# A GENERAL MODULAR DESIGN OF ELIN FILTERS BASED ON F<sup>-1</sup>NF MODELS

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**Abstract:** We present in this paper a systematic modular design method for ELIN circuits. Unlike other methods presented in literature this one can be simply and directly applied by substituting each linear block in the block diagram of the circuit by specific nonlinear modules. We give also some requirements for parameter settings. Examples are given in log – domain.

Keywords: ELIN circuits, log- domain filters, nonlinear integrators, modular design.

#### **1. Introduction**

There are various methods for designing Externally Linear Internally Nonlinear (ELIN) circuits and Log-domain techniques are mostly used. The general and systematic method is based on the state-space formulation. This one was introduced by Frey [1] and developed by other authors. Intuitive modular methods based on log-domain integrators were given [2, 3, 4]. In this paper we give a simple method for designing a linear circuit with an ELIN structure. The proceeding is based on the above named methods but it is more general, direct and does not need any intermediate flowgraph transformation. In a block diagram described by linear differential equations or transfer functions, by substituting each linear operating block by equivalent nonlinear modules the ELIN schematic results directly. We give examples in log-domain and make also some observations and clarify some conditions to be fulfilled.

### 2. Linear Block Diagram Transformations.

We give in this section a direct method to transform a linear block diagram described by transfer functions or linear differential equations into an ELIN diagram. Consider a linear relation between two variables:

r a linear relation between two variables:  
$$y_2 = \text{Lin}(y_1)$$

If  $y_i$  are signals with a nonlinear dependence on  $x_i$  of the form:

$$y_i = F(x_i); \quad x_i = F^{-1}(y_i); \quad i = 1, 2, ...$$
 (2)

we can express two signals  $x_1$  and  $x_2$  respectively by a nonlinear relationship

$$x_2 = F^{-1}(Lin(F(x_1))) = N(x_1)$$
 (3)

Taking into account the expressions (1), (2), and (3) the block diagrams in Fig. 1 result.

(1)

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Figures 2, 3 and 4 show some building block transformations based on Figure 1 and the following relations:





Fig 2. Equivalent Lin – ELIN serie – parallel connections



Fig 3. Equivalent Lin – ELIN connection for a summing input port

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Fig 4. Behavioural model of the multiple input nonlinear building block.

The simple and direct transformations from Figures 1- 4 show that departing from a block diagram, one can substitute each linear building block by a corresponding nonlinear block. Adding them input and output F- F<sup>-1</sup> block terminals the ELIN form of the t. f. is obtained.

Some equivalent L- ELIN block diagrams are shown in figure 5.



Fig 5. Some equivalent L- ELIN filter block diagrams a) Negative feedback structure b)Leap-frog structure

The above transformations are indepent of the order of the linear differential equations symbolized by L and the nature of signals y (currents or voltages) and also of the function F. If F-F<sup>-1</sup> are **exp- ln** functions, characteristic for bipolar transistors:

$$i_{\rm C} = I_{\rm S} \cdot e^{\int_{\rm BE}^{\rm V_T}}; \quad v_{\rm BE} = V_{\rm T} \cdot \ln \frac{I_{\rm C}}{I_{\rm S}}$$
(6)

the ELIN circuits are in the log- domain. In this case it is advantageous to have currents in/out  $(y \rightarrow i)$  for the linear basic block and voltages for the nonlinear blocks  $(x \rightarrow y)$ . The simplest and most frequently used N blocks are the nonlinear integrators.

#### 3. Nonlinear log-domain integrators.

Based on exponential cells represented in Fig. 6 nonlinear log- domain structures can be deduced. Integrators are mostly used in such schematics.

Their structures were already given in literature but we would make some observations in order to simplify the setting of parameters and the direct drawing-up the schematics.

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Fig 6. Exponential transconductor cells with opposite polarities a) positive e transconductor; b) negative transconductor with two symbols

#### a) Summer-integrator

Lin: 
$$\frac{\tau di_o}{dt} = k_1 \cdot i_1 + k_2 \cdot i_2 + \dots = \sum_{i=1}^{n} k_i \cdot i_j$$
 (7)

 $k_1, k_2,...$  are real numbers positive or negative,  $\tau > 0$  is the time constant i<sub>1</sub>, i<sub>2</sub>, are input currents

The corresponding nonlinear function between voltages is derived by substituting all the currents with nonlinear **exp** functions:

$$\mathbf{i}_{o} = \mathbf{I}_{o} \cdot \mathbf{e}^{\mathbf{v}_{o}} \mathbf{v}_{A}$$
;  $\frac{\mathrm{d}\mathbf{i}_{o}}{\mathrm{d}t} = \frac{\mathbf{I}_{o}}{\mathbf{V}_{A}} \cdot \mathbf{e}^{\mathbf{v}_{o}} \mathbf{v}_{A} \cdot \frac{\mathrm{d}\mathbf{v}_{o}}{\mathrm{d}t}$ ;  $\mathbf{i}_{j} = \mathbf{I}_{j} \cdot \mathbf{e}^{\mathbf{v}_{j}} \mathbf{v}_{A}$ ;  $j = 1, 2, \dots$ 

(8)

If

If one considers the voltage  $v_0$  across a capacitor C, current  $i_C$  is of the form:

$$i_{C} = C \cdot \frac{dv_{o}}{dt} = \frac{k_{1} \cdot C \cdot V_{A}}{\tau \cdot I_{O}} \cdot I_{1} \cdot e^{(v_{1} \cdot v_{o})/v_{A}} + \frac{k_{2} \cdot C \cdot V_{A}}{\tau \cdot I_{O}} \cdot I_{2} \cdot e^{(v_{2} \cdot v_{O})/v_{A}} + \dots$$
(9)

we choose 
$$\frac{\mathbf{C} \cdot \mathbf{V}_{\mathbf{A}}}{\mathbf{\tau}} = \mathbf{I}_{\mathbf{0}}$$
 (10)

current 
$$i_{C}$$
 becomes:  $\mathbf{i}_{C} = \sum_{j=1}^{n} \mathbf{k}_{j} \cdot \mathbf{I}_{j} \cdot \mathbf{e}^{(\mathbf{v}_{j} \cdot \mathbf{v}_{0}) / \mathbf{v}_{A}}$  (11)

The nonlinear voltage integrator is shown in Figure 7.

#### **Requirements and particular cases.**

- Current i<sub>C</sub> is positive, negative or zero so • that the circuit configuration should permit this current flow. Therefore equation (11) can be implemented only if it has at least two terms (n=2) and in this case  $k_1k_2 < 0$
- If the integrator to be implemented has only • one input signal, the second term in the right hand sum (7) will be considered a constant current without deteriorating the transfer function, that is:



Fig 7. Nonlinear integrator with multiple inputs

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$$\boldsymbol{\tau} \cdot \frac{\mathrm{d}\mathbf{i}_{o}}{\mathrm{d}\mathbf{t}} = \mathbf{k}_{1} \cdot \mathbf{i}_{1} + \mathbf{k}_{2} \cdot \mathbf{I}; \quad \mathbf{H}(\mathbf{s}) = \frac{\mathbf{i}_{o}(\mathbf{s})}{\mathbf{i}_{1}(\mathbf{s})} = \frac{\mathbf{k}_{1}}{\boldsymbol{\tau} \cdot \mathbf{s}}; \tag{12}$$

with 
$$k_1 \cdot k_2 < 0$$
;  $i_2 = I$ ;  $v_2 = 0$ 

Currents  $I_i$  in relations (8) and (11) are chosen so that when all the signals are zero, circuits should have appropriate biasing currents Ii:

$$\sum_{j=1}^{n} \mathbf{k}_{j} \cdot \mathbf{I}_{j} = \mathbf{0}$$
(13)

If the integrator has a negative feedback, one of the currents is  $i_0$  and the • corresponding term in the sum (11) becomes:  $k_0 I_0$ . This case is specific for a lossy integrator:

$$\tau \frac{d\mathbf{i}_{o}}{dt} + \mathbf{i}_{o} = \mathbf{k}_{1} \cdot \mathbf{i}_{1} \quad \text{or} \quad \tau \frac{d\mathbf{i}_{o}}{dt} = \mathbf{k}_{1} \cdot \mathbf{i}_{1} - \mathbf{i}_{o} \quad ; \quad \mathbf{H}(\mathbf{s}) = \frac{\mathbf{k}_{1}}{\tau \cdot \mathbf{s} + 1} \tag{14}$$
$$\mathbf{k}_{1} \cdot \mathbf{I}_{1} - \mathbf{I}_{0} = 0 \tag{15}$$

$$\cdot I_1 - I_0 = 0$$

k

The general and the resulting scheme respectively are given in Figure 8



The above proceeding permits a direct design of circuit departing not only from the state space equations but also from the block diagram of a certain function. **Example: BP Filter** 



Fig 9. Band Pass Filter a) Block diagram b) ELIN diagram c) Log-Domain circuit

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**Fig 10 Simulated** schematic for k=1,  $\tau_1 = Q/\omega_0$ ,  $\tau_2 = 1/\omega_0 Q$ , Q=2,  $f_0 = 600$  KHz,  $C_1 = C_2$ 



# Conclusion.

Using specific nonlinear voltagemode blocks a linear current-mode circuit can be directly reconfigured departing from its block diagram. This direct procedure is a general one and the reconfigured block diagram does not depend on the F function. Parameters and circuit structure depend on F. Relations between parameters and requirements for log-domain circuits have been also deduced.

Fig 11 Frequency characteristic for different  $I_O$ 

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