IMPROVING EPSAC TUNING FOR PROCESSES WITH UNSTABLE TRANSMISSION ZEROS

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Abstract: In MIMO plants the presence of right-half plane (RHP) transmission zeros deteriorates the performance of the control algorithms, often resulting in instability problems. This paper presents rules about tuning the EPSAC predictive control algorithm for the above mentioned type of processes. A quadruple-tank process with a RHP transmission zero has been used to test by simulation the proposed tuning procedure.

Keywords: Multivariable systems, Predictive control, EPSAC, Transmission zeros, Non-minimum phase, Controller tuning

1. INTRODUCTION

Predictive control is a possible and most often chosen solution, besides elaborating complicated compensation schemes, for the control of strong coupled multivariable processes that have to be controlled at different operating points [1].

Transmission zeros represent specific values of the complex frequency for which transmission through the system is blocked. When positioned in the right half complex plane they impose limitations on the achievable performances of the closed loop multivariable system [2]. Corresponding to a RHP transmission zero the inputs that will zero the outputs will go exponentially to infinity, this being the reason to refer them as being “unstable” rather than “non-minimum phase” [3].

The systems with RHP transmission zeros present difficulties when applying tuning strategies due to the instability problems. When non-minimum phase processes were controlled with the GPC algorithm, with the control horizon equal to the prediction horizon, and equal to one, and the control weighting factor equal to zero, instability appeared [4,5]. In the MIMO case instability problems are more difficult to be solved due to the hidden dynamic of the RHP transmission zeros [6].

This paper presents tuning rules for the two approaches of the EPSAC (Extended Predictive Self Adaptive Control) predictive control algorithm, [7], based on the time constant of the RHP transmission zero and on the settling times of the multivariable process coupling channels. The new tuning approaches were tested by simulation considering the quadruple tank process, [8], that has been used as a test bed also with other predictive controllers [9,10,11]. A DMC controller was applied in [9], without discussing the instability problems and the tuning procedure. In [10,11] was studied this process control with the GPC algorithm, the instability problem being solved mainly by introducing a different than zero control weighting factor.

2. EPSAC FOR MULTIVARIABLE PROCESSES

This section briefly summarizes the extension of the EPSAC-MBPC to multivariable systems, both "solidary" and "selfish" approaches, considering the case of a system with n_u inputs and n_y outputs, [7].

2.1 *Principle of MIMO EPSAC*

The following structure of the generic process model is used:

$$y_i(t) = x_i(t) + n_i(t)\,,\ i = \overline{1, n_y} \tag{1}$$

with: $y_i(t)$ - measured process outputs, $x_i(t)$ - model outputs and $n_i(t)$ - disturbances (including modeling errors, noise, etc.); t denoting the discrete-time index.

The predicted values of the outputs are then calculated with:

$$y_i(k+t\,|\,t) = x_i(k+t\,|\,t) + n_i(k+t\,|\,t)\,,\ i = \overline{1, n_y} \tag{2}$$

for $k = \overline{N_1, N_2}$ where N_1 and N_2 are the minimum and the maximum prediction horizons. The multi-step prediction problem is solved by recursion of the process models, in order to obtain $x_i(t+k\,|\,t)$, and using filtering techniques on the noise models to obtain $n_i(t+k\,|\,t)$. A detailed description is given in [7].

The future response of the process is considered to be the result of two effects:

$$y_i(t+k\,|\,t) = y_{iBase}(t+k\,|\,t) + y_{iOptimize}(t+k\,|\,t)\,,\ i = \overline{1, n_y} \tag{3}$$

The first term is regarded as the effect of the past controls, the predicted disturbances and of the basic future control scenario $\{u_{jBase}(t+k\,|\,t)\}$ for $k = \overline{0, N_u - 1}$ (N_u being the control horizon), and for $j = \overline{1, n_u}$.

The second term in (3) is the effect of the optimizing future control actions:

$$\delta u_j(t+k\,|\,t) = u_j(t+k\,|\,t) - u_{jBase}(t+k\,|\,t)\,,\ k = \overline{0, N_u - 1} \tag{4}$$

where $u_j(t+k\,|\,t)$ are the desired optimal control actions. The optimizing control actions δu_j can be considered as a series of impulses and a final step; so the cumulative effect to the i-th output of the system at time $t+k$ is:

$$y_{iOptimize}(t+k\,|\,t) = \sum_{j=1}^{n_u} [h_k^{ij}\delta u_j(t\,|\,t) + h_{k-1}^{ij}\delta u_j(t+1\,|\,t) + \ldots + g_{k-N_u+1}^{ij}\delta u_j(t+N_u-1\,|\,t)] \tag{5}$$

where h_k^{ij} and g_k^{ij} are the coefficients of the impulse and step response of the process (from input j to output i).

The key EPSAC-MBPC equations for multivariable processes are:

$$\mathbf{Y}_i = \mathbf{Y}_{iBase} + \mathbf{Y}_{iOptimize} = \overline{\mathbf{Y}}_i + \sum_{j=1}^{n_u} \mathbf{G}_{ij}\mathbf{U}_j \tag{6}$$

where, for $i = \overline{1, n_y}$ and $j = \overline{1, n_u}$:

$$\mathbf{Y}_i = [y_i(t+N_1\,|\,t) \ \ \ldots \ \ y_i(t+N_2\,|\,t)]^T,\ \overline{\mathbf{Y}}_i = [y_{iBase}(t+N_1\,|\,t) \ \ \ldots \ \ y_{iBase}(t+N_2\,|\,t)]^T,$$

$$\mathbf{G}_{ij} = \begin{bmatrix} h_{N_1}^{ij} & h_{N_1-1}^{ij} & \ldots & h_{N_1-N_u+2}^{ij} & g_{N_1-N_u+1}^{ij} \\ h_{N_1+1}^{ij} & h_{N_1}^{ij} & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ h_{N_2}^{ij} & h_{N_2-1}^{ij} & \ldots & h_{N_2-N_u+2}^{ij} & g_{N_2-N_u+1}^{ij} \end{bmatrix},\ \mathbf{U}_j = [\delta u_j(t\,|\,t) \ \ \ldots \ \ \delta u_j(t+N_u-1\,|\,t)]^T$$

The two MIMO EPSAC approaches, which differ in the choice of the control criteria, will be presented in the following subsections. Without affecting the generality of the problem will be considered in the sequel a system with 2 inputs and 2 outputs.

2.2 *Solidary Control*

The objective is to find the optimal control vectors $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$ which minimize the cost function:

$$J(\mathbf{U}) = \sum_{k=N_1}^{N_2}\left[r_1(t+k\,|\,t) - y_1(t+k\,|\,t)\right]^2 + \sum_{k=N_1}^{N_2}\left[r_2(t+k\,|\,t) - y_2(t+k\,|\,t)\right]^2 \qquad (7)$$

subject to $u_1(t+k\,|\,t) = u_1(t+N_u-1\,|\,t)$ and $u_2(t+k\,|\,t) = u_2(t+N_u-1\,|\,t)$ for $k \geq N_u$, $r_i(.|.)$, $i=1,2$, being the reference trajectories.

Defining the matrices $\mathbf{G_1} = [\mathbf{G_{11}} \quad \mathbf{G_{12}}]$, $\mathbf{G_2} = [\mathbf{G_{21}} \quad \mathbf{G_{22}}]$ (where $\mathbf{G}_{ij}$ are defined in (6)) and the vector $\mathbf{U} = \left[\mathbf{U_1}^\mathbf{T} \quad \mathbf{U_2}^\mathbf{T}\right]^\mathbf{T}$, the cost function can be transformed in the quadratic form:

$$J(\mathbf{U}) = \mathbf{U^T H U} + \mathbf{2f^T U} + \mathbf{c} \qquad (8)$$

with $\mathbf{H} = \mathbf{G_1^T G_1} + \mathbf{G_2^T G_2}$, $\mathbf{f} = -\left[\mathbf{G_1^T(R_1 - \overline{Y}_1) + G_2^T(R_2 - \overline{Y}_2)}\right]$ and

$\mathbf{c} = \mathbf{(R_1 - \overline{Y}_1)^T(R_1 - \overline{Y}_1) + (R_2 - \overline{Y}_2)^T(R_2 - \overline{Y}_2)}$.

Minimization of $J(\mathbf{U})$, with respect to $\mathbf{U}$, results in the solution:

$$\mathbf{U}^* = -\mathbf{H}^{-1}\mathbf{f} \qquad (9)$$

2.3 *Selfish Control*

In this case the objective is to find the optimal control vectors $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$, which minimize the cost functions:

$$J_1(\mathbf{U}_1) = \sum_{k=N_1}^{N_2}\left[r_1(t+k\,|\,t) - y_1(t+k\,|\,t)\right]^2 \text{ and } J_2(\mathbf{U}_2) = \sum_{k=N_1}^{N_2}\left[r_2(t+k\,|\,t) - y_2(t+k\,|\,t)\right]^2 \qquad (9)$$

subject to $u_1(t+k\,|\,t) = u_1(t+N_u-1\,|\,t)$ and $u_2(t+k\,|\,t) = u_2(t+N_u-1\,|\,t)$ for $k \geq N_u$.

Note that this objective is only applicable if the number of controls (n_u) is equal to the number of controlled variables (n_y).

The cost function, for $i=1,2$ can be written as a quadratic form in $\mathbf{U}_i$:

$$J_i(\mathbf{U}_i) = \mathbf{U}_i^\mathbf{T}\mathbf{H}_i\mathbf{U}_i + 2\mathbf{f}_i^\mathbf{T}\mathbf{U}_i + \mathbf{c}_i \qquad (10)$$

with $\mathbf{H}_i = \mathbf{G}_{ii}^\mathbf{T}\mathbf{G}_{ii}$, $\mathbf{f}_i = -\mathbf{G}_{ii}^\mathbf{T}(\mathbf{R}_i - \overline{\mathbf{Y}_i} - \mathbf{G}_{ij}\mathbf{U}_j)$, for $j \neq i$, and

$\mathbf{c}_i = (\mathbf{R}_i - \overline{\mathbf{Y}}_i - \mathbf{G}_{ij}\mathbf{U}_j)^\mathbf{T}(\mathbf{R}_i - \overline{\mathbf{Y}}_i - \mathbf{G}_{ij}\mathbf{U}_j)$.

The minimization leads to the solution, for $i=1,2$:

$$\mathbf{U}_i^* = -\mathbf{H}_i^{-1}\mathbf{f}_i \qquad (11)$$

Notice that the optimal solution for one input $\mathbf{U}_i^*$ depends on the other input $\mathbf{U}_j$.

Considering the previously defined matrices $\mathbf{G_1}$, $\mathbf{G_2}$ and the compound vector $\mathbf{U}$, the explicit solution is then given by:

$$\mathbf{U}^* = \begin{bmatrix}\mathbf{G_{11}^T G_1}\\ \mathbf{G_{22}^T G_2}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{G_{11}^T(R_1 - \overline{Y}_1)}\\ \mathbf{G_{22}^T(R_2 - \overline{Y}_2)}\end{bmatrix} \qquad (12)$$

Remark: For both approaches (Solidary and Selfish) at each sample time only two elements of $\mathbf{U}^*$ are required to compute the control action applied to the process:

$$\begin{aligned} u_1(t) &= u_{1Base}(t\,|\,t) + \mathbf{U}_1^*(1)\\ u_2(t) &= u_{2Base}(t\,|\,t) + \mathbf{U}_2^*(1) \end{aligned} \qquad (13)$$

where $\mathbf{U}_1^*(1) = \delta u_1(t\,|\,t)$ and $\mathbf{U}_2^*(1) = \delta u_2(t\,|\,t)$.

At the next sampling instant the whole procedure is repeated taking into account the new measured outputs according to the receding horizon control principle.

The cost index can also be extended taking into account constraints, this then leading to a quadratic programming problem [7].

3. QUADRUPLE TANK PROCESS APPLICATION

3.1 *Process description*

In order to analyze the controller's ability to control non-minimum phase multivariable processes it was considered the quadruple tank process presented in Figure 1 and described by the equations (14).

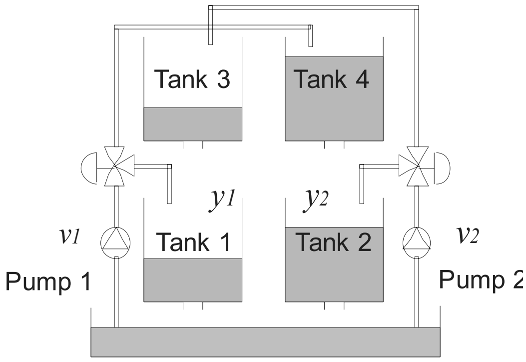


Figure 1 - Schematic diagram of the four-tank process

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} v_1
\end{aligned} \tag{14}$$

The process inputs are the voltage signals that feed the two pumps, v_1 and v_2, and the outputs are the measured levels, y_1 and y_2. A_i is the cross section of tank i; a_i is proportional to the cross section of the hole i; h_i is the water level in tank i. The corresponding flow to the voltage v_i is $k_i v_i$. The measured level signals are $y_1 = k_c h_1$ and $y_2 = k_c h_2$. The parameters γ_1 and $\gamma_2 \in (0,1)$ specify how the valves are set.

The linear model of the process, obtained by linearization around an operating point described by h_i^0, for $i = \overline{1,4}$, is given by the transfer function matrix in equation (15)

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{\gamma_1 c_1}{1+sT_1} & \dfrac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \dfrac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \dfrac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \tag{15}$$

where $c_i = T_i k_i k_c / A_i$ for $i = 1,2$ and $T_i = (A_i / a_i)\sqrt{2h_i^0 / g}$ for $i = \overline{1,4}$.

The model (15) can be expressed as a product of two polynomial matrices:

$$\mathbf{G}(s) = \mathbf{A}(s)^{-1}\mathbf{B}(s) \tag{16}$$

The matrices $\mathbf{A}(s)$, with the diagonal elements equal to the least common multipliers of the denominators of the corresponding row of $\mathbf{G}(s)$, [1], and $\mathbf{B}(s)$ can be expressed as in equations (17):

$$\begin{aligned}
\mathbf{A}(s) &= \begin{bmatrix} (1+sT_1)(1+sT_3) & 0 \\ 0 & (1+sT_2)(1+sT_4) \end{bmatrix} \\
\mathbf{B}(s) = \mathbf{A}(s)\mathbf{G}(s) &= \begin{bmatrix} \gamma_1 c_1(1+sT_3) & (1-\gamma_2)c_1 \\ (1-\gamma_1)c_2 & \gamma_2 c_2(1+sT_4) \end{bmatrix}
\end{aligned} \tag{17}$$

Analyzing the position of the transmission zeros, given by $\det \mathbf{B}(s) = 0$, results that:

- for $0<\gamma_1+\gamma_2<1$ the process has a transmission zero in the RHP, and
- for $1<\gamma_1+\gamma_2<2$ both transmission zeros of the process are in the left-half plane.

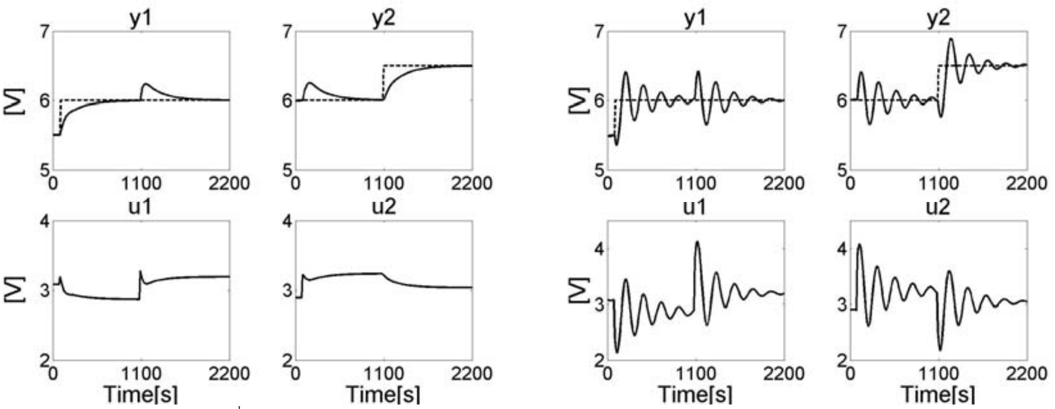
The simulation results were obtained considering a position of the valves, $(\gamma_1,\gamma_2)=(0.43,0.34)$, that placed one of the transmission zeros in the RHP, $(z_-,z_+)=(-0.0565,0.013)$. Although in this specific case we have a precise, nonlinear, model of the process, equations (14), the prediction was made based on a linear model keeping in mind that we usually have at our disposal only this type of model. The model was obtained in a certain operating point, chosen as in [8] and characterized by (v_1^0, v_2^0)=(3.15,3.15) and $(h_1^0,h_2^0,h_3^0,h_4^0)$=(12.44,13.17,4.73,4.98).

3.2 *EPSAC tuning*

A discrete model of the process, obtained for a sampling time of $T_s=10\sec$, was used for designing and tuning the EPSAC controller.

The N_1 parameter has been chosen equal to 1 since the non-minimum phase of a multivariable process does not imply an inverted response. Also the N_u parameter was chosen as small as possible, and in the following experiments it was taken equal to 1.

In [11] it was indicated that the choice of N_2 should be correlated with the time constant corresponding to the RHP zero. N_2 must be chosen greater than $2*(1/z_+)/T_s$, thus greater than 15. The result in Figure 2(a) was obtained with a solidary controller for $N_1=1$, $N_2=16$ and $N_u=1$. In [10], the GPC tuning parameters for this process were chosen: $N_1=1$, $N_2=78$, $N_u=20$ for $T_s=2\sec$. Note also the similarity between GPC and solidary EPSAC cost functions.



(a) Solidary EPSAC (b) Selfish EPSAC

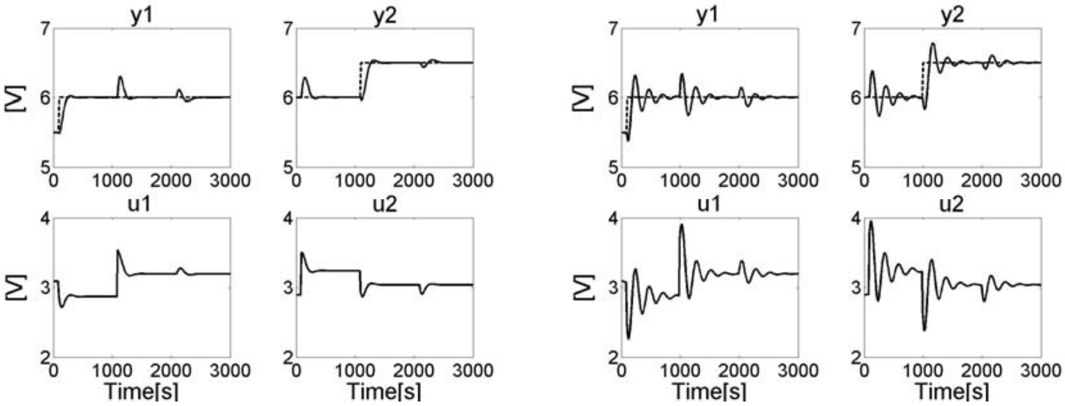
Figure 2 - Control results

However, the selfish approach to EPSAC, with $N_2=16$ did not lead to satisfactory results. This happens because the prediction horizon is not sufficiently large to properly appreciate the degree of coupling that characterizes the process and the loss of information generated by this short prediction horizon cannot be compensated by the control algorithm. So, for the selfish approach to EPSAC, the predictive horizon has to be mainly dictated by the polynomials in the matrix $\mathbf{A}(s)$ and so the N_2 parameter to be taken greater than the response time of the slower coupling channel.

The results that were obtained with $N_2=45$ are plotted in Figure 2(b), N_2*T_s being larger than the response time of transfer function from input v_1 to output y_2.

In Figure 3(a) are presented improved closed loop results that were obtained with the solidary approach to EPSAC and the parameters $N_1=1$, $N_2=18$ and $N_u=1$, considering the trade-off between the control effort and the desired time of response of the closed loop system. A step disturbance was applied to tank 3 at time $t = 2000\sec$.

The results in Figure 3(b), obtained while using a selfish controller with $N_1 = 1$, $N_2 = 55$ and $N_u = 1$, also show that a relaxation in the closed loop response specifications implies less activity of the controls and so less oscillations in the outputs.



(a) Solidary EPSAC (b) Selfish EPSAC

Figure 3 - Control results with improved tuning

4. CONCLUSIONS

This paper gives rules for multivariable EPSAC tuning in the case of processes with unstable transmission zeros. The tuning approach was successfully tested on quadruple tank process by simulation.

Although both multivariable EPSAC controllers led to good performances, the solidary controller is recommended to be used because it employs shorter prediction horizon which implies less computation.

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