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MATRIX EXPONENTIALS TO ROBOT DYNAMICS

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Abstract: Within of the more papers, the authors have developed various algorithms based on matrix exponentials from functional analysis. They have devoted, especially, to establishment of the forward kinematics equations *(DKM)* typical of the mechanical robot structure. Within of this paper, the matrix exponentials and *DKM* will lie at the basis of a new algorithm devoted to determination of the dynamics robot control functions. The new algorithm will lead to determine on the one hand of the generalized inertia forces and dynamics matrices respectively, and on the other hand of the matrix dynamics equations. That is why, we will show that the matrix exponentials have a great significance on the one hand in the kinematics and dynamics control, on the other hand in the kinematics and dynamics accuracy to whatever robot structure.

Keywords: Robotics, Modeling, Mechanics, Dynamics, and Control.

1. INTRODUCTION

The dynamics robot equations can be defined, in a simplified form, as below:

$$\overline{\theta}(t)= f\left[\overline{Q}_m(t)\right]; \qquad \overline{Q}_m(t)= f^{-1}\left\{\overline{\theta}(t);\dot{\overline{\theta}}(t);\ddot{\overline{\theta}}(t)\right\}; \qquad \overline{Q}_m(t)=\left[Q_m^i(t); i=1\rightarrow n\right]^T. \qquad (1)$$

Above $\left\{\overline{\theta}(t);\dot{\overline{\theta}}(t);\ddot{\overline{\theta}}(t)\right\}$ and $\overline{Q}_m(t)$ represent the column vectors typical of the generalized variables, and generalized driving forces respectively from every driving joint of the robot. The last is also called the dynamics control function. The mechanical robot structure *(MRS)* with *n d.o.f.*, considered as non-conservative system, has been represented in Fig.1. Unlike first equation from (1) that expresses the direct dynamics model, the second is called the inverse dynamics model *(IDM)*. The robots perform some processes with high values of the dynamic accuracy performances. That is why, the dynamic control becomes a compulsory condition.

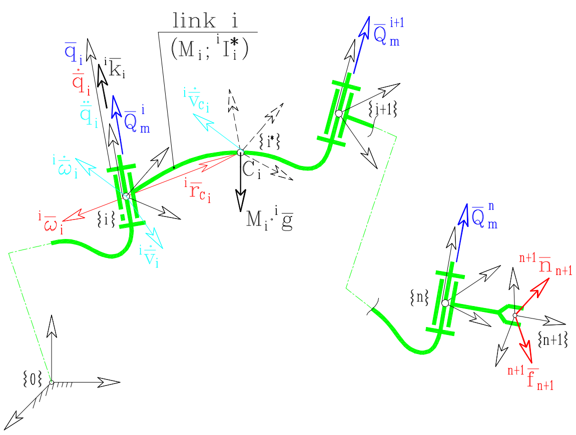

Fig. 1

The algorithm of the dynamic control functions *(IDM)* is included in *SimMEcROb Simulator* [1]. It achieves a complex study regarding the behavior of any mechanical robot structure. Thus, beside the dynamic control functions *(either generalized driving forces or dynamics equations as non-linear and differential of second order)*, the dynamics operational variable functions can be also analyzed. Within of this paper, a new dynamics algorithm will be developed. With the view of this, in keeping with [2] and [3], the matrix exponentials in forward robot kinematics are called.

2. MATRIX EXPONENTIALS TO FORWARD ROBOT KINEMATICS

The transfer equations to any open kinematic chain, either with *(R)-rotation* or *(T)-prismatic* joints, typical of the *MRS* can be expressed by means of the matrix exponentials *(ME)*. In keeping with [2] and [3], this section is devoted to short analysis of the forward kinematics algorithm *(DKM)* based onto matrix exponentials from functional analysis. This algorithm establishes on the one hand the homogeneous transformations from forward geometry *(DGM)*, and on the other hand Jacobian matrix, as well as the kinematics equations. Thus, at beginning, *MRS* is taken into initial configuration:

$$\bar{\theta}^{(0)} = [q_i = 0; i = 1 \to n]^T . \qquad (2)$$

The Fig. 2 shows that the screw parameters to every driving axis *(i)* and oriented, are defined with respect to frame *{0}* as: $\{\bar{k}_i^{(0)};\bar{v}_i^{(0)}\}$. The first, $\bar{k}_i^{(0)}$, is the unit vector of the driving axis, while the second:

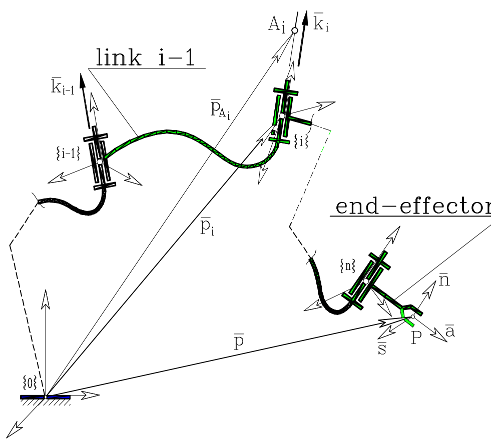

Fig. 2

$$\bar{v}_i^{(0)} = \{\bar{p}_i^{(0)}\times\}\bar{k}_i^{(0)}\cdot\Delta_i + (1-\Delta_i)\cdot\bar{k}_i^{(0)}; \qquad (3)$$

$$\Delta_i = \{\ \{1\ \ , i = R\}; \{0\ \ , i = T\}\}\ ;$$

Above $\{\bar{p}_i^{(0)}\times\}$ is the skew-symmetric matrix associated to position vector.

2.1 The Forward Geometry Algorithm

Upon homogeneous transformation (locating) matrices and their components the next notations are implemented:

$$^{j-1}_{\ \ i}[T](q_k; k = j \to i) = T_{ij-1} \in \bar{\theta} \neq \bar{\theta}^{(0)};\ ^{j-1}_{\ \ i}[T](q_k = 0; k = j \to i) = T^{(0)}_{ij-1} \in \bar{\theta}^{(0)}; \qquad (4)$$

On the basis of the mathematical models from [2], [3] and [4], in the following *DGM* algorithm, with matrix exponential expressions *(ME)*, will be short described:

- According to matrix algorithm from [1], the orienting matrix differential shows as:

$$\Delta R_{ii-1} = \{\bar{k}_i^{(0)}\times\} = \frac{1}{2\cdot s(q_i\cdot\Delta_i)}\cdot\{R_{ii-1}(\bar{k}_i^{(0)};q_i\cdot\Delta_i) - R^T_{ii-1}(\bar{k}_i^{(0)};q_i\cdot\Delta_i)\}; \qquad (5)$$

and $$dR_{ii-1}(\bar{k}_i^{(0)};q_i\cdot\Delta_i) = R_{ii-1}(\bar{k}_i^{(0)};q_i\cdot\Delta_i)\cdot\Delta R_{ii-1}\cdot dq_i . \qquad (6)$$

- Thus, the orienting matrix, between *{i-1}→{i}*, can be written as matrix exponential:

$$\left\{\begin{matrix} R_{ii-1}(q_i\cdot\Delta_i) \\ R(\bar{k}_i^{(0)};q_i\cdot\Delta_i)\end{matrix}\right\} = e^{\{\bar{k}_i^{(0)}\times\}q_i\cdot\Delta_i} = \left\{\begin{matrix} I_3 + \{\bar{k}_i^{(0)}\times\}sq_i + \{\bar{k}_i^{(0)}\times\}^2(1-cq_i) \\ I_3\cdot cq_i + \{\bar{k}_i^{(0)}\times\}sq_i + \bar{k}_i^{(0)}\cdot\bar{k}_i^{(0)T}\cdot(1-cq_i)\end{matrix}\right\}. \qquad (7)$$

- Thus, the homogeneous transformation (locating) matrix, between the two adjoining frames {i-1}→{i}, takes the next exponential expression:

$$T_{ii-1} = exp\left\{T_{ii-1}^{(0)} \cdot (U_i \cdot q_i) \cdot \left\{T_{ii-1}^{(0)}\right\}^{-1}\right\} \cdot T_{ii-1}^{(0)}; \text{ where } U_i = \pm\begin{bmatrix} \left\{{}^{i}\bar{k}_i \cdot \Delta_i \times\right\} & {}^{i}\bar{k}_i \cdot (1-\Delta_i) \\ 0\ 0\ 0 & 0 \end{bmatrix}. \quad (8)$$

- Considering (8), the homogeneous transformation matrix that expresses the position-orientation of the frame *{n}* with respect to *{0}* shows as below:

$$\begin{Bmatrix} {}_{n}^{0}[T] = T_{n0} \\ \prod_{i=1}^{n} T_{ii-1}(q_i) \end{Bmatrix} = \begin{Bmatrix} \left(\prod_{j=1}^{i-1} \delta_{jj-1}\right)^{-1} \cdot \left(\prod_{j=1}^{i} T_{jj-1}^{(0)}\right) \cdot e^{U_i \cdot q_i} \cdot \left(\prod_{j=1}^{i} T_{jj-1}^{(0)}\right)^{-1} \cdot \left(\prod_{j=1}^{i} T_{jj-1}^{(0)}\right) \\ where\ \delta_{jj-1} = \left\{\left\{T_{jj-1}^{(0)}; i \geq 2\right\}; \{I_4; i=1\}\right\} \end{Bmatrix}. \quad (9)$$

- On the basis of the Rodrigues' formula, the (3×1) column vector establishes with:

$$\bar{b}_i = \left\{I_3 \cdot q_i + \left\{\bar{k}_i^{(0)} \times\right\}[1 - c(q_i \cdot \Delta_i)] + \bar{k}_i^{(0)} \cdot \bar{k}_i^{(0)T} \cdot [q_i - s(q_i \cdot \Delta_i)]\right\} \cdot \bar{v}_i^{(0)}. \quad (10)$$

- Using the above expressions, the next matrix exponential shows as:

$$e^{A_i q_i} = exp\left\{\left(\prod_{j=1}^{i} T_{jj-1}^{(0)}\right) \cdot U_i \cdot \left(\prod_{j=1}^{i} T_{jj-1}^{(0)}\right)^{-1} = T_{i0}^{(0)} \cdot U_i \cdot \left\{T_{i0}^{(0)}\right\}^{-1}\right\} \cdot q_i = \begin{bmatrix} e^{\left\{\bar{k}_i^{(0)} \times\right\} q_i \cdot \Delta_i} & \bar{b}_i \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix}. \quad (11)$$

- Considering (9) and (10), the exponential expressions, for the homogeneous transformation matrices that express the position-orientation of the both frames *{n}* or *{n+1}* with respect to *{0}* and the inverse, under the following matrix form are obtained:

$$\begin{Bmatrix} T_{x0} = \prod_{i=1}^{x} T_{ii-1} \\ \begin{bmatrix} R_{x0} & \bar{p} \\ 0\ 0\ 0 & 1 \end{bmatrix} \\ where \\ x = \{n; n+1\} \end{Bmatrix} = \begin{Bmatrix} \prod_{i=1}^{n}\left(e^{A_i \cdot q_i}\right) \cdot T_{x0}^{(0)} = e^{\sum_{i=1}^{n} A_i q_i} \cdot T_{x0}^{(0)} = exp\left\{\sum_{i=1}^{n} A_i \cdot q_i\right\} \cdot T_{x0}^{(0)} \\ where \qquad R_{x0} = exp\left\{\sum_{i=1}^{n}\left\{\bar{k}_i^{(0)} \times\right\} q_i \Delta_i\right\} \cdot R_{x0}^{(0)} \\ \bar{p} = \sum_{i=1}^{n}\left\{exp\left\{\sum_{j=0}^{i-1}\left\{\bar{k}_j^{(0)} \times\right\} q_j \Delta_j\right\}\right\} \cdot \bar{b}_i + exp\left\{\sum_{i=1}^{n}\left\{\bar{k}_j^{(0)} \times\right\} q_i \Delta_i\right\} \cdot \bar{p}^{(0)} \delta_x \\ and \qquad \delta_x = \{\{0; x=n\}; \{1; x=n+1\}\} \end{Bmatrix}; \quad (12)$$

$$\begin{Bmatrix} T_{x0}^{-1} = \prod_{i=x}^{1} T_{ii-1}^{-1} \\ \begin{bmatrix} R_{x0}^{T} & -R_{x0}^{T} \cdot \bar{p} \\ 0\ 0\ 0 & 1 \end{bmatrix} \\ where \\ x = \{n; n+1\} \end{Bmatrix} = \begin{Bmatrix} \left\{T_{x0}^{(0)}\right\}^{-1} \cdot \prod_{i=n}^{1} exp(-A_i \cdot q_i) = \left\{T_{x0}^{(0)}\right\}^{-1} \cdot exp\left\{-\sum_{i=n}^{1} A_i \cdot q_i\right\} \\ where \qquad R_{x0}^{T} = \left\{R_{x0}^{(0)}\right\}^{T} \cdot exp\left\{-\left\{\sum_{i=n}^{1}\left\{\bar{k}_i^{(0)} \times\right\} q_i \Delta_i\right\}\right\} \\ -R_{x0}^{T} \cdot \bar{p} = \begin{Bmatrix} -\sum_{i=n}^{1}\left\{exp\left\{\sum_{j=i-1}^{0}\left\{\bar{k}_i^{(0)} \times\right\} q_j \Delta_j\right\}\right\} \bar{b}_i - \\ -exp\left\{-\left\{\sum_{i=n}^{1}\left\{\bar{k}_i^{(0)} \times\right\} q_i \Delta_i\right\}\right\} \bar{p}^{(0)} \delta_x \end{Bmatrix} \\ and \qquad \delta_x = \{\{0;\ x=n\}; \{1;\ x=n+1\}\} \end{Bmatrix}. \quad (13)$$

Remarks. *It comes out that the matrix exponentials enjoy important advantages specifically its compact form, easy geometric visualization and especially they avoid the frames typical to every kinetic link. ME are, likewise, included in the next algorithms.*

2.2 The Jacobian Matrix Exponential

Using the algorithm from [2] and [3], in the following one variant devoted to establishment of the Jacobian matrix with matrix exponentials *(ME)*, will be presented.

- At beginning the three new matrices, based on exponentials, are determined as below:

$$\underset{(3x3)}{M\,exp}(V_{i1})=exp\left\{\sum_{j=0}^{i-1}\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_j\cdot\Delta_j\right\};\qquad \underset{(3x6)}{M\,exp}(V_{i2})=\left[I_3\quad \Delta_i\cdot\left\{\bar{k}_i^{(0)}\times\right\}\right];\tag{14}$$

$$\underset{\{6x[9+3\cdot(n-i)]\}}{M\,exp}(V_{i3})=\begin{bmatrix} I_3 & [0] & [0] \\ [0] & \left\{\begin{matrix}\left[exp\left\{\sum_{m=i-1}^{k-1}\left\{\bar{k}_m^{(0)}\times\right\}q_m\delta_m\Delta_m\right\}\right] \\ k=i\to n;\ \delta_m=\{\{0;m=i-1\};\{1;m\geq i\}\}\end{matrix}\right\} & exp\left\{\sum_{k=i}^{n}\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}\end{bmatrix}.$$

- On the basis of the matrices (14) another new matrices are also implemented:

$$\underset{(6\times 6)}{M\,exp}\{J_{i1}\}=\begin{bmatrix} M\,exp\{V_{i1}\} & [0] \\ [0] & M\,exp\{V_{i1}\}\end{bmatrix};\qquad \underset{(6\times 9)}{M\,exp}\{J_{i2}\}=\begin{bmatrix} M\,exp\{V_{i2}\} & [0] \\ [0] & I_3\end{bmatrix};\tag{15}$$

$$\underset{\{9\times[12+3\cdot(n-i)]\}}{M\,exp}\{J_{i3}\}=\begin{bmatrix} M\,exp\{V_{i3}\} & [0] \\ [0] & I_3\end{bmatrix};\quad \underset{\{[12+3\cdot(n-i)]\times 1\}}{M_{iv\omega}}=\left[\bar{v}_i^T\quad \left[\bar{b}_k\ \ k=i\to n\right]^T\quad \bar{p}_n^{(0)T}\quad \Delta_i\bar{k}_i^{(0)T}\right]^T.$$

- Consequently, using (14) and (15) based onto *ME*, the Jacobian matrix can be determined by means of the expression written as below:

$${}^0J(\bar{\theta})=\left[{}^0J_i=\begin{bmatrix}{}^0J_{iv}\\ {}^0J_{i\omega}\end{bmatrix}=\{M\,exp\{J_{i1}\}\cdot M\,exp\{J_{i2}\}\cdot M\,exp\{J_{i3}\}\cdot M_{iv\omega}\};\ i=1\to n\right].\tag{16}$$

- The next matrices lie at the basis to calculus of the Jacobian matrix time derivative:

$$\underset{(6\times 6)}{M\,exp}\{\dot{J}_{i1}\}=\begin{bmatrix} M\,exp\{\dot{V}_{i1}\} & [0] \\ [0] & M\,exp\{\dot{V}_{i1}\}\end{bmatrix};\qquad M\,exp\{\dot{J}_{i3}\}=\begin{bmatrix} M\,exp\{\dot{V}_{i3}\} & [0] \\ [0] & I_3\end{bmatrix};\tag{17}$$

$$M\,exp\{\dot{V}_{j1}\}=\left\{\begin{matrix}\sum_{k=1}^{j-1}\left\{exp\left\{\sum_{l=0}^{k-1}\left\{\bar{k}_l^{(0)}\times\right\}q_l\delta_{kl}\cdot\Delta_l\right\}\right\}\cdot\left\{\bar{k}_l^{(0)}\times\right\}\dot{q}_k\Delta_k\left\{exp\left\{\sum_{m=k}^{j-1}\left\{\bar{k}_m^{(0)}\times\right\}\cdot q_m\delta_m\Delta_m\right\}\right\} \\ \delta_{kl}=\{\{0;k>j-1\};\{1;k\leq j-1\}\}\end{matrix}\right\};$$

$$M\,exp\{\dot{V}_i\}=\left\{\begin{matrix}\sum_{j=1}^{i-1}\left\{exp\left\{\sum_{k=0}^{j-1}\left\{\bar{k}_k^{(0)}\times\right\}q_k\delta_{jk}\cdot\Delta_k\right\}\right\}\cdot\left\{\bar{k}_j^{(0)}\times\right\}\dot{q}_j\Delta_j\left\{exp\left\{\sum_{m=j}^{p}\left\{\bar{k}_m^{(0)}\times\right\}q_m\delta_m\Delta_m\right\}\right\} \\ and\qquad l=0\to i;\ \ p=i-1\to n;\ \ \delta_{jk}=\{\{0;j>p\};\{1;j\leq p\}\}\end{matrix}\right\}.$$

- Thus, for $i=1\to n$, every column from Jacobian matrix time derivative, shows as:

$$\underset{(i=1\to n)}{{}^0\dot{J}_i}=Trace\left\{\begin{bmatrix} M\,exp\{\dot{J}_{i1}\}\cdot M\,exp\{J_{i2}\}\cdot M\,exp\{J_{i3}\} \\ M\,exp\{J_{i1}\}\cdot M\,exp\{J_{i2}\}\cdot M\,exp\{\dot{J}_{i3}\} \\ M\,exp\{J_{i1}\}\cdot M\,exp\{J_{i2}\}\cdot M\,exp\{J_{i3}\}\end{bmatrix}\cdot\left[M_{iv\omega}\quad M_{iv\omega}^{*}\quad \dot{M}_{iv\omega}\right]\right\}.\tag{18}$$

$$\begin{bmatrix}{}^nJ(\bar{\theta})\\ {}^n\dot{J}(\bar{\theta})\end{bmatrix}=\begin{bmatrix}\left\{R_{n0}^{(0)}\right\}^{-1}\cdot\prod_{i=n}^{1}exp\left\{-\left\{\bar{k}_i^{(0)}\times\right\}q_i\Delta_i\right\} & [0] \\ [0] & \left\{R_{n0}^{(0)}\right\}^{-1}\cdot\prod_{i=n}^{1}exp\left\{-\left\{\bar{k}_i^{(0)}\times\right\}q_i\Delta_i\right\}\end{bmatrix}\cdot\begin{bmatrix}{}^0J(\bar{\theta})\\ {}^0\dot{J}(\bar{\theta})\end{bmatrix}.\tag{19}$$

The two expression (16) and (18) or (19), called the Jacobian matrix exponentials, will be applied so as to determine the generalized inertia forces from dynamics equations.

3. GENERALIZED INERTIA FORCES

This section is devoted to establishment of the generalized inertia forces typical to every kinetic link $i = 1 \rightarrow n$ from mechanical robot structure *(MRS)*, represented in Fig. 1. In keeping with [1], they are determined by means of the following expressions:

$$ {}^i\overline{F}_i = \Delta_\theta \cdot M_i \cdot \left\{ {}^i\dot{\overline{v}}_i + {}^i\dot{\overline{\omega}}_i \times {}^i\overline{r}_{C_i} + {}^i\overline{\omega}_i \times {}^i\overline{\omega}_i \times {}^i\overline{r}_{C_i} \right\} + \left(1 - \Delta_\theta\right) \cdot M_i \prod_{j=i}^{1} {}^{j}_{j-1}[R] \cdot {}^0\dot{\overline{v}}_0 \;; \tag{20} $$

$$ {}^i\overline{N}_i = \Delta_\theta \cdot \left\{ {}^iI_i^* \cdot {}^i\dot{\overline{\omega}}_i + {}^i\overline{\omega}_i \times {}^iI_i^* \cdot {}^i\overline{\omega}_i \right\}; \qquad {}^0\dot{\overline{v}}_0 = \tau \cdot g \cdot {}^0\overline{k}_0 \,. \tag{21} $$

Applying the matrix exponentials short described in the previous sections, angular velocity and acceleration from the above expressions are substituted as below:

$$ {}^i\overline{\omega}_i = \begin{Bmatrix} R_{i0}^{-1} \cdot {}^0J_\omega\left(\overline{\theta}\right) \cdot \dot{\overline{\theta}}^{(i)} \\ R_{i0}^{-1} \cdot \sum_{j=1}^{i} {}^0J_{j\omega} \cdot \dot{q}_j \end{Bmatrix}; \qquad {}^i\dot{\overline{\omega}}_i = \begin{Bmatrix} R_{i0}^{-1} \cdot \left[{}^0J_\omega\left(\overline{\theta}\right) \quad {}^0\dot{J}_\omega\left(\overline{\theta}\right) \right] \cdot \left[\ddot{\overline{\theta}}^{(i)T} \quad \dot{\overline{\theta}}^{(i)T} \right]^T \\ R_{i0}^{-1} \cdot \left\{ \sum_{j=1}^{i} \left({}^0J_{j\omega} \cdot \ddot{q}_j + {}^0\dot{J}_{j\omega} \cdot \dot{q}_j \right) \right\} \end{Bmatrix}. \tag{22} $$

Substituting (13), (14) and (17) in (22), the next exponential expressions can be written:

$$ {}^i\overline{\omega}_i = \left\{ \left\{ R_{i0}^{(0)} \right\}^{-1} \cdot exp\left\{ -\left\{ \sum_{j=i}^{1} \left\{ \overline{k}_j^{(0)} \times \right\} q_j \Delta_j \right\} \right\} \right\} \cdot \left\{ \sum_{j=1}^{i} \left\{ exp\left\{ \sum_{k=1}^{j-1} \left\{ \overline{k}_k^{(0)} \times \right\} q_k \Delta_k \right\} \right\} \cdot \overline{k}_j^{(0)} \dot{q}_j \Delta_j \right\}; \tag{23} $$

$$ {}^i\dot{\overline{\omega}}_i = \left\{ \left\{ R_{i0}^{(0)} \right\}^{-1} \cdot exp\left\{ -\left\{ \sum_{j=i}^{1} \left\{ \overline{k}_j^{(0)} \times \right\} q_j \Delta_j \right\} \right\} \right\} \cdot \left\{ \sum_{j=1}^{i} \left\{ M\,exp\left\{ V_{j1} \right\} \cdot \ddot{q}_j + M\,exp\left\{ \dot{V}_{j1} \right\} \cdot \dot{q}_j \right\} \overline{k}_j^{(0)} \Delta_j \right\}. $$

Using (13) and (18), the linear acceleration from (20) is written as exponential expression:

$$ {}^i\dot{\overline{v}}_i = \begin{Bmatrix} R_{i0}^{-1} \cdot \left[{}^0J_v\left(\overline{\theta}\right) \quad {}^0\dot{J}_v\left(\overline{\theta}\right) \right] \cdot \left[\ddot{\overline{\theta}}^{(i)T} \quad \dot{\overline{\theta}}^{(i)T} \right]^T \\ R_{i0}^{-1} \cdot \left\{ \sum_{j=1}^{i} \left({}^0J_{jv} \cdot \ddot{q}_j + {}^0\dot{J}_{jv} \cdot \dot{q}_j \right) \right\} \end{Bmatrix} = \left\{ \left\{ R_{i0}^{(0)} \right\}^{-1} exp\left\{ -\left\{ \sum_{j=i}^{1} \left\{ \overline{k}_j^{(0)} \times \right\} q_j \Delta_j \right\} \right\} \right\} \cdot {}^0\dot{\overline{v}}_i \tag{24} $$

where
$$ {}^0\dot{\overline{v}}_i = \sum_{j=1}^{i} \left\{ \begin{matrix} M\,exp\left\{ J_{j1} \right\} \cdot M\,exp\left\{ J_{j2} \right\} \cdot M\,exp\left\{ J_{j3} \right\} \cdot M_{jv} \cdot \ddot{q}_j \; + \\ + Trace\left\{ \begin{bmatrix} M\,exp\left\{ \dot{J}_{j1} \right\} \cdot M\,exp\left\{ J_{j2} \right\} \cdot M\,exp\left\{ J_{j3} \right\} \\ M\,exp\left\{ J_{j1} \right\} \cdot M\,exp\left\{ \dot{J}_{j2} \right\} \cdot M\,exp\left\{ J_{j3} \right\} \\ M\,exp\left\{ J_{j1} \right\} \cdot M\,exp\left\{ J_{j2} \right\} \cdot M\,exp\left\{ \dot{J}_{j3} \right\} \end{bmatrix} \cdot \left[M_{jv} \quad M_{jv}^* \quad \dot{M}_{jv} \right] \right\} \cdot \dot{q}_j \end{matrix} \right\}. $$

The generalized inertia forces are also pointed out by means of the next dynamic terms:

$$ M_{ij} = \sum_{k=max\,(i;j)}^{n} Tr\left[A_{ki} \cdot {}^kI_{psk} \cdot A_{kj}^T \right]; \qquad V_{ijm} = \sum_{k=max\,(i;j;m)}^{n} Tr\left[A_{ki} \cdot {}^kI_{psk} \cdot A_{kjm}^T \right]; \tag{25} $$

$$ Q_g^i = \sum_{k=i}^{n} M_k \cdot {}^0\overline{g}^T \cdot A_{ki} \cdot {}^k\overline{r}_{C_k} \;; \qquad Q_{SU}^i = {}^nJ\left(\overline{\theta}\right)^T \cdot \begin{bmatrix} I_3 & [0] \\ \left\{ {}^n\overline{r}_{n+1} \times \right\} & I_3 \end{bmatrix} \cdot \begin{bmatrix} {}^{n}_{n+1}[R] \cdot {}^{n+1}\overline{f}_{n+1} \\ {}^{n}_{n+1}[R] \cdot {}^{n+1}\overline{n}_{n+1} \end{bmatrix}; $$

where
$$ A_{ki} = \left\{ exp\left\{ \sum_{j=0}^{i-1} A_j \cdot q_j \right\} \right\} \cdot A_i \cdot \left\{ exp\left\{ \sum_{l=i}^{k} A_l \cdot q_l \right\} \right\} \cdot T_{k0}^{(0)}; \tag{26} $$

and
$$ A_{kjm} = \left\{ exp\left\{ \sum_{l=0}^{m-1} A_l \cdot q_l \right\} \right\} \cdot A_m \cdot \left\{ exp\left\{ \sum_{i=m}^{j-1} A_i \cdot q_i \right\} \cdot A_i \cdot exp\left\{ \sum_{p=i}^{k} A_p \cdot q_p \right\} \right\} \cdot T_{k0}^{(0)}. \tag{27} $$

Therefore, by outward iterations $i = 1 \rightarrow n + 1$, the matrix exponentials are established. They lie at the basis, among of things, to establishment of the generalized inertia forces.

4. THE DYNAMICS EQUATIONS

The mechanical robot structure is covered by inward iterations, for $i = n+1 \rightarrow 1$, from the end-effector to the fixed basis. In keeping with [1], and taking into consideration the superposition effect principle, the generalized forces (force and torque) in every driving joint of the robot are established with the following matrix expressions:

$$ {}^{i}\bar{f}_i = (-1)^{\Delta_m} \cdot \frac{1-\Delta_m}{1+3\cdot\Delta_m} \cdot {}^{i}_{n+1}[R] \cdot {}^{n+1}\bar{f}_{n+1} + \Delta_m^2 \sum_{j=i}^{n} \left\{ {}^{i}_{j}[R] \cdot {}^{j}\bar{F}_j \right\}; \tag{28}$$

$$ {}^{i}\bar{n}_i = (-1)^{\Delta_m} \frac{1-\Delta_m}{1+3\Delta_m} \left\{ {}^{i}_{n+1}[R]\, {}^{n+1}\bar{n}_{n+1} + {}^{i}\bar{r}_{n+1} \times {}^{i}_{n+1}[R]\, {}^{n+1}\bar{f}_{n+1} \right\} + \Delta_m^2 \sum_{j=i}^{n} \left\{ {}^{i}_{j}[R] \left({}^{j}\bar{r}_{C_{ji}} \times {}^{j}\bar{F}_j + {}^{j}\bar{N}_j \right) \right\}.$$

Above the generalized inertia forces are substituted by (20)-(24) as exponential expressions. In keeping with [1], the robot dynamic control functions (either generalized driving forces or dynamics equations) are determined with the expressions below written:

$$ Q_m^i = \left\{ \begin{array}{l} \text{either } \left\{ \Delta_i \cdot {}^{i}\bar{n}_i^T + (1-\Delta_i) \cdot {}^{i}\bar{f}_i^T \right\} \cdot {}^{i}\bar{k}_i \\ \text{or } \Delta_m^2 \cdot \Delta_\theta \cdot \left[\left(\sum_{j=1}^{n} M_{ij} \cdot \ddot{q}_j + V_i \right) - Q_g^i \right] - (-1)^{\Delta_m} \frac{1-\Delta_m}{1+3\Delta_m} \cdot Q_{SU}^i \end{array} \right\}. \tag{29}$$

In the above expressions, the following notations have been implemented:

$$ \Delta_m = \{\{-1;(M_{SU};M_i)\};\{0;M_{SU}\};\{1;M_i\}\};\ \Delta_\theta = \left\{\left\{1;\left(\dot{\theta};\ddot{\theta}\right)\right\};\left\{0;\left(\dot{\theta};\ddot{\theta}\right)=0\right\}\right\}; \tag{30}$$

Above the forces applied upon the robot are shown by Δ_m, while the motion from driving joints is taken into study by Δ_θ, yielding the either generalized dynamics or static forces.

5. CONCLUSIONS

Within of this paper a new algorithm *(IDM)* of the dynamics control functions has been developed. At beginning, a short presentation of the matrix exponentials in forward robot kinematics has been achieved. The matrix exponentials lie at the basis of a new algorithm devoted to determination of the dynamics robot control functions. This algorithm allows the determination on the one hand of the generalized inertia forces and on the other hand the dynamics equations. That is why, the matrix exponentials have a great significance on the one hand in the kinematics and dynamics control, on the other hand in the kinematics and dynamics accuracy to whatever mechanical robot structure.

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