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INVERSE DYNAMICS OF A 3-TTRS PARALLEL MANIPULATOR USING EQUIVALENT LUMPED MASSES

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Abstract: The dynamic analysis of parallel robots requires a great deal of computing as regards the formulation of the generally nonlinear equations of motion and their solution. The paper presents the development of a new dynamic algorithm for the 3-TTRS parallel manipulator with the Lagrange formulation using equivalent lumped masses.

The Lagrange formulation with multipliers is applied in order to derive the set of equations for the manipulator motion. Due to the complexity of the geometrical model, the evaluation of Lagrange function and especially its derivatives with only six coordinates is found to be extremely involved and tedious. A much better approach is to select 9 generalized coordinates: the relative rotational angles, and the joint coordinates.

The algorithms offer the possibility of a complex dynamic study for this type of parallel manipulator in order to evaluate its dynamic capacities. The inverse dynamics plays an important role for high performance control algorithms.

Key words: Parallel robots, kinematics, dynamics, equivalent lumped masses.

1. INTRODUCTION

A great advantage of the parallel structure is that it enables the design of very fast robots by combining the action of the actuators, while the low mass of the moving elements induces small inertia forces. The problems connected to the dynamics of the parallel structures usually are much more complicated than those for the serial robots.

To solve the dynamic model, Merlet [8] uses the Lagrange formulation. He applied the inverse and forward dynamic model to the “left hand”, a prototype made at INRIA based on a kinematic chain KPS (Kardan, Prismatic, Spheric). Miller [9] presents the complete model of the DELTA robot based on Lagrange’s equations. In this case it is assumed that the bars of the robot also have moments of inertia. Guglielmetti [4] develops a dynamic model simplified on the basis of the Codourey model [1] using the kinematic relations for speeds and accelerations. Do [2] uses the Newton-Euler formalism to find the inverse dynamics of a general robot. Honneger [5] has suggested to use the dynamics equations in an adaptive control scheme for the Hexaglide robot, in which the tracking errors are used on-line to correct the parameters used in the dynamic equations. Pisla [10] proposes a generalised dynamic model for parallel robots using Lagrange equations of the first type on the basis of equivalent lumped masses.

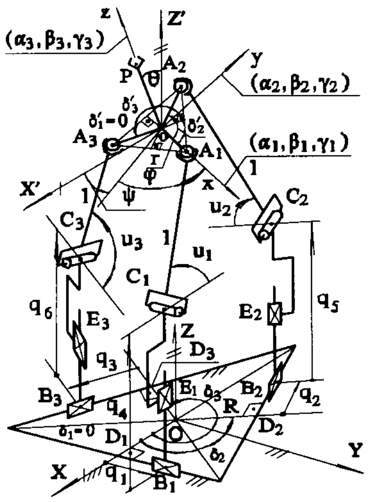

Figure 1

In this paper the inverse dynamics of a 3-TTRS parallel manipulator using Lagrange equations on the basis of equivalent lumped masses is presented. Ripianu [11] and Seyfert [12] almost in the same time have dealt with the problem to replace a mass-loaded rigid body by single rigidity connected mass point (lumped masses) without changing the mass geometry. This theory will be used in the paper.

The parallel manipulators with 3-TTRS structural formula belong to the spatial-closed-kinematic chain guided in three points mechanisms.The parallel structure mechanism consists of three identical kinematic chains between the fixed base and the mobile platform. Each chain contains two linear actuators, a rotational passive joint and a spherical passive joint (figure 1). In [6] and [7] Itul proposes its simplified dynamic model, by neglecting the guiding rods inertia.

2. KINEMATICS OF A 3-TTRS PARALLEL MANIPULATOR

To establish the relationships between the kinematic parameters of the mechanism and the parameters of the mobile platform, the parametric equations of the mobile curves will be used, on which the centers of the spherical joints are moving. These centres are called the guiding points. In the following equations, sinus and cosines for different angles are symbolized with "s" and "c".

$$\begin{cases} X_i = R\, c\delta_i - q_i\, s\delta_i - l\, cu_i\, c\delta_i \\ Y_i = R\, s\delta_i + q_i\, c\delta_i - l\, cu_i\, s\delta_i \\ Z_i = q_{i+3} + l\, su_i \end{cases} \qquad i=1,2,3 \qquad (1)$$

The absolute coordinates of the guiding points can be computed with respect to the mobile platform coordinates (the X, Y, Z coordinates and the ψ, θ, φ Euler angles):

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + [R] \cdot \overline{p}_i \qquad i=1,2,3 \qquad (2)$$

where:

$[R]$ represents the orientation matrix of the mobile platform;

$\overline{p}_i = \overline{oA_i} = [x_i, y_i, z_i]^T = [r\, c\delta'_i, r\, s\delta'_i, 0]^T$ represents the radius vector of the guiding points with respect to the working platform frame.

By replacing (1) in (2), the closure equations for the inverse geometric model are derived.

An easier solution could be obtained by eliminating the relative rotation angles from the equations (1):

$$\begin{cases} q_i = Y_i\, c\delta_i - X_i\, s\delta_i \\ q_{i+3} = Z_i \mp \sqrt{l^2 - (R - X_i\, c\delta_i - Y_i\, s\delta_i)^2} \end{cases} \qquad i=1,2,3 \qquad (3)$$

The rotation relative angles can be derived with respect to the three A_i points positions:

$$su_i = \frac{\pm\sqrt{l^2 - (R - X_i\,c\delta_i - Y_i\,s\delta_i)^2}}{l},\ cu_i = \frac{R - X_i\,c\delta_i - Y\,s\delta_i}{l},\ u_i = \text{atn2}(su_i, cu_i) \qquad i=1,2,3 \qquad (4)$$

The closure equations for the direct geometric model are deriving by equalizing the distances between the guiding points with respect to the two reference systems OXYZ and oxyz:

$$\begin{cases}(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \\ (X_3 - X_2)^2 + (Y_3 - Y_2)^2 + (Z_3 - Z_2)^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2 \\ (X_1 - X_3)^2 + (Y_1 - Y_3)^2 + (Z_1 - Z_3)^2 = (x_1 - x_3)^2 + (y_1 - y_3)^2 + (z_1 - z_3)^2\end{cases} \qquad (5)$$

The nonlinear equations (5) contain five unknowns: the relative rotation angles $u_i\ (i = 1,2,3)$. After their deriving, from the relationships (1) the platform position is derived.

To derive the kinematic model, which must contain also the kinematic parameters of the guiding points C_iA_i, the equations (1) are differentiating with respect to the time:

$$[J_i]\cdot\dot{\bar{q}}_i = \dot{\bar{P}}_i \qquad (6)$$

$$[J_i]\cdot\ddot{\bar{q}}_i = \ddot{\bar{P}}_i - l\dot{u}_i^2\,\bar{l}_i \qquad (7)$$

in which:

$\dot{\bar{q}}_i = \begin{bmatrix}\dot{q}_i \\ \dot{q}_{i+3} \\ \dot{u}_i\end{bmatrix}$ is the joint velocities vector; $\ddot{\bar{q}}_i = \begin{bmatrix}\ddot{q}_i \\ \ddot{q}_{i+3} \\ \ddot{u}_i\end{bmatrix}$ is the joint accelerations vector;

$\dot{\bar{P}}_i = \begin{bmatrix}\dot{X}_i \\ \dot{Y}_i \\ \dot{Z}_i\end{bmatrix} = \begin{bmatrix}\dot{X} \\ \dot{Y} \\ \dot{Z}\end{bmatrix} + [R]\cdot(\bar{\omega}\times\bar{p}_i)$ is the vector of the absolute velocity for the point A_i;

$\ddot{\bar{P}}_i = \begin{bmatrix}\ddot{X}_i \\ \ddot{Y}_i \\ \ddot{Z}_i\end{bmatrix} = \begin{bmatrix}\ddot{X} \\ \ddot{Y} \\ \ddot{Z}\end{bmatrix} + [R]\cdot[\bar{\varepsilon}\times\bar{p}_i + \bar{\omega}\times(\bar{\omega}\times\bar{p}_i)]$ is the vector of the absolute acceleration for the point A_i; $\bar{l}_i = [-l\,cu_i\,c\delta_i \ \ -l\,cu_i\,s\delta_i \ \ l\,su_i]^T$ is the unit vector of the C_iA_i direction;

$[J_i] = [\bar{n}_i \quad \bar{k}_0 \quad l\bar{w}_i] = \begin{bmatrix}-s\delta_i & 0 & l\,su_i\,c\delta_i \\ c\delta_i & 0 & l\,su_i\,s\delta_i \\ 0 & 1 & l\,cu_i\end{bmatrix}$ is the Jacobi matrix of the kinematic chain “i”;

$\bar{\omega}$ is the angular velocity of the working platform;
$\bar{\varepsilon}$ is the angular acceleration vector of the working platform.

3. LAGRANGE FORMULATION WITH MULTIPLIERS FOR THE INVERSE DYNAMICS

Since the parallel manipulator has six degrees of freedom, there is the temptation to choose as generalized coordinates the six joint coordinates or the Cartesian coordinates of the manipulated object. Due the complexity in evaluation of the Lagrange function and especially its derivatives, the choosing of 9 generalized coordinates (represented by the six joint coordinates q_i, q_{i+3} and the relative rotation angles u_i) is more adequate.

The following hypotheses are adopted: all the 12 joints are frictionless; the guiding bars are homogenous bars with length "l "and mass " m_b "; the manipulated object is reduced on the material point with the mass "m", which is situated in the centre of the triangle working platform; the mobile platform is reduced to a plate with the mass "M" having an equilateral triangle form with the $r\sqrt{3}$.

Since the elements i and i+3 perform translations, it is not important their mass concentration. In order to fulfill the condition:

$$\sum_{i=1}^{n} m_i \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix} = \begin{bmatrix} I_{yz} & I_{xy} & I_{xz} & Mx_c \\ I_{yx} & I_{zx} & I_{yz} & My_c \\ I_{zx} & I_{zy} & I_{xy} & Mz_c \\ Mx_c & My_c & Mz_c & M \end{bmatrix} \tag{8}$$

the guiding bar mass is replaced as follows: $\frac{1}{6} m_b$ in the points A_i and C_i and $\frac{2}{3} m_b$ in the middle of the bar and the platform bar is replaced as follows: $\frac{M}{12}$ in the points A_i and $\frac{3}{4} M$ in the middle of platform (the point o).

In this case the Lagrange equations with multiplies take the following form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{dL}{\partial q_j} = Q_j + \sum_{k=1}^{3} \lambda_{kj} \ , \quad j = 1, 2, ..., 9 \tag{9}$$

where:

Q_j is the generalized external force ($Q_1, ..., Q_6$ are the actuating forces, $Q_7 = Q_8 = Q_5 = 0$); λ_i are the Lagrange multipliers;

$A_{kj} = \dfrac{\partial f_k}{\partial q_j}$; $k = 1,2,3$; $j = 1,2,...,9$;

f_k $(q_1, q_2, ..., q_6, q_7 = u_1, q_8 = u_2, q_9 = u_3) = 0$ are the functions given by the closure equations for the direct geometric model.

The Lagrange function, accordingly to the hypothesis mode is:

$$\begin{aligned} L = {} & \frac{1}{2}\sum_{i=1}^{3} m_i \dot{q}_i^2 + \frac{1}{2}\sum_{i=1}^{3}\left(m_{i+3} + \frac{1}{6}m_b\right)\left(\dot{q}_i^2 + \dot{q}_{i+3}^2\right) + \frac{1}{2}\sum_{i=1}^{3}\frac{2}{3}m_b\,\bar{v}_{c_i}^2 + \\ & + \frac{1}{2}\sum_{i=1}^{3}\left(\frac{1}{6}m_b + \frac{1}{12}M\right)\bar{v}_i^2 + \frac{1}{2}\left(m + \frac{3}{4}M\right)\bar{v}^2 - \sum_{i=1}^{3} m_i\, g h - \sum_{i=1}^{3} m_{i+3}(q_{i+3} - d) - \\ & - \sum_{i=1}^{3}\frac{1}{6}m_b\, g\, q_{i+3} - \sum_{i=1}^{3}\frac{2}{3}m_b\, g Z_{c_i} - \sum_{i=1}^{3}\left(\frac{1}{6}m_b + \frac{1}{12}M\right) g Z_i - \left(m + \frac{3}{4}M\right) g\, Z \end{aligned} \tag{10}$$

where:

M is the mass of the working platform; m is the mass of the manipulated object

m_i, m_{i+3} are the masses of „i" and „i+3" link; m_b is the mass of the guiding rod;

$\bar{v}_i = [J_i]\,\dot{\bar{q}}_i$ is the velocity vector of the point A_i; $\bar{v}_{c_i} = \bar{v}_i \ (l =: \frac{1}{2})$ is the mass centre velocity for the guiding rod C_iA_i; $\bar{v} = \frac{1}{3}\sum_{i=1}^{3}\bar{v}_i$ is the centre velocity of the mobile platform; $Z_{c_i} = Z_i\left(l =: \frac{1}{2}\right)$ is the mass centre Z coordinate of the guiding rod C_iA_i;

$Z = \frac{1}{3}\sum_{i=1}^{3} Z_i$ is the Z coordinate of the working platform centre; h, d - the constant geometric parameters.

Taking into consideration the (10), the equations (9) become:

$$\left(m_i + m_{i+3} + m_b + \frac{M}{12}\right)\ddot{q}_i + \left(\frac{m}{9} + \frac{M}{12}\right)\left(\sum_{i=1}^{3}\dot{\bar{v}}_i\right)\cdot\frac{\partial \bar{v}_i}{\partial \dot{q}_i} = Q_i + \sum_{k=1}^{3}\lambda_k A_{ki} \qquad i = 1,2,3 \tag{11}$$

$$\left(m_{i+3} + m_b + \frac{M}{12}\right)\ddot{q}_{i+3} + \left(\frac{m_b}{2} + \frac{M}{12}\right)\left(l\ddot{u}_i cu_i - l\dot{u}^2 su_i\right) + \left(\frac{m}{9} + \frac{M}{12}\right)\left(\sum_{i=1}^{3}\dot{\bar{v}}_i\right)\cdot\frac{\partial \bar{v}_i}{\partial \dot{q}_{i+3}} +$$
$$+\left(m_{i+3} + m_b + \frac{m}{3} + \frac{M}{3}\right) = Q_{i+3} + \sum_{k=1}^{3}\lambda_k A_{k,i+3} \qquad i = 1,2,3 \tag{12}$$

$$\left(\frac{m_b}{3} + \frac{M}{12}\right)l^2\ddot{u}_i + \left(\frac{m_b}{2} + \frac{M}{12}\right)l\ddot{q}_{i+3}\, cu_i + \left(\frac{m}{9} + \frac{M}{12}\right)\left(\sum_{i=1}^{3}\dot{\bar{v}}_i\right)\cdot\frac{\partial \bar{v}_i}{\partial \dot{u}_i} +$$
$$+\left(\frac{m_b}{2} + \frac{m}{3} + \frac{M}{3}\right) g\, l\, cu_i = \sum_{k=1}^{3}\lambda_k A_{k,i+6} \qquad i = 1,2,3 \tag{13}$$

in which:

$$\bar{v}_i = \begin{bmatrix} \dot{u}_i\, l\, su_i\, c\delta_i - \dot{q}_i\, s\delta_i \\ \dot{u}_i\, l\, su_i\, s\delta_i + q_i\, c\delta_i \\ \dot{u}_i\, l\, cu_i + \dot{q}_{i+3} \end{bmatrix}; \quad \dot{\bar{v}}_i = \begin{bmatrix} \ddot{u}_i\, l\, su_i\, c\delta_i + \dot{u}_i^2\, l cu_i\, c\delta_i - \ddot{q}_i\, s\delta_i \\ \ddot{u}_i\, l\, su_i\, s\delta_i - \dot{u}_i^2\, l\, cu_i\, s\delta_i + \ddot{q}_i\, c\delta_i \\ \ddot{u}_i\, l\, cu_i - \dot{u}_i^2\, l\, su_i + \ddot{q}_{i+3} \end{bmatrix}$$

$$\frac{\partial \bar{v}_i}{\partial \dot{q}_i} = \begin{bmatrix} -s\delta_i \\ c\delta_i \\ 0 \end{bmatrix}; \quad \frac{\partial \bar{v}_i}{\partial \dot{q}_{i+3}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \frac{\partial \bar{v}_i}{\partial \dot{u}_i} = \begin{bmatrix} l\, su_i\, c\delta_i \\ l\, su_i\, s\delta_i \\ l cu_i \end{bmatrix}$$

To solve the inverse dynamic model, the λ_1, λ_2, λ_3 multipliers are computed from (13) and then the actuating forces Q_i and Q_{i+3}, (i=1,2,3) are computed from (11) and (12).

4. SIMULATION RESULTS

The algorithm for the dynamic model of the 3-TTRS parallel manipulator including the guiding rods inertia was implemented in the developed simulation program for parallel robots. For the simulation, the parallel mechanism with the fixed base at the bottom was considered. The following geometric parameters were considered: R =0,15 m, l =0,3 m, r =0,10 m, $\delta_1 = \delta_1'$ $=0^o$, $\delta_2 = \delta_2'=120^o$, $\delta_3 = \delta_3' =240^o$, M =1 kg, m =1 kg, $m_1 = m_2 = m_3 = m_4 = m_5 = m_6$ = 0,5 kg, m_b=0,2kg. The selected displacement of the working platform is a helical translation with the equations:

$$\begin{aligned} X(t) &= R\cdot\cos(\lambda(t)) \\ Y(t) &= R\cdot\sin(\lambda(t)) \\ Z(t) &= .5 + .1\frac{\lambda(t)}{2\cdot\pi} \end{aligned} \quad \text{where } \lambda(t) = \begin{cases} \frac{2}{\pi}\cdot t^2 & \text{if } 0 \le t < \frac{\pi}{2} \\ 2\cdot\left(t - \frac{\pi}{2}\right) + \frac{\pi}{2} & \text{if } \frac{\pi}{2} \le t < \pi \\ -2\cdot\frac{(t-\pi)^2}{\pi} + 2\cdot(t-\pi) + 3\cdot\frac{\pi}{2} & \text{if } \pi < t \le \frac{3\pi}{2} \end{cases}; \quad \psi = \theta = \varphi = 0$$

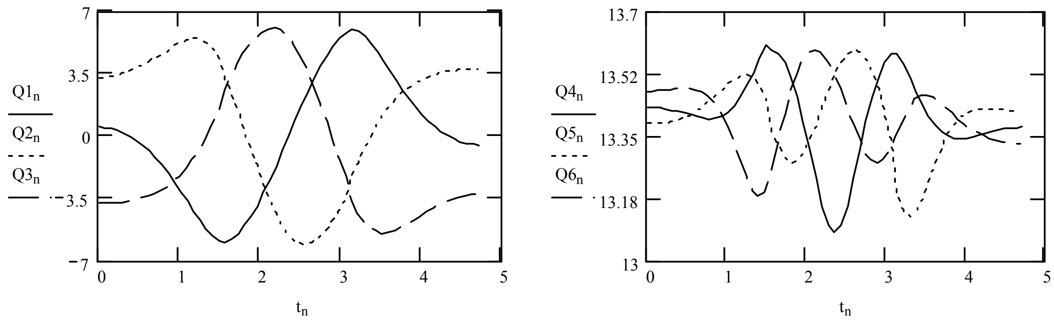

Figure 2

In the figure 2 the driving forces Q_i and Q_{i+3} (i=1,2,3) are represented using the derived dynamic model.

The obtained numerical results mean the input data for the dynamic control of the parallel manipulator. The simplified dynamic model, which was obtained in the paper using equivalent masses, leads to the deviations of the actuating forces under 0,5% for the horizontal joints and 0% for the vertical joints with respect to the dynamic model with continuous mass.

Making a comparative study with the dynamic models presented in the papers [6] and [7], the results have demonstrated that the neglecting of the guiding rods masses, even for simple moving of the platform, leads to considerable errors in the evaluation of the generalized actuating forces, especially for the vertical translation joints.

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