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QFT ROBUST CONTROL OF BIOTECHNOLOGICAL PROCESSES

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Abstract: This paper deals with the design of a robust controller based on QFT techniques (Quantitative Feedback Theory) for a biotechnological process. The model of the biotechnological process has been linearized for values of the dilution rate which assure the process efficiency. A linear model with variable parameters did result for which the QFT algorithm for robust tracking has been applied.

Keywords: Quantitative Feedback Theory, robust control, biotechnological process, stability bound

1 INTRODUCTION

It is well known that the biosynthesis processes are very complex and difficult to model and control [2]. They can be developed in continuous or discontinuous bioreactors. The usual modelling procedures, based on kinetic enzymatic schemes, lead to non-linear state models, with a large number of parameters. Moreover, the state variables that characterize the biotechnological process are not accessible to the direct measurement. A number of uncertainties arise right from the beginning of the process with respect to the substrate preparation and nature (the composition of the natural substrates is not known). These facts do induce many uncertainties in the process modelling and difficulties to their control.

This paper presents the design of a robust controller based on QFT techniques (Quantitative Feedback Theory) for a biotechnological process. The model of the biotechnological process has been linearized for values of the dilution rate which assure the process efficiency. A linear model with variable parameters did result for which the QFT algorithm for robust tracking has been applied [6].

The structure of the paper is the following: the second section presents the nonlinear model of the biotechnological process, the firth section shows the linearized model, the fourth section presents the QFT algorithm for robust tracking, in the fifth section are presented the simulation results and the last is dedicated to the conclusions.

2 THE MODEL OF THE BIOTECHNOLOGICAL PROCESS

A continuous biotechnological process described by three equations [1] has been considered in the paper.

$$\frac{dX}{dt} = (\mu - D)X \tag{1}$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{1}{Y_{X/S}}\mu X \tag{2}$$

$$\frac{dP}{dt} = -DP + (\alpha\mu + \beta)X \tag{3}$$

with

$$\mu = \frac{\mu_{\max}\left(1 - P/P_m\right)S}{K_m + S + S^2/K_i} \tag{4}$$

where

X = cell mass concentration (5.9956 g/l at the steady-state);
P = product concentration (19.1267 g/l at the steady-state);
S = substrate concentration in the culture (5.0109 g/l at the steady-state);
S_f = feed substrate concentration in the culture (20.0 g/l at the steady-state);
D = dilution rate (0.202 l/h at the steady-state);
μ = specific growth rate [l/h] given by (4), which includes the inhibition due to the substrate and the reaction product;
$Y_{X/S}$ = cell mass yield (0.4 g/g);
α = kinetic parameter (2.2 g/g);
β = kinetic parameter (0.2 l/h);
μ_{max} = maximum specific growth rate (0.48 l/h);
P_m = product saturation constant (50 g/l);
K_m = substrate saturation constant (1.2 g/l);
K_i = substrate inhibition constant (22 g/l).

The control objective is to maximize the output rate of biomass per unit volume of the culture. The non-linear behavior of the process clearly results from its static characteristic (figure 1).

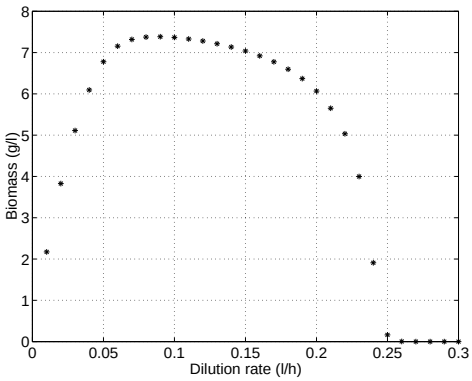

Figure 1 - The static characteristic of the process

From figure 1 it can be noticed that the control objective is accomplished for the dilution rate varying within the interval [0.09, 0.21].

3 THE LINEARIZED MODEL OF THE BIOTECHNOLOGICAL PROCESS

The linearized model has been determined for a nominal point of functioning $\{\overline{X}, \overline{S}, \overline{P}, \overline{D}\}$, where $X = \overline{X} + \Delta X$, $S = \overline{S} + \Delta S$, $P = \overline{P} + \Delta P$ and $D = \overline{D} + \Delta D$. It is

given by the following equations:

$$\frac{d\Delta X}{dt} = (\overline{\mu} - \overline{D})\Delta X + b\overline{X}\Delta S + a\overline{X}\Delta P - \overline{X}\Delta D \tag{5}$$

$$\frac{d\Delta S}{dt} = -\frac{\overline{\mu}}{Y_{X/S}}\Delta X - (\overline{D} + \frac{b\overline{X}}{Y_{X/S}})\Delta S - \frac{a\overline{X}}{Y_{X/S}}\Delta P + (S_f - \overline{S})\Delta D \tag{6}$$

$$\frac{d\Delta P}{dt} = (\overline{\mu} - \overline{D})\Delta X + b\overline{X}\Delta S + a\overline{X}\Delta P - \overline{X}\Delta D \tag{7}$$

where

$$\overline{\mu} = \frac{\mu_{\max}(1 - \overline{P}/P_m)}{k_m + \overline{S} + \overline{S}^2/k_i} \tag{8}$$

$$a = -\frac{\mu_{\max}\overline{S}}{P_m(k_m + \overline{S} + \overline{S}^2/k_i)} \tag{9}$$

$$b = \frac{\mu_{\max}(1 - \overline{P}/P_m)(k_m - \overline{S}^2/k_i)}{(k_m + \overline{S} + \overline{S}^2/k_i)^2} \tag{10}$$

The linearized model of the process can be write as a matrix:

$$\begin{bmatrix} \Delta\dot{X} \\ \Delta\dot{S} \\ \Delta\dot{P} \end{bmatrix} = \begin{bmatrix} \overline{\mu} - \overline{D} & b\overline{X} & a\overline{X} \\ -\frac{\overline{\mu}}{Y_{X/S}} & -\overline{D} - \frac{b\overline{X}}{Y_{X/S}} & -\frac{a\overline{X}}{Y_{X/S}} \\ \alpha\overline{\mu} + \beta & \alpha b\overline{X} & \alpha a\overline{X} - \overline{D} \end{bmatrix} \cdot \begin{bmatrix} \Delta X \\ \Delta S \\ \Delta P \end{bmatrix} + \begin{bmatrix} -\overline{X} \\ S_f - \overline{S} \\ -\overline{P} \end{bmatrix} \Delta D \tag{11}$$

4 QFT ALGORITHM FOR TRACKING PERFORMANCES

QFT algorithm is given by the following steps [3], [5]:

1. Synthesize the desired tracking model.
2. Specify the J linear-time-invariant plant models that define the boundary of the region of plant parameter uncertainty.
3. Obtain plant templates, at specified frequencies, that pictorially describe the region of plant parameter uncertainty on the Nichols chart.
4. Select the nominal plant transfer function $P_0(s)$.
5. Determine the stability contour (U-contour) on the Nichols chart.
6. Determine the tracking bounds on the Nichols chart.
7. Determine the optimal bounds on the Nichols chart.
8. Synthesize the nominal loop transmission function, $L_0(s)= G(s)P_0(s)$, that satisfies all the bounds and the stability contour.
9. Based on steps 1 through 8, synthesize the prefilter, $F(s)$.
10. Simulate the system in order to obtain the time response data for each of the J plants.

5 SIMULATIONS AND RESULTS

The following transfer function corresponds to the linearized model (11), presented in section 3.

$$P(s) = \frac{K(s+a)}{s^2 + bs + c} \tag{12}$$

where $K \in [6.06\ 7.39]$; $a \in [0.05\ 0.16]$; $b \in [0.30\ 0.63]$; $c \in [0.03\ 0.07]$

The tracking models for upper and lower bounds were established:

$$T_{RU}(s) = \frac{0.6584(s+30)}{s+2 \pm j \cdot 3.969} \tag{13}$$

$$T_{RL}(s) = \frac{120}{(s+3)(s+4)(s+10)} \tag{14}$$

Figure 2 presents the optimal bounds obtained by the intersection between the stability contour and the tracking bound at different frequencies [4].

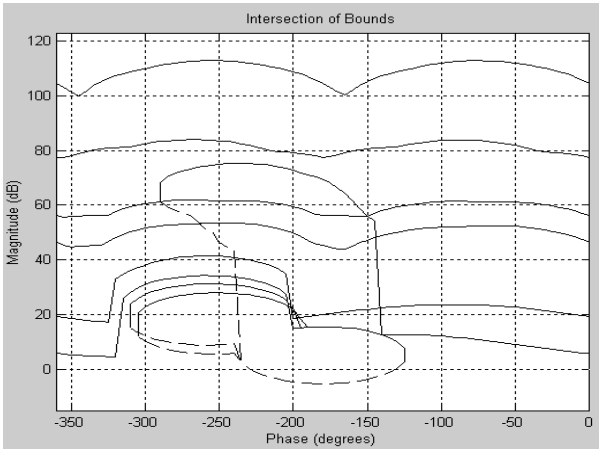

Figure 2 - The optimal bounds for stability and tracking

Figure 3 presents the loop shaping. The closed loop contour must not violate the stability contour and it must be located above the tracking bound for a given frequency. The following transfer function for the controller results:

$$G(s) = \frac{774.8(s/0.2384+1)(s/3.01+1)(s/489.7+1)}{(s/0.0147+1)(s/9.973+1)(s/247+1)(s/3863+1)} \tag{15}$$

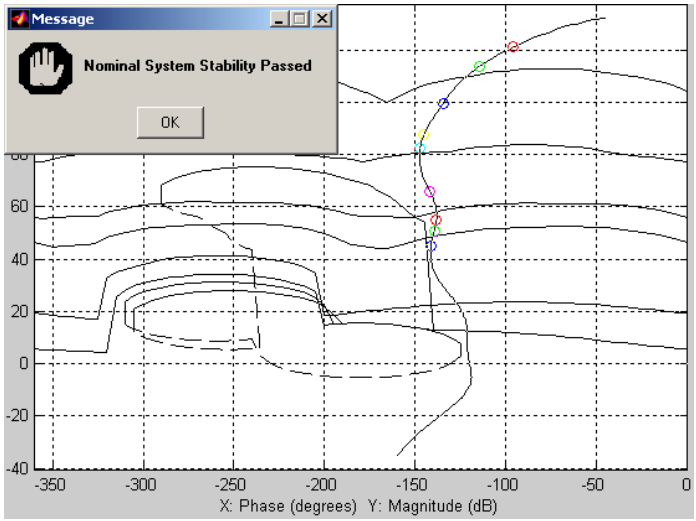

Figure 3 - Loop shaping using QFT techniques

Based on the transfer function of the closed loop system obtained above, the prefilter is designed such that the tracking specifications to be accomplished (figure 4). The transfer function of the prefilter is given by equation (16):

$$F(s) = \frac{1}{s^2 / 3.855^2 + 2 \cdot 0.6646 \cdot 3.855 \cdot s + 1} \tag{16}$$

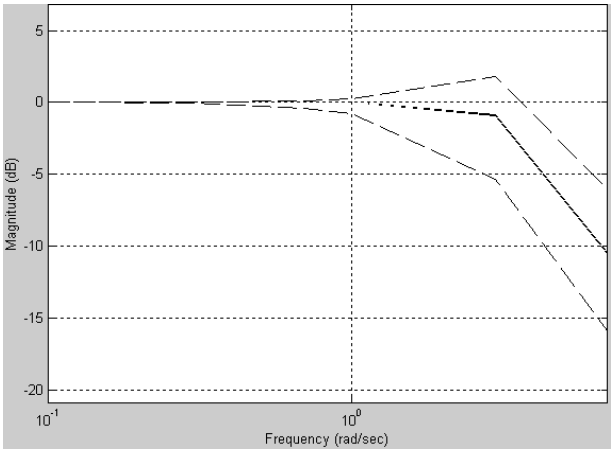

Figure 4 - The design of the prefilter

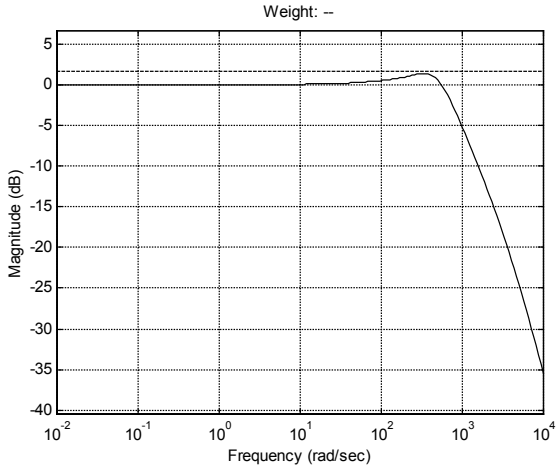

Figure 5 - The checking of the stability condition

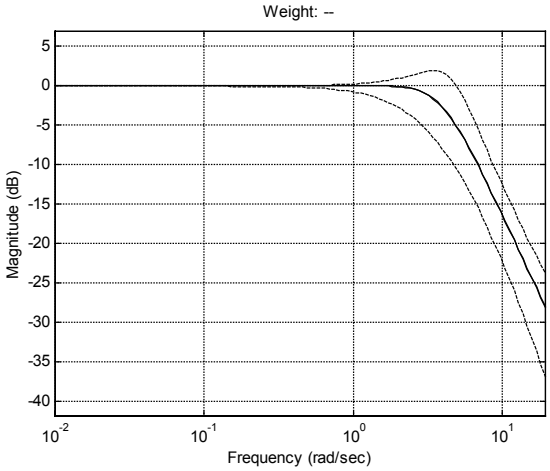

Figure 6 - The checking of the tracking conditions

Figures 5 and 6 show the checking of the stability and robustness tracking conditions. As one can notice the shaped loop respects both the stability condition (the magnitude characteristic is situated under the imposed bound) and the robustness tracking property (the magnitude characteristic of the loop is located between the upper and lower imposed bounds).

Figure 7 presents the time response of the closed loop system for three linear models of the process.

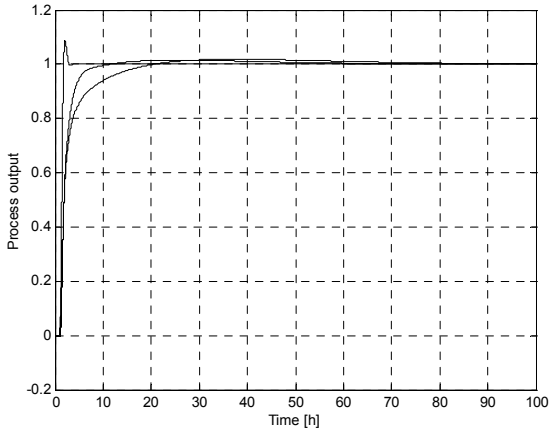

Figure 7 - The time response of the closed loop system for three linear models

6 CONCLUSIONS

The paper clearly proves the necessity of using robust control for the biotechnological processes. The biotechnological processes are characterized by many uncertainties which makes the process models to have variable parameters. QFT is a solution to design a robust controller such that the stability and robust tracking performances of the closed loop system to be accomplished.

In the present case QFT algorithm is easy to implement because the linearized model of the biotechnological process is on minimum phase. In general non-minimum phase linear models approximate the most biotechnological processes and in this case the QFT procedure is more complicated.

7. REFERENCES

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