtained from the relations

$$T(z_f, t) = P(t) + \frac{\partial H}{\partial t}$$

$$\tau(z_f, t) = P(t)N_P(t) + \frac{\partial}{\partial t}(HN_P) \tag{3.32}$$

$$c_5(P, t)[N(z_f, t) - N_P(t)] = H\frac{\partial N_P}{\partial t}$$

On expanding in power of σ and collecting, it is found that

$$T_0(z_f) = P_0 \,; \tau_0(z_f) = P_0 N_{P_0} \,; N_0(z_f) = N_{P_0} \tag{3.33}$$

Combining the first two equations from (3.32),

$$\tau_1(z_f, t) - P_0 N_{P_1} = H_0\frac{\partial N_{P_1}}{\partial t} + T_1(z_f, t)N_{P_0} \tag{3.34}$$

$$c_{50}(P)[N_1(z_f, t) - N_{P_1}(t)] = H_0\frac{\partial N_{P_1}}{\partial t}$$

and in the second order

$$\tau_2(z_f, t) - P_0 N_{P_2} - T_1(z_f, t)N_{P_1} - T_2(z_f, t)N_{P_0} = H_0\frac{\partial N_{P_2}}{\partial t} + H_1\frac{\partial N_{P_1}}{\partial t} \tag{3.35}$$

$$c_{50}(P)[N_2(z_f, t) - N_{P_2}(t)] + c_{51}(P, t)[N_1(z_f, t) - N_{P_1}(t)] = H_0\frac{\partial N_{P_2}}{\partial t} + H_1\frac{\partial N_{P_1}}{\partial t}$$

The boundary conditions at the bottom of the stripper are very similar.

4. CONCLUSIONS

A new approach to identification and control of the isotopic separation process has been presented. Using the proposed equations, parameters and boundary conditions, there are a variety of methods available for the control design. The identification is based on suitable input/output data. The validation of the generated model is the present preoccupation of the authors.

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