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Modeling – numerical simulation of a three phase, asynchronous, electrical motor

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Abstract. It presents a control block scheme for the speed of the three phase asynchronous motor, commanded in modulated tensions and frequency, and also a modeling program – its numerical simulation. The modeling was realized in the three phase system, with parameters in respect to the stator and.in the "d, q" coordinate system, considered fixed with respect to the stator. Numerical integration is operated with a multi-step method (local iterative linearisation – LIL algorithm).

1. Introduction

The appearance, in series production, of the variable frequency static converters (VFSC), that allow the supply and the control of the rotation speed of the three phase, asynchronous motor, determined the research and use of this possibility, in order to implement it in the control of the capacity of the centrifugal pumps, by modifying their driving speed. In order to analyze the three phase, asynchronous motor (AM), it has been started from its functioning equations obtained from the "T" equivalent scheme, with parameters with respect to the stator (figure 1):

$$m_M = J \cdot \frac{dn}{dt} + \mu 0 + \mu 1 \cdot n + \mu 2 \cdot n^2 \quad ;$$

$$u1 = R1 \cdot i1 + L1 \cdot \frac{di1}{dt} + R_m \cdot i_m + L_m \cdot \frac{di_m}{dt}, u1 = R1 \cdot i1 + L1 \cdot \frac{di1}{dt} + \frac{R'2}{S} \cdot i'2 + L'2 \cdot \frac{di'2}{dt}, \quad (1)$$

$$i1 = i_m + i'2,$$

The notations used in the relations (1) and those that follow, result from the figure 1. and from the couples' equation:

n , n1 [rot/min.] – rotor's speed and synchronism rotation speed of the rotating stator electromagnetic field, at supply frequency;

n2 [rot/min.] - synchronism speed of the rotor's rotating field, at supply frequency;

J [N•m•sec^2] – proportionality coefficient with polar inertia moment of the moving masses (electric motor + pump), reduced to motor's axis;

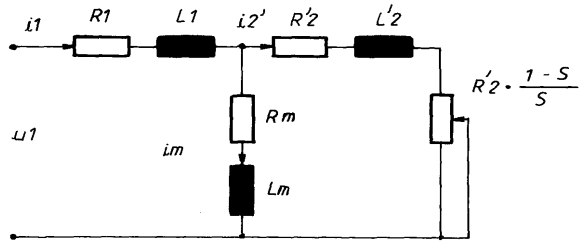

Fig. 1

m_M [N / m] - mechanic couple for driving the pump;

m_1 - number of phases of the electrical motor;

p1 - number of pole pairs of the electrical motor;
f_N - motor's functioning nominal frequency;
$\mu 0$ - friction couple (motor + pump);
$\mu 1, \mu 2$ - proportionality coeficient of the load couple belonging to the flow rate, and of the load couple for pressure;

The next values are implied:

$$\Omega_1 = \frac{2\pi \cdot n1}{60}; \quad \Omega = \frac{2\pi \cdot n}{60}; \quad S = \frac{\Omega_1 - \Omega}{\Omega_1} = \frac{n2}{n1} = \frac{n1 - n}{n1}; \tag{2}$$

$$p_{EM} = m_1 \cdot (i'2)^2 \cdot \frac{R'2}{S}; p_2 = m_1 \cdot \left(i'2\right)^2 \cdot R'^2; p_M = m_1 \cdot \left(i'2\right)^2 \cdot R'2\frac{1-S}{S} = m_M \cdot \Omega;$$

Ω1 [rad/sec] - angular velocity of the electromagnetic field of the stator, at rotation speed "n1";
Ω [rad/sec] - angular velocity of the electromagnetic field of the rotor at rotation speed "n";
S - motor's slipping;
p_{EM} [w] - electromagnetic power transmitted to the rotor;
p_2 [w] - electric power lost through heat, in the rotor;
p_M [w] - mechanical power developed by the rotor and transmitted for driving the pump;

$$\frac{1-S}{S} = \frac{\Omega}{\Omega_1 - \Omega} \quad ; \quad m_M = \frac{m_1 \cdot \left(i'2\right)^2 \cdot R'2}{\Omega_1 - \Omega} = \frac{60 \cdot m_1 \cdot \left(i'2\right)^2 \cdot R'2}{2\pi \cdot (n1 - n)} \tag{3}$$

Equilibrium equation in couples will be approximated by an equation of the form:

$$m_M = \frac{60}{2\pi} \cdot \frac{m_1 \cdot \left(i'2\right)^2 \cdot R'2}{n1 - n} = J \cdot \frac{dn}{dt} + \mu 0 + \mu 1 \cdot n + \mu 2 \cdot n^2 \tag{4}$$

where :

$$\mu 0 \left[N \cdot m\right], \mu 1 \quad \left[\frac{N \cdot m \cdot rot}{\sec}\right], \mu 2 \quad \left[\frac{N \cdot m \cdot rot^2}{\sec^2}\right],$$

are the coefficients from above;

2. The mathematical model.

There results the asynchronous motor's analog model, through 3 state variables:

$$x1 = i1; x2 = i'2; x3 = n;$$

$$\dot{x}1 = b1 \cdot u1 + a11 \cdot x1 + \left(a120 + a121 \frac{n1}{n1 - x3}\right) \cdot x2 = F_1(u1, x1, x2, x3);$$

$$\dot{x}2 = b2 \cdot u1 + a21 \cdot x1 + \left(a220 + a221 \frac{n1}{n1 - x3}\right) \cdot x2 = F_2(u1, x1, x2, x3); \tag{5}$$

$$\dot{x}3 = a333 \frac{(x2)^2}{n1 - x3} - \left(a330 + a331 \cdot x3 + a332 (x3)^2\right) = F_3(x1, x2, x3);$$

where the coefficients a11, a120, …b1, b2 are computed using the relations:

$$a11=-\frac{R1+\frac{L_m\cdot R1}{L'2}+R_m}{L1+L_m+\frac{L_m\cdot L1}{L'2}};a120=\frac{R_m}{L1+L_m+\frac{L_m\cdot L1}{L'2}};a121=\frac{-\frac{L_m}{L'2}\cdot R'2}{L1+L_m+\frac{L_m\cdot L1}{L'2}};\qquad(6)$$

$$a211=\frac{-1}{L'2}\left(R1-\frac{L1\cdot\left(R1+\frac{L_m\cdot R1}{L'2}+R_m\right)}{L1+L_m+\frac{L_m\cdot L1}{L'2}}\right);a220=\frac{-L1\cdot R_m}{L'2\left(L1+L_m+\frac{L_m\cdot L1}{L'2}\right)};\quad a221=\frac{R'2}{L'2}\left(\frac{L1\cdot\frac{L_m}{L'2}}{L1+L_m+\frac{L_m\cdot L1}{L'2}}-1\right);$$

$$b1=\frac{1+\frac{L_m}{L'2}}{L1+L_m+\frac{L_m\cdot L1}{L'2}};b2=\frac{1}{L'2}\left(1-\frac{L1\cdot\left(1+\frac{L_m}{L'2}\right)}{L1+L_m+\frac{L_m\cdot L1}{L'2}}\right);$$

$$a330=\frac{\mu 0}{J};a331=\frac{\mu 1}{J};a332=\frac{\mu 2}{K};a333=\frac{60\cdot m_1\cdot R'2}{2\cdot\pi\cdot J};$$

From the known relations:

$$U1=\sqrt{2}\cdot U1_{ef}\cdot\sin\varpi t=\sqrt{2}\cdot U1_{ef}\cdot\sin(2\pi\cdot f_1\cdot t);$$

$$U1_{ef}=k\cdot\Phi_N\cdot\omega\cdot f;U1_{Nef}=k\cdot\Phi_N\cdot\omega\cdot f_N;$$

$$\frac{U1_{ef}}{U1_{Nef}}=\frac{f}{f_N}\Rightarrow\frac{U1_{ef}}{220V}=\frac{f}{50Hz}\Rightarrow$$

$$\Rightarrow U1_{ef}=\frac{220V}{50Hz}\cdot f=4{,}4[V/Hz]\cdot f[Hz];$$

The choosing of the coefficients J, $\mu 0$, $\mu 1$, $\mu 2$ will be made such that, next will be considered that "m_M" – the mechanical couple will be influenced, in a first approximation, only by rotation speed and its variation: $\mu 0=\mu 2=0$.

In these conditions will be obtained the equation of the nominal couple " m_N ":

$$m_N=J\cdot\frac{dn}{dt}+\mu 1\cdot n\quad;\qquad\frac{L[n(t)]}{L[m_N(t)]}=\frac{1/\mu 1}{1+\frac{J}{\mu 1}\cdot s}=\frac{K_M}{1+T_M\cdot s}$$

where "*TM* " is a time constant having values between 1 - 10 sec and that is practically approximated with the value *TM* = 3 sec.

It is computed in case of the stationary regime (*dn/dt* = *0*), for which the couple is considered nominal, and imposing:

$$m_N=\mu 1\cdot n_N\quad,\ \mu 1=\frac{m_N}{n_N}\quad;\qquad J=T_M\cdot\mu 1$$

where:
$$m_N\,[N\cdot m]=\frac{P_N\ [w]}{\omega_N\ [rad/\sec]}\ .$$

Next, it is obtained the numerical model through process's state variables and starting from the analog model using the LIL algorithm. In the program LLI.DOM3, is presented the program for numerical modeling of the asynchronous three phase motor, with application for the electrical motor used for driving the circulation pump, with controllable capacity, that was used in an installation for controlling the flow rate of the heating agent to the consumer. The block scheme is presented in the figure 2:

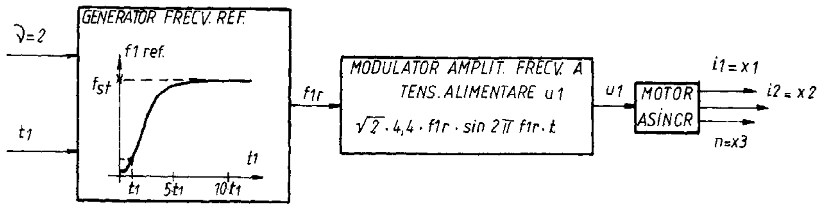

Fig. 2.

The simplified block scheme, for controlling the speed of the AM, commanded in modulated tensions in amplitude and frequency is given in figure 3, where is considered:

$$\Delta f = f_{ref} - f \quad ; \qquad f_C = f_{ref} + \Delta f \quad .$$

Δf - controlling deviation with respect to reference;

f_c - command frequency, resulted from a control error Δf ;

The use of VFSC for controlling the speed of the AM, showed the necessity of the study of its behavior, at different supply tensions, modulated in frequency and amplitude. According to these there have been made numerical modeling and simulations of the behavior of the AM, having as input: ramp, impulse and sinusoidal, frequency.

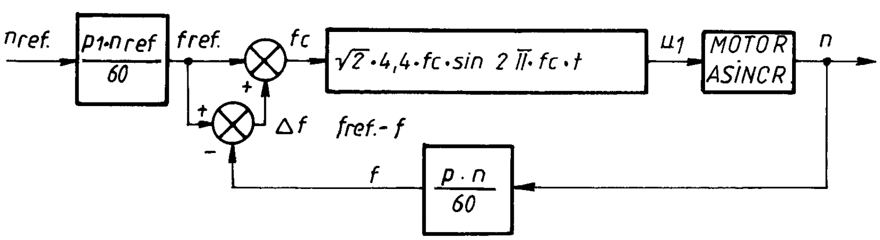

Fig. 3.

The state variables of the equation system of the electrical three phase AM, in "dq" coordinates:

$$x_1 = i_{1d} \quad ; \quad x_2 = i_{1q} \quad ; \quad x_3 = i_{2d} \quad ; \quad x_4 = i_{2q} \quad ; \quad x_5 = \omega_{el} \quad ;$$

and will result the equation system written as follows:

$$\begin{aligned}
u_{1d} &= R_1 \cdot x_1 + L_1 \cdot \dot{x}_1 + L_m \cdot \dot{x}_3 \quad ; \\
u_{1q} &= R_1 \cdot x_2 + L_1 \cdot \dot{x}_2 + L_m \cdot \dot{x}_4 \quad ; \\
0 &= R_2' \cdot x_3 + L'_1 \cdot \dot{x}_3 + L_m \cdot \dot{x}_1 + x_5 \cdot \left(L_2' \cdot x_4 + L_m \cdot x_2\right) \quad ; \qquad (7) \\
0 &= R_2' \cdot x_4 + L_2' \cdot \dot{x}_4 + L_m \cdot \dot{x}_2 - x_5 \cdot \left(L_2' \cdot x_3 + L_m \cdot x_1\right) \quad ; \\
\frac{3}{2} \cdot L_m \cdot \left(x_2 \cdot x_3 - x_1 \cdot x_4\right) \cdot p_1 &= \frac{J}{p_1} \cdot \dot{x}_5 + \mu 0 + \frac{\mu 1}{p_1} \cdot x_5 + \frac{\mu 2}{p_1^2} \cdot x_5^2 \quad ;
\end{aligned}$$

where:

$$R_1 = R_1 \quad ; \quad L_1 = L_1' + L_m \quad ; \quad L_m = L_m \quad ; \quad R_2' = R_2' \quad ; \quad L_2' = L_2' + L_m \; ;$$

The system above can be written as:

$$\dot{x} = F(u, x)$$

resulting the system:

$$\begin{aligned}
\dot{x}_1 &= b_{11} \cdot u_{1d} + a_{11} \cdot x_1 + a_{12} \cdot x_3 + a_{13} \cdot x_2 \cdot x_5 + a_{14} \cdot x_4 \cdot x_5 = F_1;\\
\dot{x}_2 &= b_{22} \cdot u_{1q} + a_{21} \cdot x_2 + a_{22} \cdot x_4 + a_{23} \cdot x_1 \cdot x_5 + a_{24} \cdot x_3 \cdot x_5 = F_2;\\
\dot{x}_3 &= b_{31} \cdot u_{1d} + a_{31} \cdot x_1 + a_{32} \cdot x_3 + a_{33} \cdot x_2 \cdot x_5 + a_{34} \cdot x_4 \cdot x_5 = F_3;\\
\dot{x}_4 &= b_{42} \cdot u_{1q} + a_{41} \cdot x_2 + a_{42} \cdot x_4 + a_{43} \cdot x_1 \cdot x_5 + a_{44} \cdot x_3 \cdot x_5 = F_4;\\
\dot{x}_5 &= a_{50} + a_{51} \cdot x_5 + a_{52} \cdot x_5^2 + a_{53} \cdot (x_2 \cdot x_3 - x_1 \cdot x_4) = F_5;
\end{aligned} \tag{7}$$

where:

$$\begin{aligned}
& b_{11} = b_{22} = -\frac{L_2}{L_m^2 - L_1 \cdot L_2}; a_{11} = a_{21} = \frac{R_1 \cdot L_2}{L_m^2 - L_1 \cdot L_2}; a_{12} = a_{22} = \frac{-R_2 \cdot L_m}{L_m^2 - L_1 \cdot L_2};\\
& a_{14} = \frac{-L_2 \cdot L_m}{L_m^2 - L_1 \cdot L_2}; a_{24} = -a_{14}; a_{13} = \frac{-L_m^2}{L_m^2 - L_1 \cdot L_2}; a_{23} = -a_{13};\\
& b_{31} = b_{42} = \frac{L_m}{L_m^2 - L_1 \cdot L_2}; a_{31} = \frac{-R_1 \cdot L_m}{L_m^2 - L_1 \cdot L_2}; a_{32} = \frac{R_2 \cdot L_1}{L_m^2 - L_1 \cdot L_2}; a_{33} = \frac{L_1 \cdot L_m}{L_m^2 - L_1 \cdot L_2};\\
& a_{34} = \frac{L_1 \cdot L_2}{L_m^2 - L_1 \cdot L_2}; a_{41} = a_{31}; a_{42} = a_{32}; a_{43} = -a_{33}; a_{44} = -a_{34};\\
& a_{50} = \frac{-\mu 0 \cdot p_1}{J}; a_{51} = -\frac{\mu 1}{J}; a_{52} = -\frac{\mu 2}{J \cdot p_1}; a_{53} = \frac{3 \cdot p_1^2 \cdot L_m}{2 \cdot J};
\end{aligned} \tag{8}$$

The details of the resisting couple " *Mm* " (ccharacterized by the angular velocity ω_m)

$$M_m = \frac{3}{2} L_m \cdot (i_{1q} \cdot i_{2q} - i_{1d} \cdot i_{2q}) \cdot p_1 = J \frac{d\omega_m}{dt} + \mu 0 + \mu 1 \cdot \omega_m + \mu 2 \cdot \omega_m^2;$$

$$\omega = \omega_{el} = \frac{2\pi \cdot p_1}{60} \cdot n; \Rightarrow M_m = J \frac{2\pi \cdot p_1}{60} \cdot \frac{dn}{dt} + \mu 0 + \mu 1 \frac{2\pi \cdot p_1}{60} \cdot n + \mu 2 \cdot \left(\frac{2\pi \cdot p_1}{60}\right)^2 \cdot n^2;$$

For approximating the transfer function, will be denoted:

$$M_{mN} = \frac{2\pi \cdot p_1}{60}\left(J \frac{dn}{dt} + \mu 1 \cdot n\right); L[M_{mN}(t)] = \frac{2\pi \cdot p_1}{60}(J \cdot s + \mu 1) \cdot L[n(t)];$$

$$\frac{L[n(t)]}{L[M_{mN}(t)]} = \frac{\dfrac{60}{2\pi \cdot p_1 \cdot \mu 1}}{1 + \dfrac{J}{\mu 1} \cdot s} = \frac{K_M}{1 + T_M \cdot s}; \left(K_M = \frac{60}{2\pi \cdot p_1 \cdot \mu 1}; T_M = \frac{J}{\mu 1} \cong (1 \cdots 10 \sec)\right)$$

It is chosen: $T_M \cong 3 \text{ sec.} = \dfrac{J}{\mu 1}$;

In stationary regime for: $\frac{dn}{dt \cong 0} \quad \Rightarrow \quad M_{mN} = \frac{2\pi \cdot p_1}{60} \cdot \mu 1 \cdot n_N$;

When choosing the integration step Δt, it is necessary the selection of the smallest time constant from the analog model (5.36.); so that will be computed the values for the coefficients " a_{11}, a_{32}, a_{51} ", that represent the time constants:

$$T_{11} = T_{21} = \frac{1}{a_{11}} = \frac{1}{a_{21}} = \frac{L_m^2 - L_1 \cdot L_2}{RR_1 \cdot L_2}; T_{32} = T_{42} = \frac{1}{a_{42}} = \frac{1}{a_{32}} = \frac{L_m^2 - L_1 \cdot L_2}{R_2 \cdot L_1}; T_{51} = \frac{1}{a_{51}} = \frac{J}{\mu 1}; T_U = \frac{1}{f_{r1}};$$

The integration step is chosen with respect to the smallest of the values computed above, that will be denoted by "T min":

$$\Delta t \cong \left(\frac{1}{100} \cdots \frac{1}{500} \right) \cdot T_{\min} \quad ;$$

The programs needed for numerical modeling and simulating of the three phase asynchronous motor are in program LLI.DOM3.

3. Conclusions

1. The numerical simulation program of an asynchronous motor is simple, easy to initialize, flexible with respect to modifications of the structure parameters.
2. The study has a generally applicable character and can be used for different applicability domains in numerical modeling and simulation by systems that contain asynchronous electrical motors with variable speed.

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