A&QT-R 2004 (THETA 14)
2004 IEEE-TTTC International Conference on Automation, Quality and Testing, Robotics
May 13 – 15, 2004, Cluj-Napoca, Romania

Robust Control of Field-Oriented Drive System with Permanent-Magnet Synchronous Servomotor

Iulian BIROU, Calin RUSU

Department of Electrical Drives and Robots, Technical University of Cluj-Napoca
Daicoviciu 15, 3400 Cluj-Napoca, Fax:+40-64-194924, E-mail: birou@edr.utcluj.ro

Abstract: A simplified transfer function of the machine is used to design various speed controllers: classic PI, H_2 and H_∞ ones. The controllers are tuned for the nominal plant, around which the mathematical model of the machine was linearised. The vector-control-based driving system was simulated, imposing different perturbations: load steps, moment of inertia, friction coefficient. Robustness was analysed.

Key words: **.** Robust control, field-oriented AC machine, PI and H_∞ speed controller.

1. INTRODUCTION

In most of the modern drive systems with AC machines which require rotor speed control, the main task is to develop a robust controller, able to achieve high dynamic performance and to maintain the system response within specified tolerances, for a large range of the reference speed values and variations of perturbances, like: load torque, total inertia moment, friction coefficient, etc. [1], [4], [5].

The designing procedure of the speed controllers can be very difficult, if a complex mathematical model of the plant (here of the AC machine) is used. But robust controllers keep the dynamic and stability performance of the controlled system even if structured or unstructured uncertainties appear. That's why, robust speed controllers will be designed using simplified models of the AC machines, and have to be used in a complex structure based on the vector-control principle [2].

2. DESIGN OF THE ROBUST H¥ CONTROLLER

We intend to apply the designing procedure of the robust H_∞ controller to the speed control loop, presented in figure 1. The speed controller was tuned for a speed step, from zero to the rated value, by imposing the following performance criteria: *stationary error* $\varepsilon_{stp} = 0$; *overshoot* $\sigma \leq 5\%$; *response time* $t_r = 0{,}15$ [sec]; *crossover band* $\Delta\omega_B \leq 150$ [rad/sec].

The design specifications in frequency domain are:

- *robust performance specifications*: minimizing the sensitivity function *S* (reducing it at least 100 times to approximate 0.3333 rad/sec).
- *robust stability specifications*: -40 dB/decade roll-off and at least -20dB at a crossover band of 100 rad/sec.

The H_∞ optimal control designing problem in the particular case of applying the small gain problem is to form an augmented plant of the process $P(s)$ with the weighting functions $W_1(s)$ and $W_3(s)$ to find an optimal stabilizing H_∞ controller, so that the infinity norm of the cost function T_{y-u} is minimized and is less then one [8], [9]:

$$\left\| T_{y-u} \right\|_\infty < 1. \tag{1}$$

Considering the robust stability and robust performance criteria, the weighting functions for the optimal H_∞ controller and the same speed control loop with the PM synchronous motor are:

$$\begin{cases} \dfrac{1}{W_1(s)} = W_1^{-1}(s) = \dfrac{1}{\gamma} \cdot \dfrac{(3s+1)^2}{100} \\ \dfrac{1}{W_3(s)} = W_3^{-1}(s) = \dfrac{150}{s+145} \end{cases}, \tag{2}$$

where γ represents the actual step value. The iterative process continues, until the graphic representation in Bode diagram of cost function T_{y-u} reach his maximum value in the proximity of 0 dB axis. In our case, for γ=39,75 we obtain the infinite norm $\left\| T_{y-u} \right\|_\infty = 0{,}9999$, and the corresponding H_∞ speed controller is:

$$H_\infty(s) = \frac{2327s^2 + 22211s + 16495}{s^3 + 822951s^2 + 548632s + 91442}. \tag{3}$$

The dynamic performances and the robust and stability performance criteria are performed. The sensitivity function S(s) and the complementary sensitivity function T(s) of the close loop for the nominal plant are:

$$T(s) = \frac{155{,}7s^2 + 1486s + 1103{,}5}{s^3 + 157{,}2s^2 + 1486{,}5s + 1103{,}6}, \tag{4}$$

and

$$S(s) = \frac{1{,}015s^3 + 1{,}5s^2 + 0{,}66s + 0{,}09}{s^3 + 157{,}2s^2 + 1486{,}5s + 1103{,}6}. \tag{5}$$

The weighting functions $W_1(s)$ and $W_3(s)$, and the sensitivity functions $S(s)$ and $T(s)$ are presented in figure 2. From this diagram results the influence of the weighting function $W_3^{-1}(s)$ to limit the peak value of $T(s)$ function. The output of the speed controller, i.e. the active current component was limited.

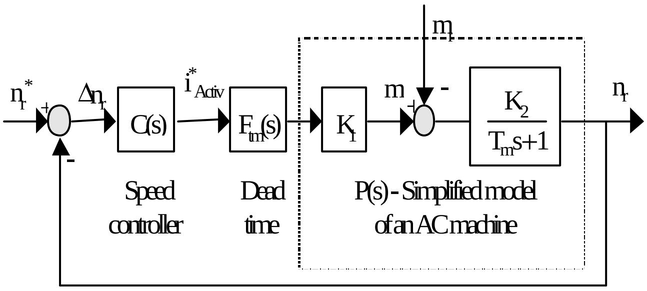

Fig. 1. Simplified speed close-loop control structure of a PM-SM.

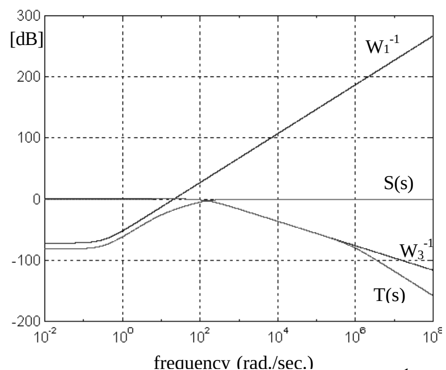

Fig. 2. Weighting functions $W_1^{-1}(s)$ $W_3^{-1}(s)$ and sensitivity functions $S(s)$ and $T(s)$.

The logarithmic Bode diagram and the Nyquist diagram of the direct-loop transfer function of the weighted process are presented in figure 3. According to them we establish the following stability parameters: crossover band $\Delta\omega_B$ = 153,7 rad./sec.; stability margins: gain margin = 130,3 dB; phase margin = 86,8°. For the same performance and robust stability specifications, a great number of weighting functions described by equation (2) can be chosen, so the solution of designing an optimal H_∞ controller is not unique [6], [10]. To analyse if the speed control structure with the H_∞ controller presented in (3) is robust stable, we apply the stability theorem for two different types of perturbation in the drive system, namely a highest variation of total inertia moment from J_{mot} to $10J_{mot}$ and a highest variation of friction coefficient from B_{mot} to $100B_{mot}$. The condition $\left\| \overline{\Delta_M}(s) T(s) \right\|_\infty < 1$ must be tested, where $\overline{\Delta_M}(s)$ represents the greatest multiplicative uncertainty for the nominal plant.

3. STABILITY ANALYSE FOR A VARIATION FROM J_{MOT} TO $10J_{MOT}$.

Considering the calculus way of the transfer function of the process, a ten times growing of the inertial moment, practically means a ten time growing of the time constant of the fixed part. The transfer functions of the nominal process and of the disturbed process are:

$$\begin{cases} P_N(s) = \dfrac{K_m}{T_m s + 1} \\ P(s) = \dfrac{K_m}{10T_m s + 1} \end{cases}, \tag{6}$$

and the existent relation between the nominal process, disturbed process, and the maximal multiplicative uncertainty is:

$$P(s) = P_N(s) \cdot (1 + \overline{\Delta_M}(s)) . \tag{7}$$

Using (7) and considering T_m=1,232 [sec], the multiplicative uncertainty in the case of ten times growing the inertial moment J, can be modeled as follows:

$$\overline{\Delta_M}(s) = \frac{-9T_m s}{10T_m s + 1} = \frac{-11{,}097s}{12{,}33s + 1} \tag{8}$$

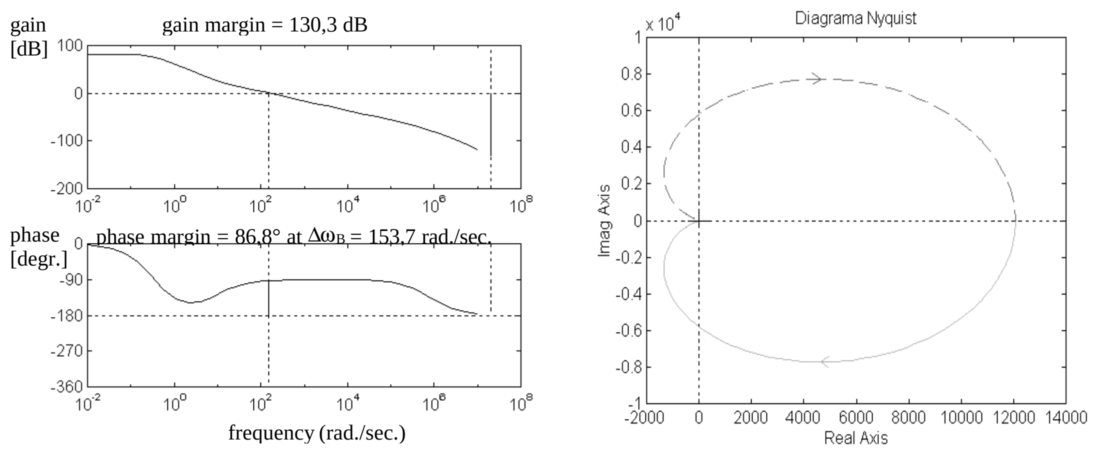

Fig. 3. Direct-loop transfer function of the weighted process in Bode and Nyquist diagram.

Known the expression of the complementary sensibility function T(s), the condition of robust stability can be determined, beeing $\left\| \overline{\Delta_M}(s)T(s) \right\|_\infty = 0{,}9359$. So, the control system with H_∞ controller remains robust stable for a variation of 10 times of the total inertial moment of the system, related to the catalogue one. Figure 4 shows the direct-loop transfer function $H_d(s)$ family curves for the PI controller and for the optimal H_∞, controller, at variations of the inertial moment of the synchronous motor, starting at J_{mot} value, from 3, 5, 7 and 10 times of this value. As we can see from the presented graphs, at the variations of J, though the PI controller doesn't go in instability, thus it is more sensitive at the parameter variations than the H_∞ controller. This shows a better robustness of the H_∞ optimal regulator.

4. STABILITY ANALYSE FOR A VARIATION FROM B_{MOT} TO $100B_{MOT}$.

Considering that the largest variation of the frictional coefficient of the mechanical system is the 100 times growing of the B_{mot} catalogue value. A 100 times growing of the frictional coefficient practically means a 100 times diminution of the time constant and of the amplification factor of the fixed part. Considering the above mentioned, the transfer functions of the nominal process and of the disturbed processes are:

$$\begin{cases} P_N(s) = \dfrac{K_m}{T_m s + 1} \\ P(s) = \dfrac{K_m/100}{(T_m/100)s + 1} \end{cases} . \qquad (9)$$

Using relation (6) and the time constant of the nominal process, the maximal multiplicative uncertainty, in the case of 100 times growing of the B frictional coefficient, can be modeled as:

$$\overline{\Delta_M}(s) = \frac{-99}{T_m s + 100} = \frac{-99}{1{,}232s + 100} . \qquad (10)$$

The robust stability condition will be expressed as:

$$\left\| \overline{\Delta_M}(s)T(s) \right\|_\infty = 0{,}9999 . \qquad (11)$$

We can find that the control system with the H_∞ controller remains robust stable for a 100 times variation of the friction coefficient.

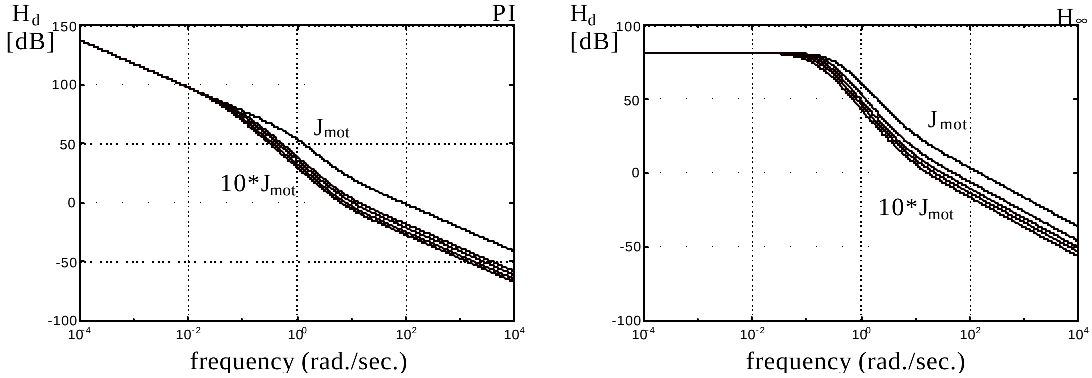

Fig. 4. Bode diagram of direct-loop transfer functions for PI and H_∞ controller, at different inertia moment values.

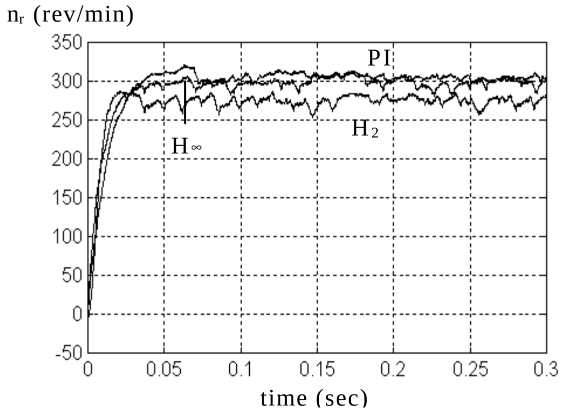

Fig. 5. Speed response at low speed step.

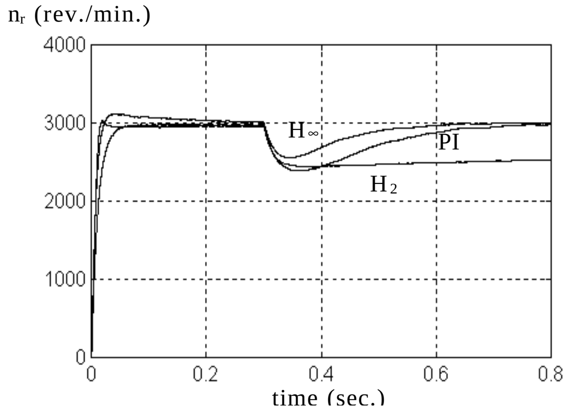

Fig. 6. Speed response at speed and load step.

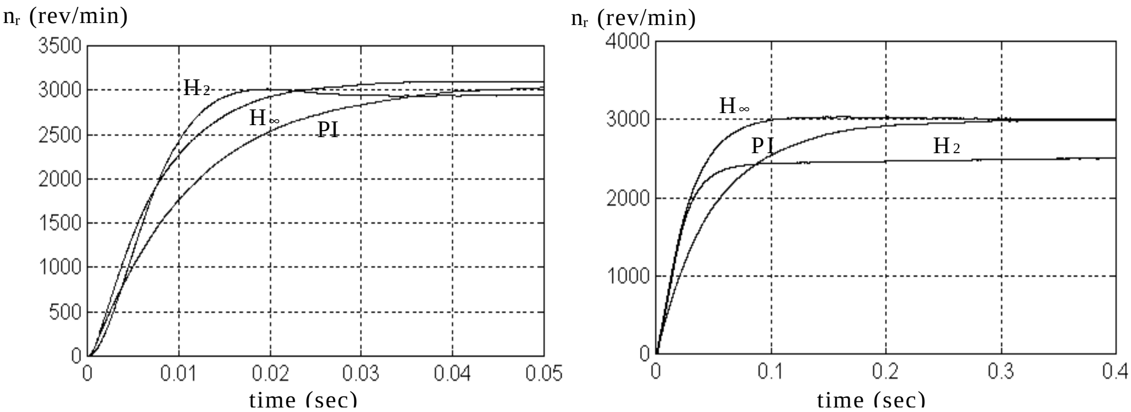

Fig. 7. Speed response for nominal process ($B=B_{mot}$ and $J_{tot}=J_{mot}$).

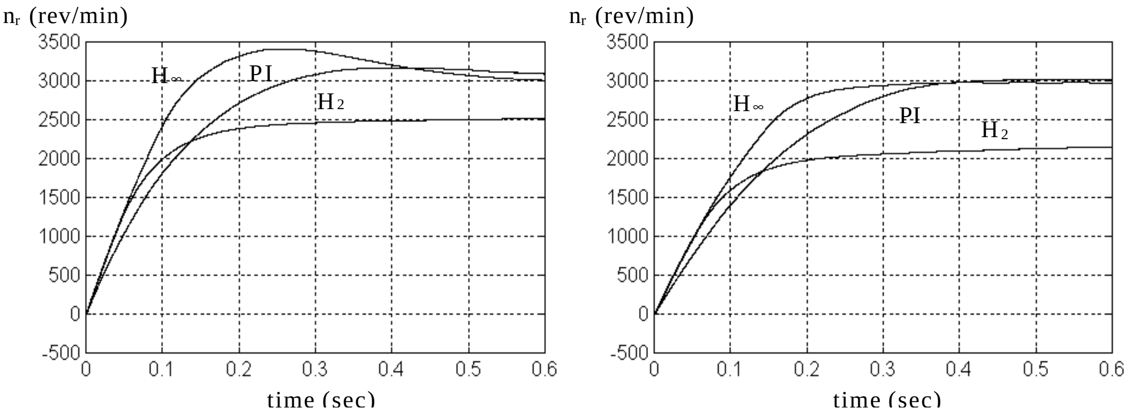

Fig. 8. Speed response for perturbed moment of inertia ($B= B_{mot}$ and $J_{tot}=11\ J_{mot}$).

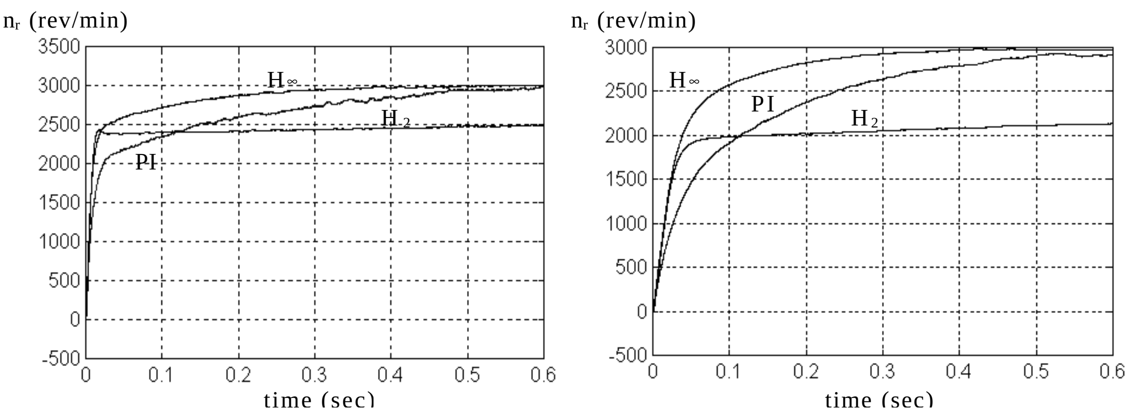

Fig. 9. Speed response for perturbed friction coefficient ($B=50\ B_{mot}$ and $J_{tot}=J_{mot}$).

CONCLUSIONS

A speed field-oriented control of the permanent magnet synchronous motor was simulated in MATLAB using the designed PI and H_∞ controllers and a robust H_2 controller [10]. The synchronous servomotor is a Stoeber ES42 one having: rated speed n = 3000 rev/min; rated torque M = 1,7 Nm; rated power P = 530 W; motor constant k = 1,05 Nm/A; PM flux Ψ_{PM} = 0,2334 Wb; inertia moment J = 1,85*10^{-4} kgm^2; friction coefficient B = 5*10^{-5} Nm(rad/sec)$^{-1}$.

In figure 5 a speed response of the nominal process was simulated for a 300 rev./min. speed step. In figure 6 we have the response for a 3000 rev./min. speed step and a nominal torque step at t=0.3 sec. Figure 7 presents the simulated results for a speed control of the synchronous machine, with PI, H_2, and H_∞ controller, having all parameters at the nominal value. In figure 8, the speed response is for a perturbed plant with B_{tot}=50B_{mot} for the same imposed speed step and nominal torque from t=0. The speed response for a similar simulation of a perturbed plant with moment of inertia variation J_{tot}=11J_{mot} is presented in figure 9. For the nominal plant the dynamic performances at a speed step are similar for all three controllers. It is normal to be so, because the controllers have been designed using a simplified model of the machine, working in the stady state nominal point. The advantage of using optimal H_∞ robust controller is evident in the presented simulations when the nominal plant is perturbed, by changing the load torque, the total moment of inertia or the friction coefficient

In conclusion, we consider that the H_∞ optimal robust controller ensures good dynamical performances and stability for a domain of variation large enough of the parameters that can be modified in the process. In applications where electromechanical parameter variations or load perturbations appear (such as robot control), performant drive systems with AC machines can be considered, by using robust speed (or position) controllers.

5. REFERENCES

[1] I. Birou, M. Imecs, (1998) „Robust control of fiel-oriented alternative current drive system,“ In *Proc. Of the 8th PEMC'98 Conference*, Praga, Czech Republic, Sept. 8-10, Vol. 4, pp. 4.222- 4.227.
[2] A. Kelemen, M. Imecs (1991) *Vector Control of AC Drives. Vol. II.: Vector Control of Synchronous Machine Drives*. Omikk Publisher, Budapest.
[3] V. Ionescu, C. Oara, M. Weiss, (1998) *Generalized Ricatti Theory and Robust Control: A Popov Function Approach*, John Wiley & Sons Ltd., Chichester, New York.
[4] P. C. Krause, O. Wasynczuk, S.D. Sudhoff (1995), *Analysis of Electrical Machinery*. In IEEE Press, IEEE Power Engineering Society, IEEE Inc.
[5] W. Leonhard (1985), *Control of Electrical Drives*, Springer-Verlag, Berlin.
[6] G. Zames (1996), "Input-output feedback stability and robustness". *In IEEE Control Systems*, Vol. 16, No. 3, June, pp. 61-66.
[7] P. Vas (1994), *Vector Control of AC Machines*. Clarendon Press, Oxford.
[8] H. Kwakernaak (1993), "Robust control and H_∞ optimization". Tutorial paper. *In Automatica*. Vol. 29, No. 2, Great Britain, pp. 255-273.
[9] J.C. Doyle, K. Glover, P.P. Khargonekar, B.A. Francis (1989) „State space solutions to standard H_2 and H_∞ control problems,“ In *IEEE Transactions on Automation and Control*, Vol. 34, pp. 831-847.
[10] R.Y. Chiang, M.G. Safonov (1992), *Robust Control Toolbox. User's Guide*. The Math Works Inc., Massachusetts.