NUMERICAL SIMULATION VARIANT OF MASTICATION PROCESSES USING DISTRIBUTED PARAMETERS

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ABSTRACT

Subsequently to paper [5], "*Analogical Modeling Variant Of Mastication Processes Using Distributed Parameters*" the partial derivative equation (pde) established for analogical modeling of mastication processes has been numerically integrated.

The obtained numeric solution is interpreted and the advantages of the method are highlighted.

KEYWORDS: mastication forces, numerical simulation, partial derivative equations

1. INTRODUCTION

As shown in paper [5], a possible variant for an analogical model of mastication can be approximated by the following pde :

$$\tilde{a}_{00}y + \tilde{a}_{10}\frac{dy}{dt} + \tilde{a}_{01}\frac{dy}{dp} + \tilde{a}_{20}\frac{d^2y}{dt^2} + \tilde{a}_{11}\frac{d^2y}{dtdp} + \tilde{a}_{02}\frac{d^2y}{dp^2} = u \qquad (1)$$

in which (u) represents the total masticatory force, while the six expressions in the left side of the equation represent the following mastication force components :

1. $(\tilde{a}_{00} y)$ is the force of elastic deformation, which is proportional with the deformation value (y) ;
2. $\left(\tilde{a}_{10} \frac{dy}{dt}\right)$ is the force of plastic deformation (absorbtion), proportional with the speed of the deformation, $\left(\frac{dy}{dt}\right)$;
3. $\left(\tilde{a}_{20} \frac{d^2 y}{dt^2}\right)$ is the dynamic force, proportional with the acceleration $\left(\frac{d^2 y}{dt^2}\right)$ or the deceleration $\left(-\frac{d^2 y}{dt^2}\right)$ of the deformation in the food bolus ;
4. $\left(\tilde{a}_{10} \frac{dy}{dt}\right)$ is the necessary force for horizontal condensing of the raising number of smaller emerging boluses. It is considered to be proportional with the horizontal speed $\left(\frac{dy}{dt}\right)$ of the deformation (y) ;
5. $\left(\tilde{a}_{02} \frac{d^2 y}{dp^2}\right)$ is also a component of the force for horizontal condensing of the raising number of smaller emerging boluses, but proportional with the horizontal acceleration $\left(\frac{d^2 y}{dp^2}\right)$ of the deformation (y) ;
6. $\left(\tilde{a}_{11} \frac{d^2 y}{dtdp}\right)$ is a mixed force considered to be simultaneously dependent of both the speed $\left(\frac{dy}{dt}\right)$ and the horizontal speed $\left(\frac{dy}{dp}\right)$ of the deformation (y).

The coefficients (a_{00}, a_{10}, ...) and (b_0) are calculated by the COEFA program, as exemplified in [5].

We should also remember [5] the following relationships between (w) and (y), expressing the deformation of the food bolus in three different ways:

$$y = y_{st} - w = y_{00} + (w_{00} - w) = y_{ff} - (w - w_{ff}) \qquad (2).$$

2. NUMERIC INTEGRATION OF PDE (1)

The numeric integration of pde (1) is based on the EDPMAS program, which uses the numeric integration method of Local Iterative Linearization (L.I.L.).

The input of the program is given by declaring the following values : y_{00}, y_{st}, t_0, t_f, p_0, p_f, b_0, a_{00}, a_{10}, a_{01}, a_{20}, a_{11}, a_{02}, λ_T and λ_P. The integration step Δt is approximated to around $0{,}01(t_f - t_0)$, and the result extraction step $\Delta\theta$ is chosen around $0{,}1(t_f - t_0)$.

If (1) is divided by ($\tilde{a}_{00}$), we can write:

$$a_{00} y + a_{10}\frac{dy}{dt} + a_{01}\frac{dy}{dp} + a_{20}\frac{d^2y}{dt^2} + a_{11}\frac{d^2y}{dtdp} + a_{02}\frac{d^2y}{dp^2} = b_0 u \qquad (3)$$

in which $a_{00}=1$; $a_{10}=\frac{\tilde{a}_{10}}{\tilde{a}_{00}}$; $a_{01}=\frac{\tilde{a}_{01}}{\tilde{a}_{00}}$; $a_{11}=\frac{\tilde{a}_{11}}{\tilde{a}_{00}}$; $a_{02}=\frac{\tilde{a}_{02}}{\tilde{a}_{00}}$; $b_0=\frac{1}{\tilde{a}_{00}}$.

The two variants, (1) and (3) can show complementary advantages.

The start of calculations has been ensured by Taylor series, while knowing the initial conditions (IC) of the status vector, respectively :

$$\mathbf{x}_{CI} = \mathbf{x}(t_0, p) = \frac{x_{00CI}}{x_{10CI}} = \frac{x_{00}(t_0, p)}{x_{10}(t_0, p)} \qquad (4)$$

in which

$$x_{00CI} = x_{00}(t_0, p) = y(t_0, p) \qquad (5)$$

$$x_{10CI} = x_{10}(t_0, p) = \left.\frac{dy(t,p)}{dt}\right|_{t=t_0} \qquad (6)$$

for the usual value $t_0 = 0$.

This starting algorithm allows the calculation of the regressive sequences $\mathbf{x}_{k-1} = \mathbf{x}_{IC} = \mathbf{x}(t_0, p)$; $\mathbf{x}_{k-2} = \mathbf{x}(t_0-\Delta t, p)$ and $\mathbf{x}_{k-3} = \mathbf{x}(t_0-2\Delta t, p)$, which start the program. The obtained numeric solution represents a curve family for the average deformation $y = y(t_k, p=\text{constant})$ and for the average diameter $w = w(t_k, p=\text{constant})$, knowing that $w= y_{st}-y$. The reference moment $t_k = k\,\Delta t$, in which the secvence (k) is incremented progressively (k = 0, 1, 2, …). These solutions (y and w) can be established for different active lengths where mastication can take place between dental arches, respectively for $p=_0$, p_1, p_2, …, p_f, where $0< p_1< p_2< \ldots < p_f$.

In this way curve families with an aspect similar to the dotted ones shown in figures 1 and 2 result: w(t, p=constant) and y(t, p=constant).

3. RESULTS

For the two examples in [5] the program EDPMAS will be used to show two significant evolutions of a food bolus deformation y=y(t, p=constant) and of its diameter $w=y_{st}-y$.

Example 1

$y_{00} = 0{,}0005$ m ; $w_{00} = 0{,}02$ m ; $w_{ff} = 0{,}0008$ m ; $y_{st} = y_{00} + w_{00}$; $t_0=0$; $t_f = 20$ sec. ; $p_0=0$; $p_f = 0{,}06$ m ; $\lambda_T = 2$; $\lambda_P = 1{,}5$; $a_{00}=1$; $a_{10}=1{,}596349$; $a_{01}= 3{,}808819 \cdot 10^{-3}$; $a_{20}=9{,}817215$; $a_{11}= 3{,}19112$;

$a_{02}=9{,}571476 \cdot 10^{-5}$; $u_0=1$.

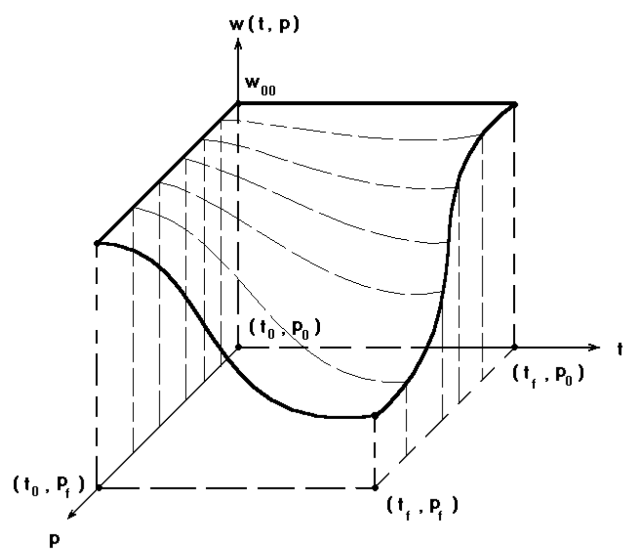

Figure 1. Family of curves w(t, p=constant).

p [m]	**0,06 m**		**0,04 m**		**0,02 m**		**0 m**	
t [sec.]	**y^m**	**w^m**	**y^m**	**w^m**	**y^m**	**w^m**	**y^m**	**w^m**
0	$5*10^{-4}$	$2*10^{-2}$	$5*10^{-4}$	$2*10^{-2}$	$5*10^{-4}$	$2*10^{-2}$	$5*10^{-4}$	$2*10^{-2}$
4	$7,5*10^{-3}$	$1,3*10^{-2}$	$7*10^{-3}$	$1,3*10^{-2}$	$4,8*10^{-3}$	$1,5*10^{-2}$		
8	$1,41*10^{-2}$	$6,33*10^{-3}$	$1,3*10^{-2}$	$7,4*10^{-3}$	$8,8*10^{-3}$	$1,2*10^{-2}$		
12	$1,75*10^{-2}$	$2,93*10^{-3}$	$1,6*10^{-2}$	$4,2*10^{-3}$	$1,09*10^{-2}$	$9,6*10^{-3}$		
16	$1,9*10^{-2}$	$1,4*10^{-3}$	$1,7*10^{-2}$	$2,8*10^{-3}$	$1,2*10^{-2}$	$8,7*10^{-4}$		
20	$1,96*10^{-2}$	$8,1*10^{-4}$	$1,8*10^{-2}$	$2,2*10^{-3}$	$1,22*10^{-2}$	$8,3*10^{-4}$		

Table 1. Results obtained for the first example

The time constants are T_1=2,(22) seconds ; T_2=4,(44) seconds and the inflexion moment t_i=3,080654 seconds. The length constants are P_1= 0,008 m ; P_2= 0,012 m and the inflexion distance $p_i=0,9731161*10^{-3}$ m.

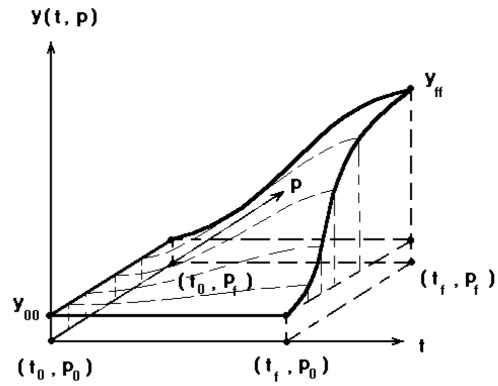

Figure 2. Family of curves y(t, p=constant).

Example 2

y_{00} = 0 m ; w_{00} = 0,03 m ; w_{ff} = 0,0012 m ; $y_{st} = y_{00}+ w_{00}$; t_0=0 ; t_f = 25 sec. ; p_0=0 ; p_f = 0,05 m ; λ_T = 1,5 ; λ_P = 2 ; a_{00}=1 ; a_{10}=8,405286 ; a_{01}= 0,0167737 ; a_{20}=17,04323 ; $a_{11}= 5,752954*10^{-2}$; $a_{02}=6,293086*10^{-5}$; u_0=1 ; $b_0=\frac{0,03}{u_0}$.

p [m]	**0,05 m**		**0,03 m**		**0,01 m**		**0 m**	
t [sec.]	**y^m**	**w^m**	**y^m**	**w^m**	**y^m**	**w^m**	**y^m**	**w^m**
0	0	$3*10^{-2}$	0	$3*10^{-2}$	0	$3*10^{-2}$	0	$3*10^{-2}$
5	10^{-2}	$2*10^{-2}$	$9*10^{-3}$	$2,1*10^{-2}$	$3,6*10^{-3}$	$2,6*10^{-2}$		
10	$2,03*10^{-2}$	$9,6*10^{-3}$	$1,8*10^{-2}$	$1,2*10^{-2}$	$7,3*10^{-3}$	$2,3*10^{-2}$		
15	$2,6*10^{-2}$	$4,4*10^{-3}$	$2,3*10^{-2}$	$7,2*10^{-3}$	$9,2*10^{-3}$	$2,1*10^{-2}$		
20	$2,8*10^{-2}$	$2,1*10^{-3}$	$2,5*10^{-2}$	$5,2*10^{-3}$	10^{-2}	$2*10^{-2}$		
25	$2,88*10^{-2}$	$1,2*10^{-3}$	$2,56*10^{-2}$	$4,4*10^{-3}$	$1,03*10^{-2}$	$1,96*10^{-2}$		

Table 2. Results obtained for the second example

The time constants are T_1=3,(33) seconds ; T_2=5 seconds and the inflexion moment t_i=4,054651 seconds. The length constants are $P_1= 0,(55)*10^{-3}$ m ; $P_2= 1,(11)*10^{-2}$ m and the inflexion distance $p_i=7,701635*10^{-3}$ m.

4. CALCULATION OF THE MASTICATION FORCES

The total mastication force results from (3):

$$u = \frac{1}{b_0} \cdot \left(a_{00} y + a_{10} \frac{dy}{dt} + a_{01} \frac{dy}{dp} + a_{20} \frac{d^2 y}{dt^2} + a_{11} \frac{d^2 y}{dtdp} + a_{02} \frac{d^2 y}{dp^2} \right) \tag{7}$$

in which $b_0 = \frac{1}{\tilde{a}_{00}} = \frac{y_{ff} - y_{00}}{u_0}$; u_0=1 and a_{00}=1.

The six force components will be asociated and calculated for the two examples shown in tables 1 and 2.

For the data of the first example, the six force components and their sum (u) for p = p_f = 0,06 m are shown in table 3.

t [sec] / u... [N]	0	4	8	12	16	20
u_{00}	$2,5*10^{-2}$	0,37	0,71	0,88	0,95	0,98
u_{10}	$7,8*10^{-3}$	0,17	$3,7*10^{-2}$	$4,3*10^{-2}$	0,2	$7,6*10^{-3}$
u_{01}	$7,4*10^{-7}$	$2,1*10^{-3}$	$4,1*10^{-3}$	$5,1*10^{-3}$	$5,6*10^{-3}$	$5,7*10^{-3}$
u_{20}	0,94	0,18	$2,8*10^{-3}$	0,18	$9*10^{-3}$	$4,4*10^{-3}$
u_{11}	$2,5*10^{-2}$	0,53	0,3	0,13	$5,9*10^{-2}$	$2,4*10^{-2}$
u_{02}	-10^{-5}	$- 4,3*10^{-3}$	$- 8,2*10^{-3}$	-10^{-2}	$-1,1*10^{-2}$	$-1,1*10^{-2}$
U	1	1,25	1,11	1,23	1,03	1,01

Table 3. The six force components and their sum for the first example, for p = p_f= 0,06 m

For the data of the second example, the six force components and their sum (u) for p = p_f= 0,05 m are shown in table 4.

t [sec] / u... [N]	0,05	5	10	15	20	25
u_{00}	$7,3*10^{-5}$	0,33	0,68	0,85	0,93	0,96
u_{10}	$2,4*10^{-2}$	0,71	$- 2,4*10^{-2}$	0,19	0,076	$3,02*10^{-2}$
u_{01}	$2,5*10^{-6}$	$1,13*10^{-2}$	$2,3*10^{-2}$	0,029	$3,15*10^{-2}$	$3,25*10^{-2}$
u_{20}	0,95	- 0,047	3,9	- 0,11	$7,05*10^{-2}$	$-3,3*10^{-5}$
u_{11}	$3,4*10^{-4}$	$9,9*10^{-3}$	$- 3,3*10^{-4}$	$2,6*10^{-3}$	$1,06*10^{-3}$	$4,2*10^{-4}$
u_{02}	$- 6,8*10^{-7}$	$- 3,8*10^{-3}$	$- 7,7*10^{-3}$	$-9,66*10^{-3}$	$-1,05*10^{-2}$	$-1,08*10^{-2}$
U	0,98	1,02	4,6	0,95	1,1	1,01

Table 4. The six force components and their sum for the second example, for p = p_f= 0,05 m

In order to simplify the presentation of results, calculated force values have been reduced to at most 4 digits and are only shown for 6 moments of time.

5. CONCLUSIONS

1. The pde (1) can represent a possible interpretation of mastication forces in time and space coordinates.
2. Through numerical integration of pde (1), based on the local iterative linearisation method, curve families can be obtained, w(t, p=constant) and y(t, p=constant), corresponding to statistical average values of the diameter, respectively of the deformation of the food bolus. This ensures a remarcable diversity in choosing and adapting the modeling process, as seen in tables 1 and 2.
3. The six mastication force components as well as their sum can be calculated.
4. By modifying the total mastication force (u), as well as the input values : y_{00}, w_{00}, y_{ff}, t_0, t_f, p_0, p_f, λ_T and λ_P, numerous temporo-spatial freedom degrees are ensured, favoring a flexible study of mastication processes.
5. Interpretation of y, w, u… and u shown in tables 1, 2, 3 and 4 can become a complex problem of interdisciplinary research, involving specialists in the fields of both the theory of partial derivative equations and dentistry.
6. This study and [5] represent a less conventional introductive method in the study of mastication processes. The studies can be continued by introducing two or three spatial coordinates, besides the temporal coordinate. This will result into an even more remarcable generalisation in the study of mastication processes.

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