AQTR 2004 (THETA 14)
2004 IEEE-TTTC - International Conference on Automation, Quality and Testing, Robotics
May 13 – 15, 2004, Cluj-Napoca, Romania

THE CAPACITANCE OF AN ELECTRIC SIGNAL TRANSDUCER

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Abstract The paper presents an interesting application of the inversion method in electrostatics. The problem of electrical images and the capacitance for two spheres which intersect on the angle is solved. The two spheres system electrical field and the potential can be calculated. So the parameters of this electric signal transducer are well-determined.

Key words: electric charge, inversion method, electrical images, capacitance.

1.INTRODUCTION

The principle of the inversion method [1] is briefly the following:

The true punctiformal electric charges q_i are situated in the points Q_i, which are at the d_i distances from the point O in a homogeneous dielectric. The potential established by these charges in the P_j points which are at the R_j distances from the point O is V_j.

O is picked as a inversion pole and p^2 as the power of inversion.

In the Q_i' points, which are the reverses of the Q_i points, the values of the charges are:

$$q_i^{'} = \frac{p}{d_i} q_i \tag{1}$$

The reverse of P_j points are P_j'.

Knowing that [1]:

$$Q_i^{'} P_j^{'} = Q_i P_j \frac{p^2}{d_i R_j} \tag{2}$$

the potential established in the P_j' points by the q_i' charges (it supposes that the q_i charges are eliminated) is:

$$V_j^{'} = \frac{R_j}{p} V_j \tag{3}$$

The Σ equipotent surface (by the V_Σ potential) is being transformed in the inverse surface Σ' which is not equipotent.

If in the O pole it is put a supplementary charge:

$$q_0 = -4\pi\varepsilon p V_\Sigma \qquad (4)$$

than the Σ' surface becomes equipotent, having the potential zero.

If we suppose that $V_j = V_\Sigma$ then:

$$V_j^{"} = V_j^{'} + \frac{q_0}{4\pi\varepsilon p^2} R_j = V_\Sigma \frac{R_j}{p} + \frac{-4\pi\varepsilon p V_\Sigma}{4\pi\varepsilon p^2} R_j = 0 \qquad (5)$$

In these conditions the relation with the initial problem is being complicated. If the supplementary charge is not used, a new inversion of the q_i' system has as a result the initial q_i system.

2. THE IMAGES CHARGES AND THE CAPACITANCE FOR A SYSTEM OF TWO CONDUCTOR SPHERES WHICH MAKE A φ=π/n ANGLE

We consider the conductor spheres of O_1 and O_2 centre and radius R_1 and R_2 respectively, which make a φ angle. This means that in the cross section of fig. 1, $\angle O_1PO_2 = \pi - \varphi$.

We consider known R_1, R_2, φ and the total charge q_t of a conductor formed by the two spheres.

We can consider exactly the same that instead of q_t charge is known the V potential of conductor.

We propose to find the position and the value of images charges function of q_t (or function of V) and to calculate the conductor capacity.

For this purpose an inversion of P pole and $p^2 = 4R_2^2$ power is made.

The sphere of R_2 radius is transformed in the plan π_2, $PO_2 \perp \pi_2$, that is tangent in T_2 of the sphere (fig. 1).

The sphere of radius R_1 is transformed into the plan π_1 so that $\pi_1 \perp PO_1$ and:

$$PT_1 = \frac{4R_2^2}{2R_1} \qquad (6)$$

The circle of intersection of two spheres have PQ diameter and is transformed into the straight line $\pi_1 \cap \pi_2$ which is perpendicular to the drawing plan in O.

The sphere surface was equipotent, with V potential, but the surface of plans π_1 and π_2 is not equipotent.

If we would put a charge (4) in the P point:

$$q = -4\pi\varepsilon p V = -4\pi\varepsilon 2R_2 V \qquad (7)$$

so the surface π_1 and π_2 becomes at zero potential.

Results that in the absence of the q charge the potential of π_1 and π_2 surfaces is the same with the potential established by the electrical images of the q charge related the π_1 and π_2 plans.

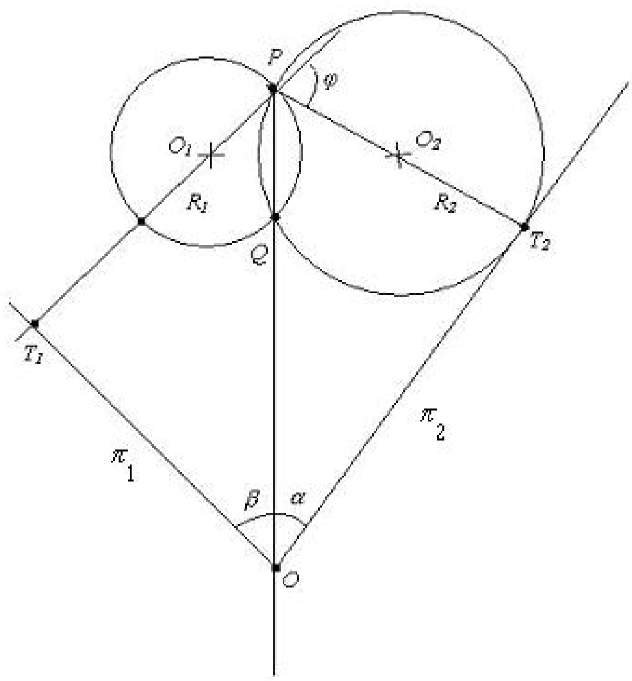

Fig. 1 – The two conductor spheres

This is a known problem (fig.2).

The number of images charges is 2n-1.

These charges are in the points $A_1, A_2, \ldots, A_{2n-1}$ which are situated on the circle of OP radius.

The first charge has the value –q is situated in A_1 and $\angle POA_1=2\alpha$, the second charge has the value q is situated in A_2 and $\angle POA_2=2\alpha+2\beta$, generally the charge of 2m-1 order (m=1,2, …, n) has the value –q, is situated in A_{2m-1} and $\angle POA_{2m-1}=$
$2\,[m\alpha+(m-1)\beta]=2(m\varphi-\beta)$, the charge of 2m order (m=1, 2, …, n-1)has the value q, is situated in A_{2m}, and $\angle POA_{2m}=2m(\alpha+\beta)=2m\varphi$.

The initial situation is obtained through a new inversion, a same pole and a same power.

The charges from $A_1, A_2, \ldots, A_{2n-1}$ are transformed into an images charges q_1', q_2', …, q'_{2n-1} situated in $A'_1, A'_2, \ldots, A'_{2n-1}$.

The point $A_1, A_2, \ldots, A_{2n-1}$ are situated in the circle with O centre and OP radius, so the reverses points $A'_1, A'_2, \ldots, A'_{2n-1}$ are situated in the straight line which is perpendicular from PO.

The reverse point of A_1 is exactly O_2 (because $PA_1 \cdot PO_2=4R_2 \cdot R_2=4R_2^2$) so the images charges are situated at the O_1O_2 straight line.

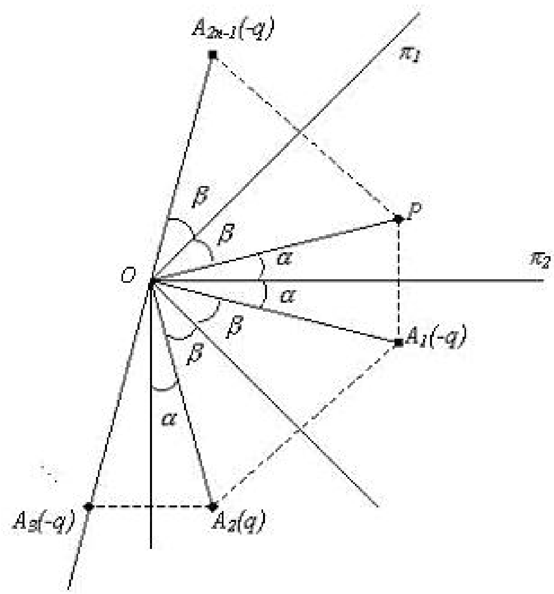

Fig. 2 – The electrical images of the q charge related the π_1 and π_2 plans

Now we make the calculus of the values and the position of images charges. We note $\lambda=R_2/R_1$.

Is calculated successively with fig. 1:

$$O_1O_2 = R_1\sqrt{\lambda^2 + 2\lambda\cos\varphi + 1} = R_1 c \tag{8}$$

We noted:

$$c = \sqrt{\lambda^2 + 2\lambda\cos\varphi + 1} \tag{9}$$

and

$$OP = r = \frac{2\lambda}{\sin\varphi} R_1 c \tag{10}$$

$$\sin\alpha = \frac{\sin\varphi}{c} \tag{11}$$

$$\sin\beta = \lambda\frac{\sin\varphi}{c} \tag{12}$$

With fig.2:

$$PA_{2m-1} = \frac{4R_2}{\sin\varphi}\cdot c\cdot\sin(m\varphi - \beta) \tag{13}$$

$$PA_{2m} = \frac{4R_2}{\sin\varphi} \cdot c \cdot \sin(m\varphi) \tag{14}$$

The position of images charges on O_1O_2 is given by:

$$PA'_{2m-1} = \frac{R_2 \sin\varphi}{c} \frac{1}{\sin(m\varphi - \beta)} \tag{15}$$

m=1, 2, …, n

The value of image charge of A'_{2m-1} results from (1):

$$q'_{2m-1} = -q \frac{\sin\varphi}{2c} \frac{1}{\sin(m\varphi - \beta)} \tag{16}$$

m=1, 2, …, n

We observe that for m=1, $PA'_1=R_2$ and for m=n, $PA'_{2n-1}=R_1$.

The position of even images charges is given by:

$$PA'_{2m} = \frac{R_2 \sin\varphi}{c} \frac{1}{\sin(m\varphi)} \tag{17}$$

m=1, 2, …, n-1

The value of image charge of A'_{2m} is:

$$q'_{2m} = q \frac{\sin\varphi}{2c} \frac{1}{\sin(m\varphi)} \tag{18}$$

m=1, 2, …, n-1

The system after the two inversions is identically with the initial system, and at the substitution of the plan charges (plan having zero potential) with the images charges, the total charge is not changed [1], [2].

So the total charge q_t is the sum of images charges:

$$q_t = \frac{q \sin\varphi}{2c} \left[\sum_{m=1}^{n-1} \frac{1}{\sin(m\varphi)} - \sum_{m=1}^{n} \frac{1}{\sin(m\varphi - \beta)} \right] \tag{19}$$

We note

$$S = \sum_{m=1}^{n} \frac{1}{\sin(m\varphi - \beta)} - \sum_{m=1}^{n-1} \frac{1}{\sin(m\varphi)} \tag{20}$$

or

$$S = \frac{1}{\sin\beta} + \sum_{m=1}^{n-1} \left(\frac{1}{\sin(m\varphi - \beta)} - \frac{1}{\sin(m\varphi)} \right) \tag{21}$$

and it results:

$$q = -q_t \frac{2c}{S \sin\varphi} \tag{22}$$

So the final form from the value of images charges is:

$$q'_{2m-1} = q_t \frac{1}{S\sin(m\varphi - \beta)} \tag{23}$$

$$q'_{2m} = -q_t \frac{1}{S\sin(m\varphi)} \tag{24}$$

The V potential of the spheres can be calculated at (7), (22):

$$V = -\frac{q}{8\pi\varepsilon R_2} = q_t \frac{c}{4\pi\varepsilon R_2 S\sin\varphi} \tag{25}$$

or directly with the images charges:

$$\begin{aligned} V &= \sum_{m=1}^{n} \frac{q'_{2m-1}}{4\pi\varepsilon PA'_{2m-1}} + \sum_{m=1}^{n-1} \frac{q'_{2m}}{4\pi\varepsilon PA'_{2m}} = \\ &= \sum_{m=1}^{n} \frac{q_t c}{4\pi\varepsilon R_2 S\sin\varphi} - \sum_{m=1}^{n-1} \frac{q_t c}{4\pi\varepsilon R_2 S\sin\varphi} = \\ &= q_t \frac{c}{4\pi\varepsilon R_2 S\sin\varphi} \end{aligned} \tag{26}$$

The conductor capacitance is from (26):

$$C = \frac{q_t}{V} = 4\pi\varepsilon R_2 \frac{S\sin\varphi}{c} \tag{27}$$

or with the detailed form with (9) and (21):

$$C = 4\pi\varepsilon R_2 \frac{\sin\varphi}{\sqrt{\lambda^2 + 2\lambda\cos\varphi + 1}} \left\{ \frac{1}{\sin\beta} + \sum_{m=1}^{n-1} \left[\frac{1}{\sin(m\varphi - \beta)} - \frac{1}{\sin(m\varphi)} \right] \right\} \tag{28}$$

Obviously there can be made er particularisation.

CONCLUSIONS

The central point of the paper is the supplementary charge which assures a zero potential on the transformed surface and the dealing mode for this charge.

The paper demonstrates that the inversion in electrostatics is a beautiful and very efficiently method.

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