DETERMINING THE TRAJECTORY OF THE COMPENSATING COUNTERWEIGHT FOR EQUILIBRATING A ROBOT ARM

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Abstract. In order to do the manipulation operation, the dynamical performances of the robot are increased by a solution of equilibrating the arm with a compensating counterweight. In the paper we determine the trajectory equation of the compensating counterweight if the mass of the arm is equal with the mass of the counterweight and using the equality between the forces that appear in the linking chain between the arm and the counterweight.

Keywords: robot arm, counterweight, equilibration, trajectory.

1. INTRODUCTION

The robot arm is composed of the translation module, the orientation module and the gripper (fig. 1), and was 3D-modeled with the SolidWorks application.

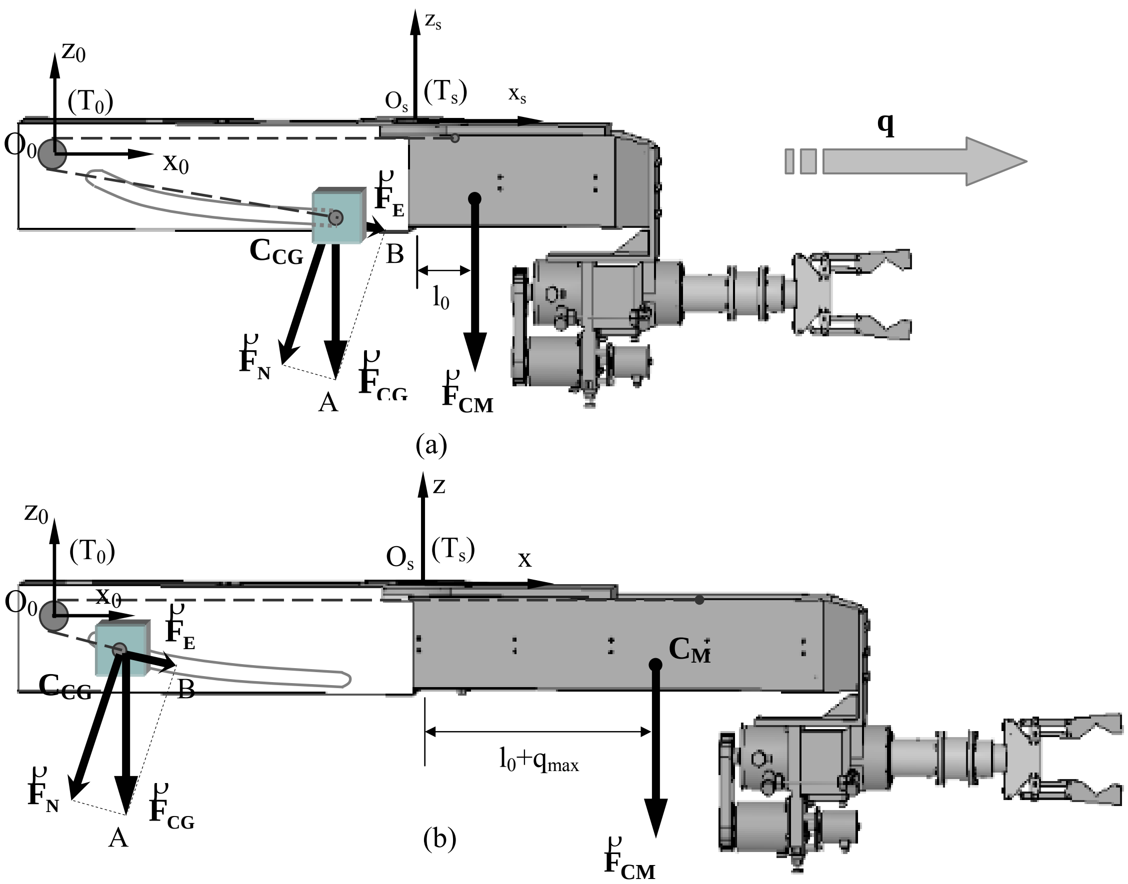

Fig. 1. Equilibration of the robot arm for the (a) minimum position; (b) maximum position

The SolidWorks program allows finding certain characteristics of the designed components and ensembles among which, for equilibration and stability, the positions of the centers of mass and the values of the inertial torques are important. For equilibrating the designed arm of the industrial robot (fig. 1) with a compensating counterweight [2, 4, 5, 6, 7], the solution has to take into account the materials the ensemble parts are made of. Because the compensating mass depends on the mass of the robot arm and because it increases the ensemble mass, it is better if the arm has a mass as small as possible, while keeping its mechanical strength and its rigidity [1]. Therefore, for constructing the arm, some super-light alloys with small density are taken into account.

2. DETERMINING THE CURVE OF THE COUNTERWEIGHT

We consider that the value of the force $\overline{F}_{CM}$ represents the mass of the robot arm ensemble, and the mass of the gripper includes also the mass of the manipulated object.

According to figure 1, by decomposing the gravity force $\overline{F}_{CG}$ of the counterweight, the $\overline{F}_E$ component along the linking chain has to balance the force $\overline{F}_{CM}$.

For obtaining the equilibrium state, we determine the shape ***z* = *f(x)*** of the gutter on which the counterweight will glide. The coordinates of the points on the curve will be computed with respect to the reference system $\{T_0\}$, which has the origin in the center of the wheel of the linking chain.

By applying the sinus theorem in the ABC_{CG} triangle and using the trigonometric relations between its angles and sides, we obtain:

$$z' = -\frac{x}{\frac{F_{CG}}{F_E}\sqrt{x^2+z^2}+z} \tag{1}$$

We denote $k = \frac{m_{CG}}{m_{CM}}$; therefore

$$z' = -\frac{x}{k\sqrt{x^2+z^2}+z} \tag{2}$$

The curve $z = f(x)$, described by the gravity center of the counterweight, is obtained by solving the differential equation (2). For this, we consider that k is a constant and we successively apply the substitutions: $u = \frac{z}{x}$, $u = \frac{1-v^2}{2v}$, and $w = v^2$, and we obtain:

$$\ln C_1 x = -\frac{1}{2}\int \frac{\left((k-1)w+k+1\right)(w+1)}{\left((k-1)w^2+6w-(k+1)\right)w}\,\mathrm{d}w. \tag{3}$$

We further consider that $k = 1$, in which case the right-hand integral from the relation (3) becomes:

$$-\frac{1}{2}\int \frac{2(w+1)}{(6w-2)w}\,\mathrm{d}w = -\frac{1}{2}\int\left(\frac{4}{3w-1}-\frac{1}{w}\right)\mathrm{d}w = -\frac{4}{3}\cdot\frac{1}{2}\ln|3w-1| + \frac{1}{2}\ln|w| =$$
$$= \frac{1}{6}\left(\ln w^3 - \ln\left((3w-1)^4\right)\right) = \frac{1}{6}\ln\frac{w^3}{(3w-1)^4}. \tag{4}$$

Taking into account that $w = v^2$, it follows that: $\ln C_1 x = \frac{1}{3} \ln \frac{v^3}{\left(3v^2 - 1\right)^2}$, or

$x = C \frac{v}{\sqrt[3]{\left(3v^2 - 1\right)^2}}$ (for $v \neq \pm \frac{1}{\sqrt{3}}$).

But we made the substitution $z = x \cdot u = x \cdot \frac{1 - v^2}{2v}$. Consequently, we obtain the following system:

$$\begin{cases} x = C \dfrac{v}{\sqrt[3]{\left(3v^2 - 1\right)^2}} \\ z = C \dfrac{1 - v^2}{2 \cdot \sqrt[3]{\left(3v^2 - 1\right)^2}} \end{cases}, \text{ where } v \in (-\infty, \infty) - \left\{ -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\} \qquad (5)$$

This represents the parametric equation for the curve $z = f(x)$, if $k = 1$.

The value of the constant C is determined from the initial conditions of the motion.

3. NUMERICAL RESULTS

We determined, using the Maple application [3], the shape of the gutter onto which the counterweight must glide, for various initial conditions.

We denote with (x_0, z_0) the coordinates of the initial position C_{CG_0} of the counterweight, when the translation module of the robot arm is in the minimum position. For the figures 2, 4, 7, 9 - 17, we consider that $x_0 = 0.45$ [m], and z_0 varies as follows:

- $z_0 = -0.2$ [m] – fig. 2, 4
- $z_0 = -0.25$ [m] – fig. 7, 9
- $z_0 = -0.26$ [m] – fig. 10, 12
- $z_0 = -0.27$ [m] – fig. 11, 13
- $z_0 = -0.3$ [m] – fig. 14, 16
- $z_0 = -0.35$ [m] – fig. 15, 17

In the figures 2, 3, 6, 7, 10, 11, 14, 15, 18, and 19, we plotted the entire parametric curve from the relation (5) in the given initial conditions, on the definition domain. Instead, in the figures 4, 5, 8, 9, 12, 13, 16, 17, 20, and 21, we plotted only that branch of the curve that is the solution for the problem. For each analyzed situation, we determined the constant C that satisfies the initial conditions, and for the found value, we plotted the curve. Also, for each situation, we computed the maximum distance q_{max} that can be covered by the translation module, as the difference between the length of the segment $\overline{O_0 C_{CG_0}}$ and of the segment $\overline{O_0 C_{CG_f}}$ (where C_{CG_f} is the final position of the counterweight).

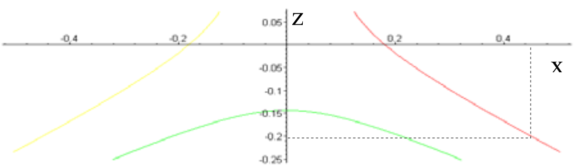

Fig. 2. The parametric curve for $x_0 = 0.45$ [m], $z_0 = -0.20$ [m] ($C \approx -0.28712014$)

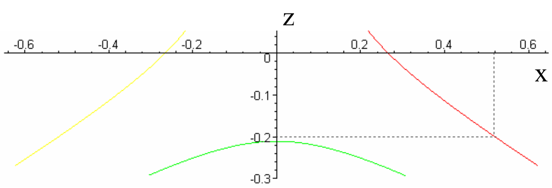

Fig. 3. The parametric curve for $x_0 = 0.52$[m], $z_0 = -0.2$ [m] ($C \approx -0.4213025$)

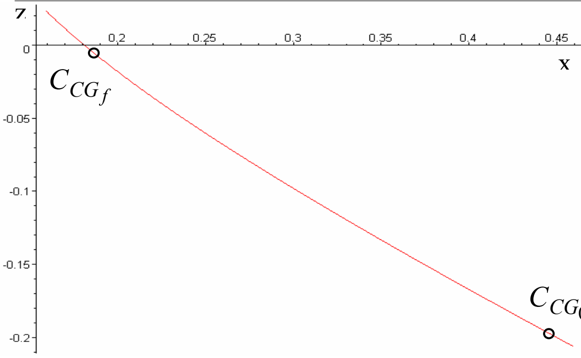

Fig. 4. The curve of the gutter for $x_0 = 0.45$ [m], $z_0 = -0.20$ [m] ($q_{max} \approx 0.31$ [m])

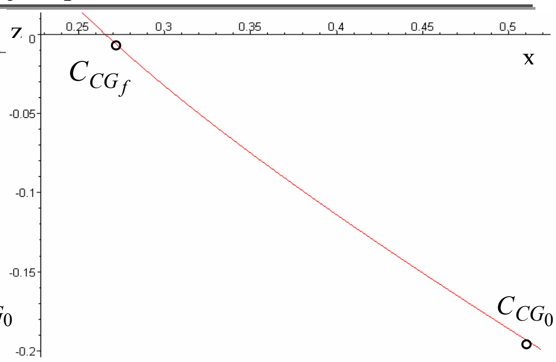

Fig. 5. The curve of the gutter for $x_0 = 0.52$ [m], $z_0 = -0.2$ [m] ($q_{max} \approx 0.29$ [m])

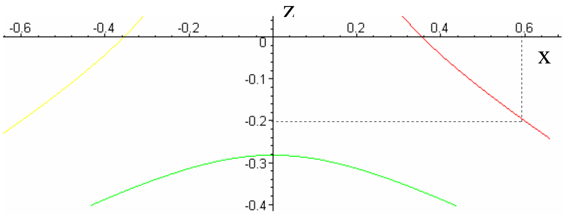

Fig. 6. The parametric curve for $x_0 = 0.6$ [m], $z_0 = -0.2$ [m] ($C \approx -0.56454721$)

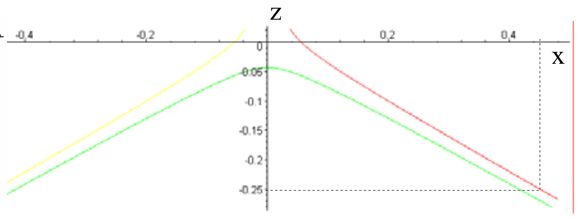

Fig. 7. The parametric curve for $x_0 = 0.45$[m], $z_0 = -0.25$ [m] ($C \approx -0.08743358$)

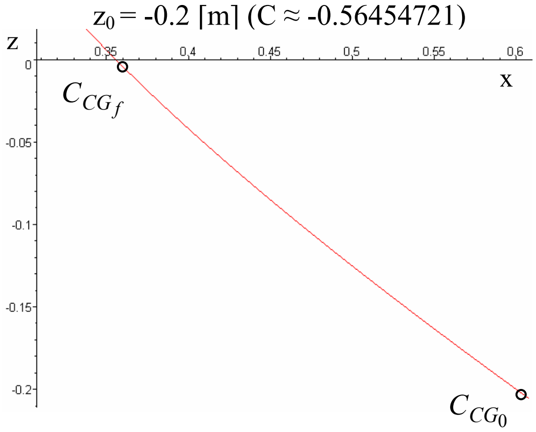

Fig. 8. The curve of the gutter for $x_0 = 0.6$ [m], $z_0 = -0.2$ [m] ($q_{max} \approx 0.27$ [m])

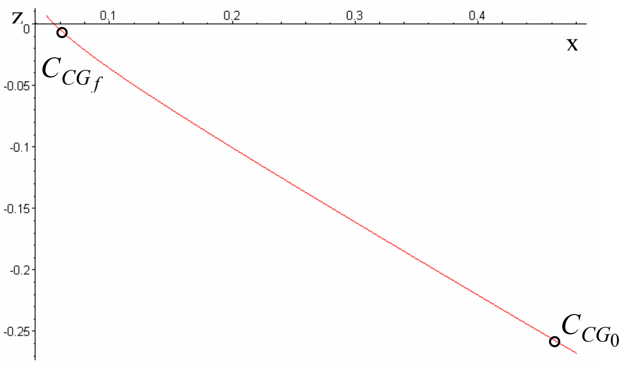

Fig. 9. The curve of the gutter for $x_0 = 0.45$ [m], $z_0 = -0.25$ [m] ($q_{max} \approx 0.35$ [m])

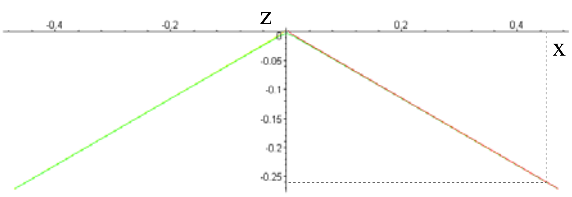

Fig. 10. The parametric curve for $x_0 = 0.45$[m], $z_0 = -0.26$ [m] ($C \approx -0.006379749$)

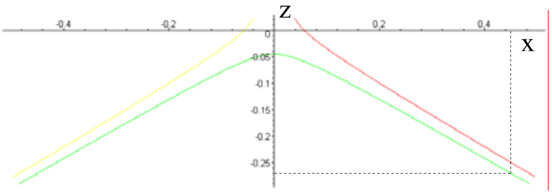

Fig. 11. The parametric curve for $x_0 = 0.45$ [m], $z_0 = -0.27$ [m] ($C \approx -0.090282841$)

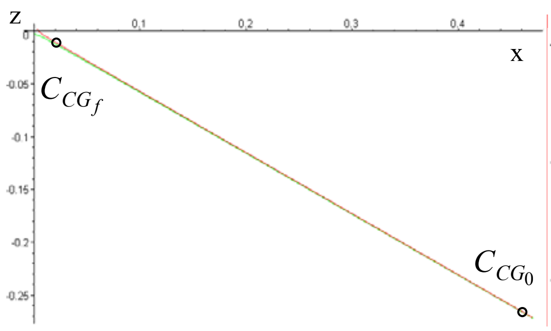

Fig. 12. The curve of the gutter for $x_0 = 0.45$ [m], $z_0 = -0.26$ [m] ($q_{max} \approx 0.5$ [m])

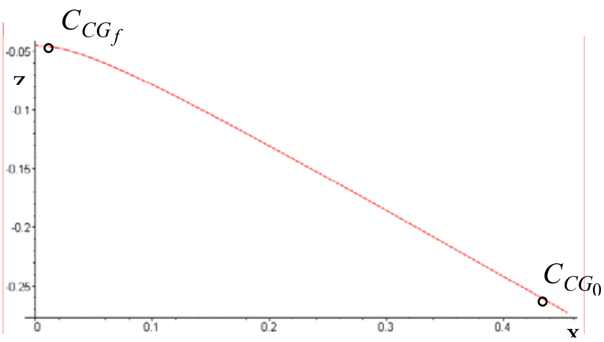

Fig. 13. The curve of the gutter for $x_0 = 0.45$ [m], $z_0 = -0.27$ [m] ($q_{max} \approx 0.47$ [m])

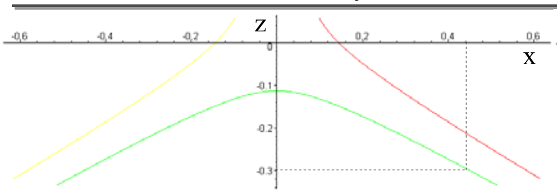

Fig. 14. The parametric curve for $x_0 = 0.45$ [m], $z_0 = -0.3$ [m] ($C \approx -0.22749808$)

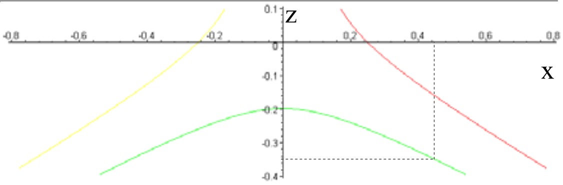

Fig. 15. The parametric curve for $x_0 = 0.45$ [m], $z_0 = -0.35$ [m] ($C \approx -0.39603165$)

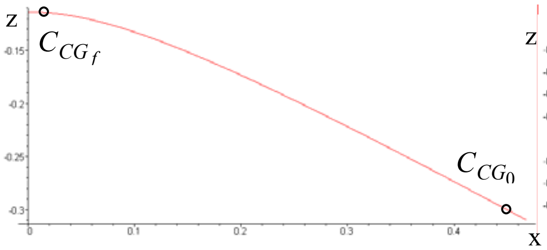

Fig. 16. The curve of the gutter for $x_0 = 0.45$ [m], $z_0 = -0.3$ [m] ($q_{max} \approx 0.42$ [m])

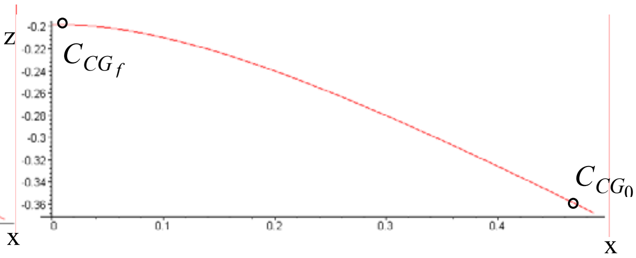

Fig. 17. The curve of the gutter for $x_0 = 0.45$ [m], $z_0 = -0.35$ [m] ($q_{max} \approx 0.35$ [m])

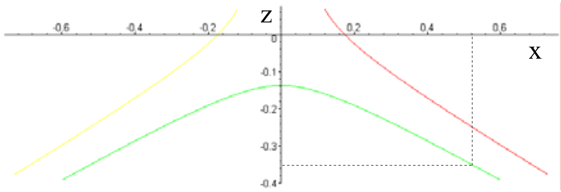

Fig. 18. The parametric curve for $x_0 = 0.52$ [m], $z_0 = -0.35$ [m] ($C \approx -0.27556867$)

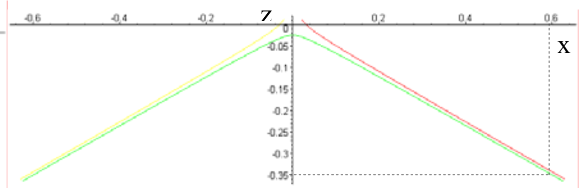

Fig. 19. The parametric curve for $x_0 = 0.6$ [m], $z_0 = -0.35$ [m] ($C \approx -0.049439743$)

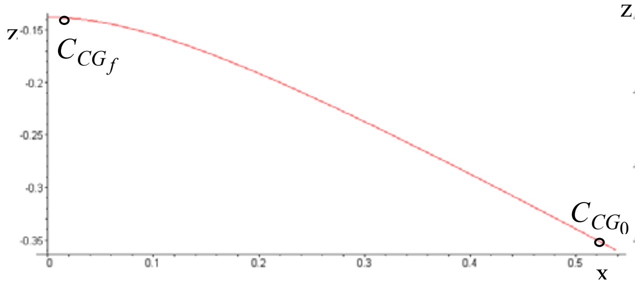

Fig. 20. The curve of the gutter for $x_0 = 0.52$ [m], $z_0 = -0.35$ [m] ($q_{max} \approx 0.48$ [m])

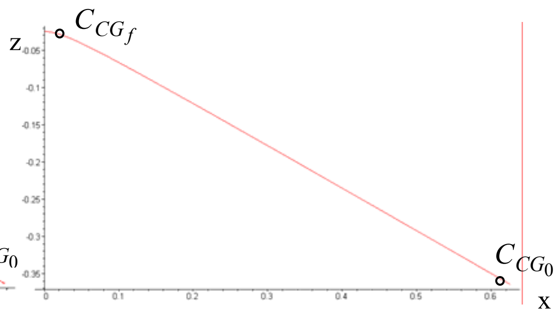

Fig. 21. The curve of the gutter for $x_0 = 0.6$ [m], $z_0 = -0.35$ [m] ($q_{max} \approx 0.66$ [m])

For each case, from among the three branches of the parametric curve, the branch that describes the curve of the gutter is the one that contains the initial point.

We can see that, for the same value of the x_0, the parametric curve is approaching an oblique asymptote, if the value of z_0 decreases. For example, for $x_0 = 0.45$, the curve is approaching an oblique asymptote when z_0 decreases from the value –0.20 to around the value –0.26. In the same situations, the constant C increases from the value -0.28712 ($z_0 = -0.20$) to the value -0.006379 ($z_0 = -0.26$). Also, the maximum distance q_{max} onto which the translation module can move increases from 0.31 ($z_0 = -0.20$) to the value 0.5 ($z_0 = -0.26$). The curve approached the oblique asymptote very much, so that for the value $z_0 = -0.27$, the curve for the gutter of the counterweight is on the other side of the asymptote. If the value of z_0 further decreases down to $z_0 = -0.35$, the maximum distance q_{max} will also decrease, because the curve on which the counterweight glides passed onto another branch of the parametric curve, which is concave and intersects the O_0z axis. Therefore, the maximum distance allowed for the motion of the translation module has the value 0.47 for $z_0 = -0.27$, and 0.35 for $z_0 = -0.35$, respectively. The value of the constant C decreases from –0.0902878 to –0.39603165 (for the same value $x_0 = 0.45$).

We study also the situation when the value of z_0 is kept constant and x_0 varies. For this, we consider the situations when $z_0 = -0.2$, and $z_0 = -0.35$ respectively.

In the first case, considering the values for x_0: $x_0 = 0.45$ (fig. 2, 4), $x_0 = 0.52$ (fig. 3, 5), and $x_0 = 0.6$ (fig. 6, 8), we can see that the branch of the curve that corresponds to the gutter moves away from the oblique asymptote and the maximum distance q_{max} decreases from 0.31 ($x_0 = 0.45$) to 0.27 ($x_0 = 0.6$), while the value of the constant *C* decreases from –0.28712 to –0.564547.

For the case when $z_0 = -0.35$ (fig. 15, 17-21), the curves are placed beneath the oblique asymptote and intersect the O_0z axis. If the value x_0 increases from 0.45 to 0.6, then the value of the maximum distance q_{max} increases from 0.35 to 0.66. For the same increase of the value x_0, the value of the constant *C* increased from –0.39603 to -0.04944.

4. CONCLUSIONS

For analyzing the equilibration with a compensating counterweight, in this paper we determined various parametric curves that depend on the variations of the values for x_0 and z_0. The plot of these parametric curves has three branches and two oblique asymptotes. The solution for the curve of the gutter was the one that passes through the point (x_0, z_0). The equation of the curve, and the maximum distance q_{max} respectively, were comparatively determined for the same value of x_0 and various values z_0. Also, we have studied the situations when we kept constant the value of z_0 and x_0 was variable.

5. REFERENCES

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