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THE ACCELERATION ENERGY TO ROBOT DYNAMICS

Iuliu NEGREAN*, Dan-Călin NEGREAN**

**Technical University of Cluj-Napoca, Department of Mechanics and Computer Programming, Bd. Muncii 103-105, Cluj-Napoca, Romania, Email address:inegrean@tempus.east.utcluj.ro*
***Technical University of Cluj-Napoca, Faculty of Automation and Computer Science, Str. Gh. Barițiu 26, Cluj-Napoca, Romania, Email address: ndc_blacky@yahoo.com*

Abstract: The algorithm of the dynamic control functions *(IDM)* achieves a complex study regarding the behavior of any mechanical robot structure. Beside the dynamic control functions, determined with various models, the dynamic functions of the operational variables can be also analyzed. In the frame of this paper, on the basis of new formulations the expressions of the acceleration energy, and then its mapping within of the matrix dynamics equations will be analyzed. The above algorithm *(IDM)* is included in *SimMEcROb Simulator* devoted to study concerning the assessment of the kinematics performances, dynamics, and accuracy respectively for whatever mechanical robot structure, regardless of its type and geometrical form.

Keywords: Robotics, Modeling, Mechanics, Dynamics, and Control.

1. INTRODUCTION

The robot control can be defined, in a simplified form, by means of the equation:

$$\overline{Q}(t) \Leftrightarrow \overline{\theta}(t) \Leftrightarrow \overline{X}(t) \ . \quad (1)$$

The robots perform some technological processes the high values for the kinematics and dynamics accuracy performances are required in. That is why, the robot dynamics control becomes a compulsory condition. The fundamental equations of the dynamics in the form of symbolic show as below:

$$\overline{\theta}(t) = f\left[\overline{Q}_m(t)\right]; \qquad \overline{Q}_m(t) = f^{-1}\left\{\overline{\theta}(t); \dot{\overline{\theta}}(t); \ddot{\overline{\theta}}(t)\right\}; \qquad \overline{Q}_m(t) = \left[Q_m^i(t); i = 1 \rightarrow n\right]^T . \quad (2)$$

Above, $\overline{Q}_m(t)$ represents the column vector of the generalized driving forces from each joint of robot, also called the matrix function of the dynamic control. First equation from (2) refers to the direct dynamics model and the second to the inverse *(IDM)*.

The algorithm of the dynamic control functions *(IDM)* is included in *SimMEcROb Simulator*, [2] and [3]. It achieves a complex study regarding the kinematics behavior and dynamics respectively to whatever mechanical robot structure. Thus, beside the dynamic control functions, the dynamic functions of the operational variables can be likewise analyzed. Within of this paper a matrix algorithm based on the acceleration energy will be described in a new formulation. In the first time, the forward kinematics formalism will be described. Its main matrix expressions will be called in the acceleration energy, and then dynamics equations of the mechanical robot structure.

2. THE FORWARD KINEMATICS FORMALISM

So as to define the direct kinematics modeling *(DKM)*, in the Fig. 1 it has been represented in a symbolical form the mechanical robot structure *(MRS)* with *n d.o.f.* The *(n)* kinetic links are joined by driving joints of fifth order symbolized as *{R-rotation; T-translation}*, and considered mechanically perfect. In keeping with the forward kinematics formalism [2] and [3] the *DKM* equations can be written in a symbolical form as follows:

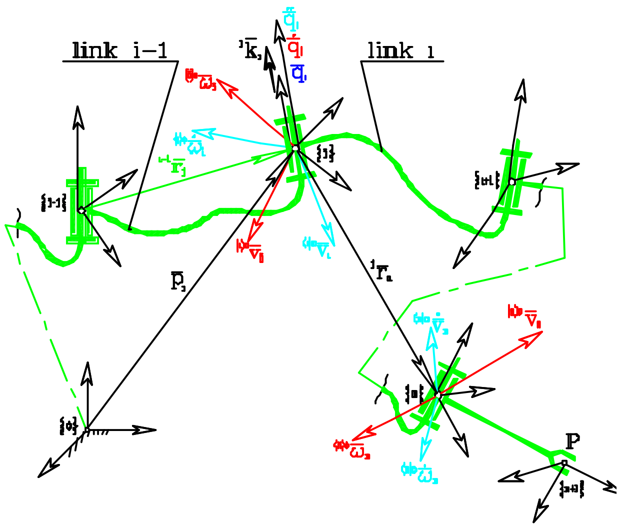

Fig. 1

$$^{(n)0}\overline{X}(t) = f\left[\overline{\theta}(t)\right] \quad ^{(n)0}\dot{\overline{X}} = f\left(\overline{\theta};\dot{\overline{\theta}}\right)$$

$$^{(n)0}\ddot{\overline{X}} = f\left(\overline{\theta};\dot{\overline{\theta}};\ddot{\overline{\theta}}\right); \tag{3}$$

$$\left\{\overline{\theta};\ \dot{\overline{\theta}};\ \ddot{\overline{\theta}}\right\} \& \left\{^{(n)0}\overline{X};\ ^{(n)0}\dot{\overline{X}};\ ^{(n)0}\ddot{\overline{X}}\right\}.$$

First set of vectors, above shown, describes the motion from every driving joint, while the second characterizes the motion of the end-effector in the Cartesian space. Their components are expressed by the symbols:

$$\left\{\overline{\theta}(t);\ \dot{\overline{\theta}}(t);\ \ddot{\overline{\theta}}(t)\right\} = \left[\ \left\{q_i(t);\ \dot{q}_i(t);\ \ddot{q}_i(t)\right\};\ i = 1 \rightarrow n\ \right]^T; \tag{4}$$

$$^{(n)0}\overline{X}(t) = \begin{bmatrix} \overline{p}(t) \\ \ldots\ldots \\ \overline{\psi}(t) \end{bmatrix} = \left\{\begin{matrix} \left[\left[f_j\left(q_i(t)\cdot\delta_i;\ i = 1 \rightarrow n\right);\ j = 1 \rightarrow 6\right]^T \\ \delta_i = \left\{\{1;\ j = 1 \rightarrow 3\};\{\Delta_i;\ j = 4 \rightarrow 6\}\right\} \end{matrix}\right\};\quad \begin{matrix} ^{(n)0}\dot{\overline{X}} = \left[^{(n)0}\overline{v}_n^T\ \ ^{(n)0}\overline{\omega}_n^T\right]^T; \\ ^{(n)0}\ddot{\overline{X}} = \left[^{(n)0}\dot{\overline{v}}_n^T\ \ ^{(n)0}\dot{\overline{\omega}}_n^T\right]^T. \end{matrix} \tag{5}$$

According to *SimMEcROb* [3], by applying the polynomial interpolating functions of either *4-3-4th* or *3rd* order, the matrix of the kinematics control functions is established:

$$M_{\theta n}^{CF} = \left\{\begin{matrix} \left[\left[\ \dot{\theta}_k(\tau)\ \ \dot{\overline{\theta}}_k(\tau)\ \ \ddot{\overline{\theta}}_k(\tau);\quad k = 1 \rightarrow m\ \right]^T \\ \left\{\left[q_{jk}(\tau)\ \ \dot{q}_{jk}(\tau)\ \ \ddot{q}_{jk}(\tau)\right];\ k = 1 \rightarrow m\right\}^T \end{matrix}\right\}; \tag{6}$$

where $k = 1 \rightarrow m$ is the configuration number taken into study, while $\tau_{k-1} \le \tau \le \tau_k$ is the actual time variable between the two successively configurations of the robot structure.

In keeping with [3] and [4], *the Jacobian matrix algorithm* will be short described so as to determine the *DKM* equations. The main steps are shown below.

- From *DGM Algorithm*, the following homogeneous transformation matrices are called:

$$\left\{\ {}^{i-1}_{i}[T][q_i(t)];\ \ {}^{0}_{i}[T]\left[q_j(t);\ j = 1 \rightarrow i\right];\ \ {}^{i}_{n}[T]\left[q_j(t);\ j = i+1 \rightarrow n\right];\quad i = 1 \rightarrow n\ \right\};$$

$$^{j}_{i}[T][q_k(t);\ \ j = 1 \rightarrow i-1;\ \ k = j+1 \rightarrow i;\ \ i = 1 \rightarrow n\ ]. \tag{7}$$

- The next loops: $\{i = 1 \rightarrow n;\ j = 1 \rightarrow i;\ k = 1 \rightarrow j\}$ are opened. In order to describe the expressions typical of the above loops, the next notations have been implemented:

$$\Delta_i = \left\{\{1;\ i = R\};\ \ \{0;\ i = T\}\right\}; \qquad \overline{q}_i = \tau_i \cdot q_i \cdot {}^{i}\overline{k}_i\ \ ;\quad \tau_i = \pm 1; \tag{8}$$

Above, the operator Δ_i shows the driving joint type from the *MRS*, and ${}^{i}\overline{k}_i$ the unit vector of the driving axis expressed with respect to proper frame *{i}*.

• At beginning, the matrix-deriving operator is defined by means of the next expression:

$$U_i = \tau_i \cdot \begin{bmatrix} \left\{ {}^i\bar{k}_i \cdot \Delta_i \times \right\} & {}^i\bar{k}_i \cdot (1-\Delta_i) \\ 0\ 0\ 0 & 0 \end{bmatrix}; \qquad (9)$$

where $\left\{ {}^i\bar{k}_i \cdot \Delta_i \times \right\}$ the skew-symmetric matrix associated to unit vector ${}^i\bar{k}_i$ is called.

• The differential matrices of first and second order are defined with the expressions:

$$A_{ij} = A_{ji} = \left\{ \begin{matrix} {}^0_j[T] \cdot U_j \cdot {}^j_i[T] \\ \begin{bmatrix} A_{ij}(R) & A_{ij}(\bar{p}) \\ 0\ 0\ 0 & 0 \end{bmatrix} \end{matrix} \right\}; \qquad A_{ijk} = A_{ikj} = \left\{ \begin{matrix} {}^0_k[T] \cdot U_k \cdot {}^k_j[T] \cdot U_j \cdot {}^j_i[T] \\ \begin{bmatrix} A_{ijk}(R) & A_{ijk}(\bar{p}) \\ 0\ 0\ 0 & 0 \end{bmatrix} \end{matrix} \right\}. \qquad (10)$$

• Every column from the Jacobian matrix and its time derivative of first order, expressed with respect to frame either *{0}* or *{n}*, are determined with the next expressions:

$${}^0J_i = \begin{bmatrix} A_{ni}(\bar{p}) \\ \cdots\cdots \\ {}^0_i[R]\,{}^i\bar{k}_i \cdot \Delta_i \end{bmatrix}; \qquad {}^0\dot{J}_i = \begin{bmatrix} \sum_{j=1}^{n} A_{nij}(\bar{p}) \cdot \dot{q}_j \\ \cdots\cdots \\ \left\{ \sum_{j=1}^{i} A_{ij}(R) \cdot \dot{q}_j \right\} \cdot {}^i\bar{k}_i \cdot \Delta_i \end{bmatrix} \qquad (11)$$

• The Jacobian matrix and its time derivative, in the two frames *{0}* and *{n}*, is written as:

$${}^0J(\bar{\theta}) = \begin{bmatrix} {}^0J_V(\bar{\theta}) \\ {}^0J_\Omega(\bar{\theta}) \end{bmatrix} = \begin{bmatrix} {}^0J_i = \begin{bmatrix} {}^0J_{iv} \\ {}^0J_{i\omega} \end{bmatrix} i = 1 \to n \end{bmatrix}; \ {}^nJ(\bar{\theta}) = \left\{ \begin{matrix} \begin{bmatrix} {}^0_n[R]^T & [0] \\ [0] & {}^0_n[R]^T \end{bmatrix} \cdot {}^0J(\bar{\theta}) \\ {}^nR \cdot {}^0J(\bar{\theta}) \end{matrix} \right\}; \qquad (12)$$

$${}^0\dot{J}(\bar{\theta}) = \begin{bmatrix} {}^0\dot{J}_V(\bar{\theta}) \\ {}^0\dot{J}_\Omega(\bar{\theta}) \end{bmatrix} = \begin{bmatrix} {}^0\dot{J}_i = \begin{bmatrix} {}^0\dot{J}_{iv} \\ {}^0\dot{J}_{i\omega} \end{bmatrix} i = 1 \to n \end{bmatrix}; \ {}^n\dot{J}(\bar{\theta}) = \left\{ \begin{matrix} \begin{bmatrix} {}^0_n[R]^T & [0] \\ [0] & {}^0_n[R]^T \end{bmatrix} \cdot {}^0\dot{J}(\bar{\theta}) \\ {}^nR \cdot {}^0\dot{J}(\bar{\theta}) \end{matrix} \right\}; \qquad (13)$$

$$\underset{(6\times C_n^2)}{{}^0J_B(\bar{\theta})} = \begin{bmatrix} J_B^{ij} = \begin{bmatrix} 2 \cdot A_{nij}(\bar{p}) \\ \cdots\cdots \\ A_{ij}(R) \cdot {}^i\bar{k}_i \Delta_i \end{bmatrix} & \begin{matrix} i = 1 \to n-1 \\ j = i+1 \to n \end{matrix} \end{bmatrix}; \ \underset{(6\times n)}{{}^0J_C(\bar{\theta})} = \begin{bmatrix} J_C^{ii} = \begin{bmatrix} A_{nii}(\bar{p}) \\ \cdots\cdots \\ A_{ii}(R) \cdot {}^i\bar{k}_i \Delta_i \end{bmatrix} & i = 1 \to n \end{bmatrix};$$

where $\left\{ {}^0J_V(\bar{\theta}); {}^0J_\Omega(\bar{\theta}) \right\}$ the linear and angular transfer sub-matrix have been called.

• Using the Jacobian matrix with its time derivative, above written, the *DKM* equations with respect to frame *{0}* and *{n}*, are defined by means of the next matrix expressions:

$$\begin{bmatrix} \left[{}^{(n)0}\bar{v}_n^T \ \ {}^{(n)0}\bar{\omega}_n^T \right]^T \\ \cdots\cdots \\ \left[{}^{(n)0}\dot{\bar{v}}_n^T \ \ {}^{(n)0}\dot{\bar{\omega}}_n^T \right]^T \end{bmatrix} = \begin{bmatrix} [0] & {}^{(n)0}J(\bar{\theta}) & [0] & [0] \\ \cdots & \cdots & \cdots & \cdots \\ {}^{(n)0}J(\bar{\theta}) & [0] & {}^{(n)0}J_B(\bar{\theta}) & {}^{(n)0}J_C(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \left[\ddot{\bar{\theta}}^T \ \ \dot{\bar{\theta}}^T \right]^T \\ \cdots\cdots \\ \left[\left[\dot{\bar{\theta}} \cdot \dot{\bar{\theta}} \right]^T \ \ \left[\dot{\bar{\theta}}^2 \right]^T \right]^T \end{bmatrix}. \qquad (14)$$

Remarks. *The DKM equations (14), established on the basis of the Jacobian matrix algorithm above described, will express the motion of the end-effector in the Cartesian space. The same results will be likewise called in the next section devoted to establishment of the acceleration energy for whatever mechanical robot structure taken into study.*

3. THE ACCELERATION ENERGY

It is known that, the dynamics equations to whatever mechanical system with *n d.o.f.*, as example mechanical robot structure *(MRS)*, are defined by means of the formalisms typical of the analytical mechanics [1] and [2]. But, the same equations can be likewise expressed by extending of the dynamics study upon the *acceleration energy*. In keeping with analytical mechanics, its defining expression into first form shows as:

$$E_a = \frac{1}{2}\cdot\int \dot{v}_j^2 \cdot dm = \frac{1}{2}\cdot\int {}^j\dot{\bar{v}}_j^T \cdot {}^j\dot{\bar{v}}_j \cdot dm\,; \qquad {}^j\dot{\bar{v}}_j = {}^j\dot{\bar{v}}_{C_j} + {}^j\dot{\bar{\omega}}_j \times {}^j\bar{r}_{C_j} + {}^j\bar{\omega}_j \times {}^j\bar{\omega}_j \times \bar{r}_{C_j}\,. \tag{15}$$

Considering Fig. 2, the acceleration of the elementary mass dm, continue distributed in the kinetic link *(j)* from *(MRS)*, see Fig. 3, is symbolized by ${}^j\dot{\bar{v}}_j$, while $\left\{ {}^j\dot{\bar{v}}_{C_j}\,; {}^j\bar{\omega}_j\,; {}^j\dot{\bar{\omega}}_j \right\}$ represent the parameters typical to a general motion of the same link, supposed as rigid.

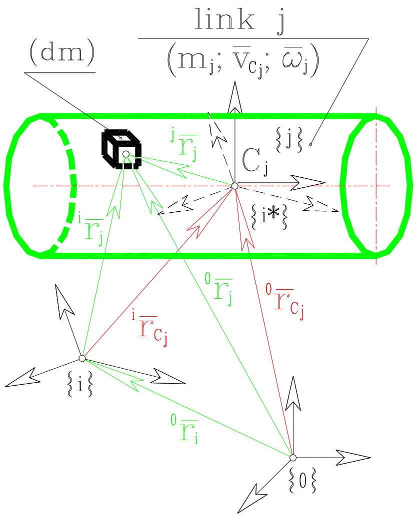

Fig. 2

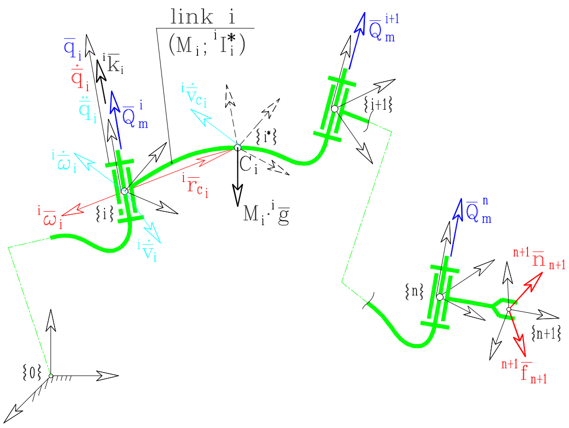

Fig. 3

Performing the calculus in (15), the defining expression of the acceleration energy takes a new general form written below as:

$$E_a = \left\{ \begin{array}{c} (-1)^{\Delta m}\cdot\dfrac{1-\Delta_M}{1+3\cdot\Delta_M}\cdot\left\{\dfrac{1}{2}\cdot M_j\cdot{}^j\dot{\bar{v}}_{C_j}^T\cdot{}^j\dot{\bar{v}}_{C_j}\right\}+ \\ +\Delta_M^2\cdot\left\{\dfrac{1}{2}\cdot{}^j\dot{\bar{\omega}}_j^T\cdot{}^jI_j^*\cdot{}^j\dot{\bar{\omega}}_j+{}^j\bar{\omega}_j^T\cdot{}^jI_{pj}^*\cdot{}^j\bar{\omega}_j\times{}^j\dot{\bar{\omega}}_j+\dfrac{1}{2}\cdot{}^j\bar{\omega}_j^T\cdot\left\{{}^j\bar{\omega}_j^T\cdot{}^jI_j^*\cdot{}^j\bar{\omega}_j\right\}{}^j\bar{\omega}_j\right\} \end{array} \right\}. \tag{16}$$

Above $\Delta_M = \{\{-1; General\ \ Motion\}; \{0; Translational\ \ Motion\}; \{1; Rotation\ \ Motion\}\}$;

and ${}^jI_j^* = \int \left\{{}^j\bar{r}_j\times\right\}\left\{{}^j\bar{r}_j\times\right\}^T\cdot dm\,;\qquad {}^jI_{pj}^* = \int {}^j\bar{r}_j\cdot{}^j\bar{r}_j^T\cdot dm\,.$ (17)

They are known as either inertia tensors or matrices of the mechanical inertia moments.

Taking into study the symbols from Fig. 2 and Fig. 3, the same acceleration energy will be also determined under form of a new matrix expression as follows:

$$\left\{ \begin{array}{c} dE_a^i \\ \dfrac{1}{2}\cdot{}^0\ddot{\bar{r}}_i^T\cdot{}^0\ddot{\bar{r}}_i\cdot dm \end{array} \right\} = \left\{ \begin{array}{c} \dfrac{1}{2}\cdot Trace\left[{}^0\ddot{\bar{r}}_i\cdot{}^0\ddot{\bar{r}}_i^T\cdot dm\right] = \dfrac{1}{2}\cdot Tr\left[{}^0\ddot{\bar{r}}_i\cdot{}^0\ddot{\bar{r}}_i^T\cdot dm\right] \\ \dfrac{1}{2}\cdot Tr\left[{}^0_i\left[\ddot{T}\right]{}^i\bar{r}_i\cdot{}^i\bar{r}_i^T\cdot dm\cdot{}^0_i\left[\ddot{T}\right]^T\right] \end{array} \right\}. \tag{18}$$

Mapping the mass integral on the whole link *(i)*, the integral expression is written as:

$$E_a^i = \begin{Bmatrix} \frac{1}{2}\cdot Tr\left[{}^0_i\left[\ddot{T}\right]\cdot\int {}^i\bar{r}_i \cdot {}^i\bar{r}_i^T \cdot dm \cdot {}^0_i\left[\ddot{T}\right]^T \right] \\ \frac{1}{2}\cdot Tr\left[{}^0_i\left[\ddot{T}\right] {}^i I_{psi} \cdot {}^0_i\left[\ddot{T}\right]^T \right] \end{Bmatrix}; \quad \begin{matrix} {}^0_i\left[\ddot{T}\right]^{(T)} = \sum_{j=1}^{i} A_{ij}^{(T)} \cdot \ddot{q}_j + \sum_{j=1}^{i}\sum_{k=1}^{i} A_{ijk}^{(T)} \cdot \dot{q}_j \cdot \dot{q}_k\ ; \\ {}^i I_{psi} = \int {}^i\bar{r}_i \cdot {}^i\bar{r}_i^T \cdot dm\ . \end{matrix}; \tag{19}$$

Performing the calculus in (19), it yields the next expression of the acceleration energy:

$$E_a = \begin{Bmatrix} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{m=1}^{i} Tr\left[A_{ij} \cdot {}^i I_{psi} \cdot A_{im}^T\right]\ddot{q}_j \cdot \ddot{q}_m + \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i}\sum_{m=1}^{i} Tr\left[A_{ij} \cdot {}^i I_{psi} \cdot A_{ikm}^T\right]\ddot{q}_j \cdot \dot{q}_k \cdot \ddot{q}_m + \\ +\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i}\sum_{l=1}^{i}\sum_{m=1}^{i} Tr\left[A_{ijk} \cdot {}^i I_{psi} \cdot A_{ilm}^T\right]\cdot \dot{q}_j \cdot \dot{q}_k \cdot \dot{q}_l \cdot \ddot{q}_m \end{Bmatrix}. \tag{20}$$

In keeping with [3], the next notations, and then dynamics matrices are implemented:

$$M\left(\bar{\theta}\right) = \left[M_{ij} = M_{ji} = \sum_{k=max(i;j)}^{n} Tr\left[A_{ki} \cdot {}^k I_{psk} \cdot A_{kj}^T\right] \quad \begin{matrix} i=1\to n \\ j=1\to n \end{matrix} \right]; \tag{21}$$

$$V\left(\bar{\theta};\dot{\bar{\theta}}\right) = \left[\dot{\bar{\theta}}^T \cdot \left[V_{ijm} = V_{imj} = \sum_{k=max(i;j;m)}^{n} Tr\left[A_{ki} \cdot {}^k I_{psk} \cdot A_{kjm}^T\right] \quad \begin{matrix} j=1\to n \\ m=1\to n \end{matrix} \right]\cdot \dot{\bar{\theta}}\ ;\ i=1\to n \right]^T;$$

$$D\left(\bar{\theta};\dot{\bar{\theta}}\right) = \left[\dot{\bar{\theta}}^T \cdot \left[D_{ijlm} == \sum_{k=max\ (i;j;l;m)}^{n} Tr\left[A_{kij} \cdot {}^k I_{psk} \cdot A_{klm}^T\right] \quad \begin{matrix} l=1\to n \\ m=1\to n \end{matrix} \right]\cdot \dot{\bar{\theta}} \quad \begin{matrix} i=1\to n \\ j=1\to n \end{matrix} \right];$$

$$B\left(\bar{\theta}\right) = \left[V_{ijm} = V_{imj} = \sum_{k=max(i;j;m)}^{n} Tr\left[A_{ki} \cdot {}^k I_{psk} \cdot A_{kjm}^T\right] \quad \begin{matrix} i=1\to n \\ j=1\to n-1 \\ m=j+1\to n \end{matrix} \right]; \tag{22}$$

$$C\left(\bar{\theta}\right) = \left[V_{ijj} = \sum_{k=max(i;j)}^{n} Tr\left[A_{ki} \cdot {}^k I_{psk} \cdot A_{kjj}^T\right] \quad \begin{matrix} i=1\to n \\ j=1\to n \end{matrix} \right].$$

Consequently, the new matrix expression of the acceleration energy, finally, shows as:

$$E_a = \begin{Bmatrix} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} \cdot \ddot{q}_i\, \ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{m=1}^{n} V_{ijm} \cdot \ddot{q}_i \cdot \dot{q}_j\, \dot{q}_m + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{m=1}^{n} D_{ijlm} \cdot \dot{q}_i \cdot \dot{q}_j \cdot \dot{q}_l \cdot \dot{q}_m \\ \frac{1}{2}\cdot \ddot{\bar{\theta}}^T \cdot M\left(\bar{\theta}\right)\cdot \ddot{\bar{\theta}} + V\left(\bar{\theta};\dot{\bar{\theta}}\right)\cdot \ddot{\bar{\theta}} + \frac{1}{2}\cdot \dot{\bar{\theta}}^T \cdot D\left(\bar{\theta};\dot{\bar{\theta}}\right)\cdot \dot{\bar{\theta}} \end{Bmatrix}. \tag{23}$$

The above expressions lie at the basis of the dynamics equations for any robot structure.

4. THE MATRIX DYNAMICS EQUATIONS

Applying the virtual work principle in dynamics [2], the following are obtained:

$$Q_m^i = \begin{Bmatrix} {}^0 J_i^T \cdot \left\{ {}^0\overline{\mathscr{F}}_{X_i} + {}^0\overline{\mathscr{F}}^*_{X_i} \right\} \\ \tau \cdot Q_g^i + \tau \cdot Q_{SU}^i + {}^0 J_i^T \cdot {}^0\overline{\mathscr{F}}^*_{X_i} \end{Bmatrix} = \begin{Bmatrix} \tau \cdot Q_g^i + \tau \cdot Q_{SU}^i + \\ + \left[\left\{A_{ni}\left(\bar{p}\right)\right\}^T \quad \left\{ {}^0_i[R]\, {}^i\bar{k}_i \cdot \Delta_i \right\}^T \right]\cdot {}^0\overline{\mathscr{F}}^*_{X_i} \end{Bmatrix}; \tag{24}$$

$$Q_g^i = \sum_{j=i}^{n} M_j \cdot {}^0\bar{g}^T \cdot A_{ji} \cdot {}^j\bar{r}_{C_j}\ ; \quad Q_{SU}^i = {}^0 J_i^T \cdot \begin{bmatrix} I_3 & [0] \\ \left\{\bar{p}_{n+1n}\times\right\} & I_3 \end{bmatrix}\cdot \begin{bmatrix} {}^0_{n+1}[R] & [0] \\ [0] & {}^0_{n+1}[R] \end{bmatrix}\cdot \begin{bmatrix} {}^{n+1}\bar{f}_{n+1} \\ {}^{n+1}\bar{n}_{n+1} \end{bmatrix}; \tag{25}$$

Above, Q_m^i generalized driving forces, while Q_g^i as well Q_{SU}^i generalized forces from every joint, due to proper weights and manipulating payload respectively, are called.

In keeping with [2], the column matrix of the generalized inertia forces is the following:

$$ {}^{0}\overline{\mathscr{F}}^{*}_{X_i} = \begin{bmatrix} \sum_{j=i}^{n} {}^{0}_{j}[R] & [0] \\ \sum_{j=i}^{n} \left({}^{0}\bar{r}_{C_j} - \bar{p}_n \right) \times {}^{0}_{j}[R] & \sum_{j=i}^{n} {}^{0}_{j}[R] \end{bmatrix} \cdot \begin{bmatrix} M_j \cdot {}^{j}\dot{\bar{v}}_{C_j} \\ \dots\dots\dots\dots \\ {}^{j}I^{*}_{j} \cdot {}^{j}\dot{\bar{\omega}}_j + {}^{j}\bar{\omega}_j \times {}^{j}I^{*}_{j} \cdot {}^{j}\bar{\omega}_j \end{bmatrix}; \tag{26} $$

Considering Appell's equations, the partial derivatives of the acceleration energy, with respect to generalized accelerations, take the explicit and matrix form written below as:

$$ \frac{\partial E_a}{\partial \ddot{q}_i} = \begin{Bmatrix} Q^{i}_{m} - \left(\tau \cdot Q^{i}_{g} + \tau \cdot Q^{i}_{SU} + Q^{i}_{fc} \right) = \left[\{A_{ni}(\bar{p})\}^{T} \quad \left\{ {}^{0}_{i}[R] {}^{i}\bar{k}_i \cdot \Delta_i \right\}^{T} \right] \cdot {}^{0}\overline{\mathscr{F}}^{*}_{X_i} \\ \sum_{k=i}^{n} \sum_{j=1}^{k} Tr\left[A_{ki} \cdot {}^{k}I_{psk} \cdot A^{T}_{kj} \right] \cdot \ddot{q}_j + \sum_{k=i}^{n} \sum_{j=1}^{k} \sum_{m=1}^{k} Tr\left[A_{ki} \cdot {}^{k}I_{psk} \cdot A^{T}_{kjm} \right] \cdot \dot{q}_j \cdot \dot{q}_m \end{Bmatrix}; \tag{27} $$

$$ \frac{\partial E_a}{\partial \ddot{\bar{\theta}}} = \begin{Bmatrix} Q_m(\bar{\theta}) - \left\{ \tau \cdot Q_g(\bar{\theta}) + \tau \cdot Q_{SU}(\bar{\theta}) + Q_{fc}(\bar{\theta}) \right\} = {}^{0}J(\bar{\theta})^{T} \cdot {}^{0}\overline{\mathscr{F}}^{*}_{X}(\bar{\theta}) \\ M(\bar{\theta}) \cdot \ddot{\bar{\theta}} + V(\bar{\theta}; \dot{\bar{\theta}}) \end{Bmatrix}; \tag{28} $$

Thus, using the acceleration energy, the matrix dynamics equations in the state space and configuration respectively, can be expressed by means of the following:

$$ Q_m(\bar{\theta}) = \Delta^{2}_{m} \cdot \left\{ \left[M(\bar{\theta}) \cdot \ddot{\bar{\theta}} + V(\bar{\theta}; \dot{\bar{\theta}}) \right] \cdot \Delta_\theta + \tau \cdot Q_g(\bar{\theta}) \right\} + \tau \cdot (-1)^{\Delta m} \cdot \frac{1 - \Delta_m}{1 + 3\,\Delta_m} \cdot Q_{SU}(\bar{\theta}); \tag{29} $$

$$ Q_m(\bar{\theta}) = \Delta^{2}_{m} \left\{ \left[M(\bar{\theta}) \ddot{\bar{\theta}} + B(\bar{\theta}) \cdot \left[\dot{\bar{\theta}} \dot{\bar{\theta}} \right] + C(\bar{\theta}) \cdot \left[\dot{\bar{\theta}}^{2} \right] \right] \Delta_\theta + \tau\, Q_g(\bar{\theta}) \right\} + \tau\, (-1)^{\Delta m} \frac{1 - \Delta_m}{1 + 3\,\Delta_m} Q_{SU}(\bar{\theta}) . \tag{30} $$

In the matrix dynamics expressions, above written, $\tau = \pm 1$, the forces applied upon the mechanical robot structure are shown by $\Delta_m = \{-1; 0; 1\}$, while the motion from driving joints is taken into study by $\Delta_\theta = \{1; 0\}$, yielding the generalized dynamics or static forces.

5. CONCLUSIONS

The above algorithm of the dynamic control functions is included in *SimMEcROb Simulator* devoted to study concerning the assessment of the kinematics performances, dynamics, and accuracy respectively for whatever mechanical robot structure regardless of its type and form. In this paper, on the basis of new formulations, the acceleration energy, and then the matrix dynamics equations have been established. They will have an important significance in the assessment of the dynamics accuracy.

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