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# BLOCK SCHEMES FOR ANALOGICAL MODELLING AND NUMERICAL SIMULATION THROUGH LOCAL-ITERATIVE LINEARIZATION

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**Abstract**. Continuing papers [1], [2] and [3], are presented a number of eight representative block schemes of analogical modelling and numerical simulation through local-iterative linearization (L.I.L), for processes defined by partial differential equations (pde) and ordinary differential equations (ode). Presentation succession is:

a) Pde nonlinear – analytical model
b) Pde nonlinear – numerical model
c) Pde linear – analytical model
d) Pde linear – numerical model
e) Ode nonlinear – analytical model
f) Ode nonlinear – numerical model
g) Ode linear – analytical model
h) Ode linear – numerical model

The paper shows the unitary and systematized aspect of L.I.L method and some restrictions and precautions imposed in this way.

**Key words**: Ordinary differential equations, partial derivative equations, state variable, local-iterative linearization.

## 1. INTRODUCTION

We remind that in papers [1], [2] and [3] the analytical model of a process defined by pde is presented in the following form:

$$\tilde{\mathbf{x}} = \tilde{\mathbf{F}}(\mathbf{u}, \mathbf{x}, \tilde{\mathbf{x}}), \quad (1)$$

$$\overset{v}{\mathbf{x}} = \mathbf{F}(\mathbf{u}, \mathbf{x}, \tilde{\mathbf{x}}), \quad (2)$$

$$\mathbf{y} = \mathbf{F}_y\ (\mathbf{u}, \mathbf{x}, \tilde{\mathbf{x}}), \quad (3)$$

In the case of a process defined by ode the analytical model has the form:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{u}, \mathbf{x}), \quad (4)$$

$$\mathbf{y} = \mathbf{F}_y\ (\mathbf{u}, \mathbf{x}), \quad (5)$$

The Cauchy continuity conditions are considered assumed, and the initial conditions known. For (pde) are supplementary defined – if is the case - boundary conditions or other specific conditions. By $\mathbf{u}$(v,…), $\mathbf{x}$(v,…), $\tilde{\mathbf{x}}$ (v,…) and $\mathbf{y}$(v,…) are denoted the input vectors, state vectors, complementary state vectors and output vectors. These vectors have a least one variable (v), which can be the time (t) or another variable, for example spatially (Cartesian), arbitrary noted by (p), (q) and (r) and $\left(\overset{\vee}{\mathbf{x}}\right)$ from (2) and (4) corresponds to $\left[\frac{\partial}{\partial \mathrm{v}}(\mathbf{x})\right]$.

The complementary state vector $(\tilde{\mathbf{x}})$, makes sense only for pde, which justifies the simple forms (4) and (5) for ode.

Nonlinear systems (1), (2), …, (5) can be personalized for time invariant linear pde, in the following forms:

$$\tilde{\mathbf{x}} = \mathbf{\Gamma} \cdot \mathbf{x} + \tilde{\mathbf{\Gamma}} \cdot \tilde{\mathbf{x}} + \mathbf{\Delta} \cdot \mathbf{u}, \tag{6}$$

$$\overset{\vee}{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \tilde{\mathbf{A}} \cdot \tilde{\mathbf{x}} + \mathbf{B} \cdot \mathbf{u}, \tag{7}$$

$$\mathbf{y} = \mathbf{C} \cdot \mathbf{x} + \tilde{\mathbf{C}} \cdot \tilde{\mathbf{x}} + \mathbf{D} \cdot \mathbf{u}. \tag{8}$$

which, for time invariant linear systems can be personalized in a well known form:

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}, \tag{9}$$

$$\mathbf{y} = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot \mathbf{u}, \tag{10}$$

Details regarding interpretation of vectors and matrices defined by (1), (2),…, (10) are exposed in [1].

The method of local-iterative linearization L.I.L is applied for (2), (4), (6 and (9) and leads, after the integration operation (specific for the method), taking into account (v) variable, to an iterative solution. It is considered at step (k), respective "moment" (k·Δv), the integration step (Δv) is sufficiently small, finally representing the general condensed form of numerical model.

$$\tilde{x}_k = \tilde{F}_k, \tag{11}$$

$$x_k = g_v \cdot F_k + h_k \tag{12}$$

$$y_k = F_{yk} \tag{13}$$

Interpretation and calculus details are largely presented in [1], [2] and [3]. Note that, in scalar coordinates $(F_k, x_k)$, relation (12) represents a line equation, available for the segment in the neighbourhood of the pivot variable $v_k = k \cdot \Delta v$. Vector $(g \cdot F_k)$ and the vector ($\mathbf{h}_k$) in relation (12), represent the forced and free component of the numerical solution. This contains only the regressive sequences (k-1; k-2; …; k-ω), where (ω) is the last regressive sequence taken into account from Taylor series.

The following general form corresponds to the condensated general form of the numerical model from (11), (12) and (13):

$$\tilde{\mathbf{x}}_{kv} = \tilde{\mathbf{F}}(\mathbf{u}_{kv}, \mathbf{x}_{kv\varepsilon}, \tilde{\mathbf{x}}_{kv\varepsilon}), \tag{14}$$

$$\mathbf{x}_{kv} = g_v \cdot \mathbf{F}(\mathbf{u}_{kv}, \mathbf{x}_{kv\varepsilon}, \tilde{\mathbf{x}}_{kv}) + \mathbf{h}_{xFkv}, \tag{15}$$

$$\mathbf{y}_{kv} = \mathbf{F}_y\ (\mathbf{u}_{kv}, \mathbf{x}_{kv}, \tilde{\mathbf{x}}_{kv}), \tag{16}$$

where the scalar

$$g_v = \frac{\sigma_{100}}{\sigma_{110}} \cdot \Delta v \tag{17}$$

and

$$h_{xFkv} = \frac{1}{\sigma_{110}} \sum_{j=1} \left[\Delta v \cdot \sigma_{10j} \cdot F\left(u_{k_v - j}, x_{k_v - j}, \tilde{x}_{k_v - j}\right) - \sigma_{11j} x_{k_v - j}\right]. \tag{18}$$

The (ε) indices from (14), (15) highlights the extrapolated form, also through the regressive sequences (k-1) , (k-2),…, (k-ω), and scalar coefficients ($\sigma_{10j}$) and ($\sigma_{11j}$) presents fractional expressions [1], [2] and [3].

It can be observed that ($\tilde{\mathbf{x}}_{kv}$) from the left member of (14), is transferred in the right side of (15), and ($\tilde{\mathbf{x}}_{kv}$) and ($\mathbf{x}_{kv}$) obtained are transferred in the right member of (16).

The forms (11), (12),…,(18) available for (pde) can be personalized for (ode), through eliminating the complementary state vector ($\tilde{\mathbf{x}}$ …).

## 2. BLOCK SCHEMES FOR ANALYTICAL AND NUMERICAL MODELS THROUGH L.I.L

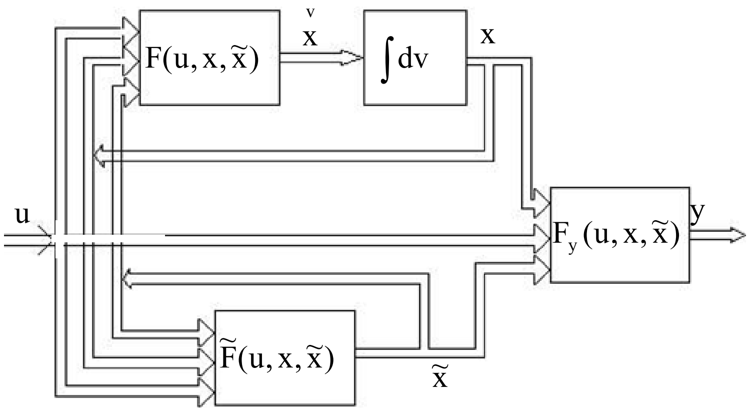


**Figure 1.**

These block schemes are obtained by personalizing the general analytical model (1), (2), (3) and the general numerical model (11), (12), (13), for nonlinear or linear forms of (pde), respectively nonlinear or linear forms of (ode).

These blocks are grouped as follows:

**a) Nonlinear pde – analytical model**

Is based on the system of equations (1), (2) and (3) from which resulted the block scheme presented in figure 1.

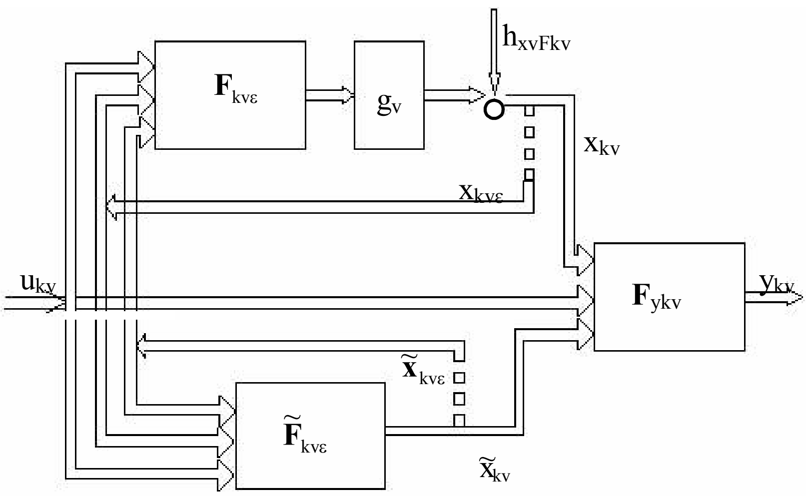


**Figure 2.**

**b) Nonlinear pde - numerical model**

Is based on the system of equations (14), (15) and (16) from which resulted the block scheme presented in figure 2.

The two dotted segments, highlight extrapolation operation, at sequences (k-1), (k-2),…,(k-ω).

**c) Linear pde – analytical model**

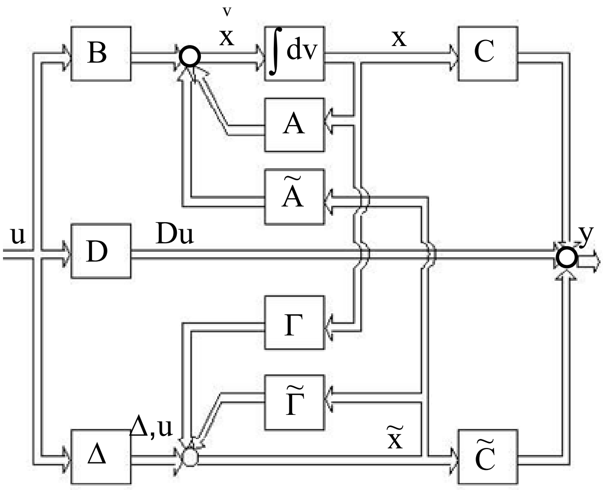


**Figure 3.**

Is based on the system of equations (6), (7) and (8) from which resulted the block scheme presented in figure 3.

**d) Linear pde – numerical model**

Is based on the system of equations (6), (7) and (8) where (6) and (8) are considered at the current sequence (kv), and (7) is integrated – as L.I.L. – in the neighbourhood of sequence (kv). Finally results:

$$\tilde{\mathbf{x}}_{kv} = \mathbf{\Gamma}\cdot\mathbf{x}_{kv\varepsilon} + \tilde{\mathbf{\Gamma}}\cdot\tilde{\mathbf{x}}_{kv\varepsilon} + \mathbf{\Delta}\cdot\mathbf{u}_{kv}, \tag{19}$$

$$\mathbf{x}_{kv} = g_v\cdot\left(\mathbf{A}\cdot\mathbf{x}_{kv\varepsilon} + \tilde{\mathbf{A}}\cdot\tilde{\mathbf{x}}_{kv} + \mathbf{B}\cdot\mathbf{u}_{kv}\right) + \mathbf{h}_{xukv}, \tag{20}$$

$$\mathbf{y}_{kv} = \mathbf{C}\cdot\mathbf{x}_{kv} + \tilde{\mathbf{C}}\cdot\tilde{\mathbf{x}}_{kv} + \mathbf{D}\cdot\mathbf{u}_{kv}, \tag{21}$$

The (ε) indices from (19), (20) highlight the extrapolated form, also through the regressive sequences (k-1) , (k-2),…, (k-ω), and

$$\mathbf{h}_{xukv} = \frac{1}{\sigma_{110}}\cdot\sum_{j=1}^{\omega}\left[\Delta v\cdot\sigma_{10j}\cdot\left(\mathbf{A}\cdot\mathbf{x}_{kv-j} + \tilde{\mathbf{A}}\cdot\tilde{\mathbf{x}}_{kv-j} + \mathbf{B}\cdot\mathbf{u}_{kv-j}\right) - \sigma_{11j}\cdot\mathbf{x}_{kv-j}\right] \tag{22}$$

For the above equations the corresponding scheme is the one in figure 4, where the two dotted segments highlight the extrapolation effects who generates vectors ($\tilde{x}_{kv}$) and ($x_{kv}$). All the component blocks (**A**), (**B**),…,(Δ) represent constant matrices, excepting ($g_v$) which corresponds to (17).

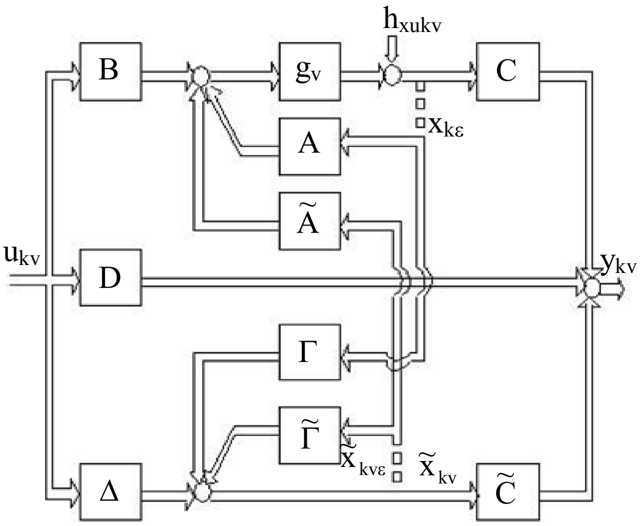


**Figure 4.**

**e) Nonlinear ode – analytical model**

Is based on the system of equations (4), (5) from which resulted the block scheme presented in figure 5.

**f) Nonlinear ode – numerical model**

Is based on the system of equations (23), (24) and (25):

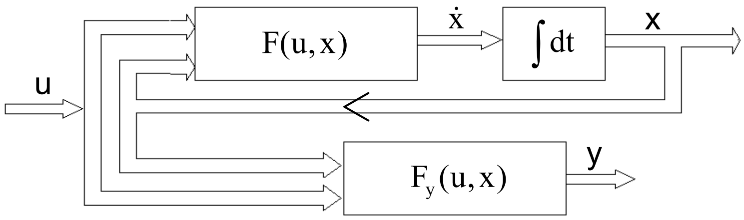


**Figure 5.**

$$\mathbf{x}_k = g_t \cdot \mathbf{F}(\mathbf{u}_k, \mathbf{x}_{k\varepsilon}) + \mathbf{h}_{xuk}, \quad (23)$$

$$\mathbf{y}_k = \mathbf{F}_y\ (\mathbf{u}_k, \mathbf{x}_k), \quad (24)$$

$$\mathbf{h}_{xuk} = \frac{1}{\sigma_{110}} \cdot \sum_{j=1}^{\omega} \left[\Delta t \cdot \sigma_{10j} \cdot \mathbf{F}(u_{k-j}, x_{k-j}) - \sigma_{11j} \cdot \mathbf{x}_{k-j}\right] \quad (25)$$

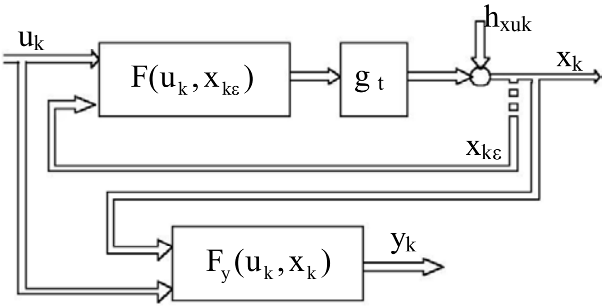


**Figure 6.**

From this system of equations results the block scheme presented in figure 6, where the dotted segment represents the extrapolation operation for generating vector ( $\mathbf{x}_{kv}$ ).

**g) Linear ode – analytical model**

Is based on the system of equations (9), (10) from which resulted the block scheme presented in figure 7

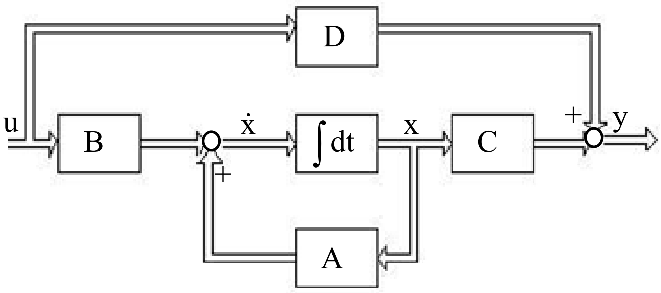


**Figure 7.**

**h) Linear ode – numerical model**

Is based on the system of equations (9), (10), representing:

$$\mathbf{x}_k = g_v \cdot (\mathbf{A} \cdot \mathbf{x}_{k\varepsilon} + \mathbf{B} \cdot \mathbf{u}_k) + \mathbf{h}_{xuk} \quad (26)$$

$$\mathbf{y}_k = \mathbf{C} \cdot \mathbf{x}_k + \mathbf{D} \cdot \mathbf{u}_k \quad (27)$$

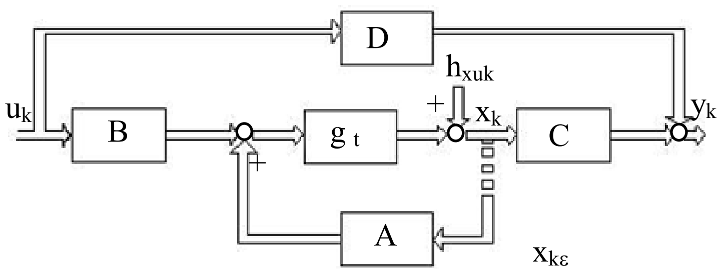


**Figure 8.**

$$\mathbf{h}_k = \frac{1}{\sigma_{110}} \sum_{j=1} \left[ \Delta v \cdot \sigma_{10j} \cdot \left( \mathbf{A} \cdot \mathbf{x}_{k-j} + \mathbf{B} \cdot \mathbf{u}_{k-j} \right) - \sigma_{11j} \mathbf{x}_{k-j} \right] \qquad (28)$$

For the above equations we have the block scheme presented in figure 8, where the dotted segment highlight the extrapolated form ($\mathbf{x}_{k\varepsilon}$) of the state vector ($\mathbf{x}_k$).All the component blocks (**A**), (**B**),…,(**D**) represent constant matrices, excepting ($g_v$) which corresponds to (17).

## 3. INITIAL CONDITIONS AND CALCULUS BEGINNING

From the state vector (**x**), known for the initial conditions ($\mathbf{x}_{IC}$), is calculated the complementary state vector ($\tilde{\mathbf{x}}_{IC}$). So, for the necessary initial conditions, we can establish the vector ($\mathbf{y}_{IC}$), representing – for general condensed form – the numerical model (11), (12), …, (18).

At the beginning of the calculus, the regressive sequences ($k_v$ - j) from (18), can be calculated, by a Runge – Kutta procedure (for example in a usual order IV form), or through Taylor series, noting that the last regressive sequence taking into account stands between limits ($3 \le \omega \le 5$).

## 4. CONCLUSIONS

This papers stands for a presentation of a mathematical formalism and a graphical representation as unitary and systemized as possible, which results through sequential discretization, based on local-iterative linearization method. Is approached a large domain, aiming the processes defined by pde (nonlinear and linear) and ode (nonlinear and linear), highlighting the possibility of successive personalizing, from pde – nonlinear to ode – linear.

This paper does not present the followings: defining $(\tilde{x}), (\tilde{A}), (\tilde{C}), (\Gamma), (\tilde{\Gamma})$ and $(\Delta)$; large explanations concerning the calculus starts; refers about the convergence of the method and the integration step ($\Delta v$); restrictions and precautions imposed in the neighbourhood of some special categories of nonlinearities.

All these aspects have been largely treated in [7], [3], [2] and especially in [1].

LITERATURE

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