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MATHEMATICAL TECHNIQUES FOR OPTIMIZATION OF THE POWER FACTOR IN ENERGY PROCESSING SYSTEMS

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Abstract. This paper describes a new mathematical method to calculate the maximum of the power factor in the electroenergetical circuits. The relative values which characterize the tension and current harmonics have been used. The given example will prove, comparatively, the efficiency and the application of this method.

Key words: harmonics of tension and current, relative values, power factor, Hessian matrix, proper values.

1. INTRODUCTION

The electroenergetical systems contain tension and current harmonics within the acceptable limits. These limits can be established by a set of relative values of a tension and current harmonics which characterise the distribution network and the receptors [1] One of the important factors that established the quality of the electrical energy and the performances of an electroenergetical system is the power factor. It is very useful to determine the maximum of the power factor in correlation with the relative values that characterise the tension and current harmonics.

An optimum case may be considered in order to easily measure and compare it with the acceptable values of non sinusoidal regime.

2. DEFINITION OF THE POWER FACTOR IN STEADY STATE LINEAR NETWORKS

Let's consider an electroenergetical system in non sinusoidal regime and a linear receptor. The receptor being linear and supplied with a nonsinusoidal tension, the number of current harmonics is equal with the number of the tension harmonics. To illustrate the relative contributions of all harmonics of current and voltage, we can use two formulae equivalents [2] of the Fourier series. For example, the characterization at the input terminals of the linear receptor introducing a φ_k phase angle on each harmonic, supplied with a nonsinusoidal voltage *u(t)* whose the development in a Fourier's series is truncated only at n terms, can be done as follows:

$$u(t)=U\sqrt{2}\sum_{k=1}^{n} b_k sin(k\omega t+\gamma_k)=U_1\sqrt{2}\sum_{k=1}^{n}\mu_k sin(k\omega t+\gamma_k), \tag{1}$$

and the current

$$i(t)=I\sqrt{2}\sum_{k=1}^{n} a_k sin(k\omega t+\gamma_k-\varphi_k)=I_1\sqrt{2}\sum_{k=1}^{n}\varepsilon_k sin(k\omega t+\gamma_k-\varphi_k). \tag{2}$$

The relative values b_k and a_k represent the ration between the r.m.s. of each k harmonic of tension, respectively current, and the r.m.s. of the tension (U) respectively current (I):

$$b_k=\frac{U_k}{U}, \tag{3}$$

$$a_k=\frac{I_k}{I}, \quad . \tag{4}$$

and, finally, the others relative values μ_k and ε_k represent the ratio between the r.m.s. of each k harmonic of tension and, respectively, current, and the r.m.s. of the fundamental of tension (U_1) respectively current (I_1), [3]:

$$\mu_k=\frac{U_k}{U_1}, \tag{5}$$

$$\varepsilon_k=\frac{I_k}{I_1}. \tag{6}$$

The following expressions are always true [4]:

$$\sum_{k=1}^{n} a_k^2=\sum_{k=1}^{n} b_k^2=1, \tag{7}$$

$$b_i^2=\frac{\mu_i^{\,2}}{\sum_{k=1}^{n}\mu_k^{\,2}}, \quad a_i^{\,2}=\frac{\varepsilon_i^{\,2}}{\sum_{k=1}^{n}\varepsilon_k^2}, \quad \forall i=1,...,n, \tag{8}$$

$$\varepsilon_1=\mu_1=1. \tag{9}$$

If we take into consideration the classical definition of the power factor

$$K_P=\frac{P}{S}, \tag{10}$$

we obtain, for a linear receptor, a lot of analogue relations expressed of the relative values of harmonics of tension b_k, μ_k, or current a_k, ε_k [5]. For example:

$$K_P = cos\,\varphi \sum_{k=1}^{n} \frac{\mu_k}{\sqrt{(cos^2\varphi + k^2 sin^2\varphi)\sum_{k=1}^{n}\mu_k^2}} \tag{11}$$

where $cos\,\varphi$ is the nominal power factor of receptor.

3. MATHEMATICAL OPTIMIZATION OF THE POWER FACTOR

We consider the expression (11) of the power factor and we want to find its maximum. Several important mathematics properties can be employed [6].

Let's take a function

$$K_P(\mu) = K_P(1,\mu_2,\mu_3,...,\mu_n) : X^{n-1} \rightarrow X,\; X = [0,1] \subset R. \tag{12}$$

For all the points μ_0, it is easy to prove that the function $K_P(\mu)$ is continuous and is a C^0 class:

$$\forall \mu_o = (1,\mu_{2o},\mu_{3o},...,\mu_{no}) \in X^{n-1},\; \lim_{\mu\rightarrow\mu_0} K_P(\mu) = K_P(\mu_0),\quad and\; K_P \in C^0(X^{n-1}) \tag{13}$$

In a similar way, we can demonstrate that the function $K_P(\mu)$ is a C^1 class:

all the partial derivation $\frac{\partial K_P}{\partial \mu_k} : X^{n-1} \rightarrow X,\; 2 \le k \le n$ are continuous and $K_P \in C^1(X^{n-1})$, (14)

and C^2 class:

all the partial derivation $\frac{\partial}{\partial \mu_j}(\frac{\partial K_P}{\partial \mu_k}) : X^{n-1} \rightarrow X,\; 2 \le j,k \le n$ are continuous and $K_P \in C^2(X^{n-1})$. (15)

If the function $K_P(\mu)$ is of class C^1, then the local extremes of $K_P(\mu)$ are among the solutions situated in X, of the non linear system:

$$\frac{\partial K_P}{\partial \mu_2}(1,\mu_2,\mu_3,...,\mu_n) = 0,...,\frac{\partial K_P}{\partial \mu_n}(1,\mu_2,\mu_3,...,\mu_n) = 0, \tag{16}$$

Moreover, if $K_P(\mu)$ is function of class C^2 and $a = (1,\mu_{2,max},...,\mu_{n,max})$ is a local extreme (critical point) of $K_P(\mu)$, which means that $dK_P(a) = 0$, in case the quadratic form $d^2K_P(a)$ is positively (negatively) defined, then the point a is a minimum (maximum) local point for $K_P(\mu)$. From the linear algebra, it is known that the quadratic form $d^2K_P(a)$ is positively (negatively) defined of its associate Hessian matrix

$$H=\left[\frac{\partial K_P}{\partial\mu_j\partial\mu_k}(a)\right]_{2\le j,k\le n},\dim H=n-1, \tag{17}$$

has all its proper values strictly positive (negative). The matrix H being symmetrical all its proper values are real, that the roots of characteristic equation

$$P(\lambda)=\det(\lambda U_{n-1}-H)=0, \tag{18}$$

are always real. For solved the non linear system equations (18), we calculate

$$\frac{\partial K_P}{\partial\mu_j}=\cos\varphi\left[\mu_j\sum_{\substack{i=1\\i\ne j}}^{n}\left(-\frac{\mu_i^2}{\sqrt{\cos^2\varphi+i^2\sin^2\varphi}}\right)+\frac{\sum_{\substack{i=1\\i\ne j}}^{n}\mu_i^2}{\sqrt{\cos^2\varphi+j^2\sin^2\varphi}}\right], \tag{19}$$

where $\mu_1=1, 2\le j\le n$.

We obtain the local extremes:

$$\mu_{j,max}=\frac{\sum_{\substack{i=1\\i\ne j}}^{n}\mu_i^2}{(\cos^2\varphi+j^2\sin^2\varphi)\sum_{\substack{i=1\\i\ne j}}^{n}\frac{\mu_i^2}{\cos^2\varphi+i^2\sin^2\varphi}}, 2\le j\le n. \tag{20}$$

4. EXAMPLE

Let's consider a linear receptor, supplied with nonsinusoidal tension with k=3 and $\mu_1=1$.

The critical points is determined for the system (16),

$$\frac{\partial K_P}{\partial\mu_2}(1,\mu_2,\mu_3)=0,\frac{\partial K_P}{\partial\mu_3}(1,\mu_2,\mu_3)=0, \tag{21}$$

and results:

$$\mu_2 = \frac{1+\mu_3^2}{\sqrt{B}(\frac{1}{\sqrt{A}} + \frac{\mu_3}{\sqrt{C}})}, \tag{22}$$

$$\mu_3 = \frac{1+\mu_2^2}{\sqrt{C}(\frac{1}{\sqrt{A}} + \frac{\mu_2}{\sqrt{B}})}, \tag{23}$$

where: $A = cos^2 \varphi + sin^2 \varphi;\ B = cos^2 \varphi + 4 sin^2 \varphi;\ C = cos^2 \varphi + 9 sin^2 \varphi.$ (24)

We get the equation:

$$\sqrt{AB}\mu_2^5 - \mu_2^4 + 2\sqrt{AB}\mu_2^3 + (C - \frac{C}{B} - 2)\mu_2^2 + (C\sqrt{\frac{B}{A}} + \sqrt{AB} - \frac{2C}{\sqrt{AB}})\mu_2 - (\frac{C}{A} + 1) = 0, \tag{25}$$

and we keep only the positive and sub unitary solutions $\mu_{2,max}$. These solutions will help to determine the positive and sub unitary values of $\mu_{3,max}$ of relation (23), and then the Hessian can be calculated:

$$H = \begin{bmatrix} \frac{\partial^2 K_P}{\partial \mu_2^2}(a) & \frac{\partial^2 K_P}{\partial \mu_2 \partial \mu_3}(a) \\ \frac{\partial^2 K_P}{\partial \mu_3 \partial \mu_2}(a) & \frac{\partial^2 K_P}{\partial \mu_3^2}(a) \end{bmatrix}, \tag{26}$$

where $a = (1, \mu_{2,max}, \mu_{3,max})$. Then the proper values of H can be determined as roots of characteristic equation:

$$P(\lambda) = det(\lambda U_2 - H) = 0, \tag{27}$$

the function K_p having a local maximum only if these roots are negative.

If we write the MATHCAD calculation sequence for the relations (22), (23), (24), (25), (26) and (27) where the sole numeric input variable is $cos\,\varphi$, then we obtain the results shown in table 1. We find that the function K_P has the points of local maximum for all the values $cos\,\varphi \in (0,1;0,8)$.

cosφ	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
μ_2	0,6398	0,6503	0,6687	0,6968	0,7374	0,7951	0,8768	0,9911	1,1441
μ_3	0,3572	0,3631	0,3736	0,3899	0,4141	0,4503	0,5059	0,5965	0,7603
λ_1	-0,728	-0,76	-0,797	-0,837	-0,88	-0,928	-0,991	-1,143	-1,475
λ_2	-0,709	-0,725	-0,747	-0,779	-0,822	-0,883	-0,978	-1,096	-1,3241

Table1.

5. CONCLUSIONS

It is remarkably useful, both in theory and appliance, to determine the maximum of the power factor depending on the relative values of the tension and current harmonics. Consequently, then has been found a calculation method that can apply to all circumstances, while the criteria to establish the maximum have been mathematically demonstrated.

The relative values b, μ, a and ε can be easily measured and the calculation algorithm of the maximum is not difficult to apply to any combination of harmonics. The method describes an efficient method to find out the quality of the electrical energy, through knowing the relative values b, μ, a and ε and the maximum of the power factor.

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