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OPTIMAL CONTROL OF D.C. MOTORS FOR FLEXIBLE MANUFACTURING SYSTEM

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ABSTRACT:
In this paper work, it is determined the optimal command, optimal trajectory and optimal supplying voltage when is imposed an optimal criteria that will assure minimal energy losses through Joule effect while the d. c. motor with constant flux and separate excitation is reversing. We impose the following hypothesizes: motor reversing time is fixed, resistant moment is proportional with speed and electromagnetic inertia is ignored.

1. INTRODUCTION

For the electric drive systems, reducing energy consumption, which is achieved through the energy losses decreasing or recuperative methods application, represents an essential problem.

The problem of this paper work is to decrease energy losses if the motor is changing its rotation sense, kinetics and network electric energy and is transmitted(transformed) itself in heat. For the performance index deduction, will start with energy losses definition of a separate excitation d. c. motor while is reversing itself:

$$W = \int_0^{t_r} R_a \cdot i^2 dt \qquad (1)$$

where:

- R_a is equivalent resistance of resistive elements from the induced circuit;
- i is the electric current through the induced circuit;
- t_r is the reversing time.

These paper work calculations are taking care of the fact that the numeric command gear which implements the motor optimal command must have a sizes variation scale between $\pm$5V and this implicates reported sizes operation.

Taking care of normal starting time: $T = J \cdot \frac{\omega_N}{M_N}$, time relative co-ordinate: $\tau = \frac{t}{T}$, and electrical current: $i = \frac{i}{i_N}$ and using reported co-ordinates, relation (1) becomes:

$$W = \int_0^{\tau_r \cdot T} \rho \cdot i^2 d\tau \qquad (2)$$

where:

- $\rho = R_a i_N^2 T$- is a constant;
- τ_r is the relative reversing time: $\tau_r = \frac{t_r}{T}$

  Impose τ_r=1 and ρ is a constant who doesn't influence the extreme, then the energy losses evaluation using Joule effect is done using the following performance:

$$W = \int_0^{T} i^2 d\tau \qquad (3)$$

2. THE OPTIMIZATION PROBLEM FORMULATION

For a correct optimization problem formulation it must change relation's (3) form. So in the first phase, motor moving equation is placed in reported co-ordinates; been made the hypothesis that masses inertia, that are in move (J), is independent of rotors angler position inside stator, and that resistant moment is proportional with speed. In these conditions the motor moving equation becomes:

$$\mu = \mu_0 + k1 \cdot \nu + \dot{\nu} \qquad (4)$$

where:

- $\mu = \frac{M_e}{M_N}$ is motor developed electromagnetic couple reported co-ordinate;
- $\mu_0 = \frac{M_0}{M_N}$ is couple reported co-ordinate determined by frictions;
- $\nu = \frac{\omega}{\omega_N}$ is speed reported co-ordinate;
- k_1 is a constant.

  Results the differential equation (4) is linear and will have the finale and initial conditions:

$\nu(0)=0$ and $\nu(T)=0$ (5)

On the other site will consider the electromagnetic couple expression given in reported co-ordinates:

$$\mu = K_a \cdot i \qquad (6)$$

where:

- $K_a = K_m \cdot \frac{i_N}{M_N}$ a constant with $K_m = 0.97 \cdot K_e ; K_e = K \cdot \phi_0$

  where:
  - K is a motor electrical constant ;
  - ϕ_0 is value of flux (constant).

So, from relations (4) and (6) we will have:

$$i = \frac{1}{K_a} \cdot \left(\mu_0 + k_1 \cdot \nu + \dot{\nu} \right) \qquad (7)$$

But when the motor is reversing, the resistive moment its changes sense once with the change of rotation sense when the rotation speed passes through zero. This phenomenon is expressed by signum function:

$$sign(\nu) = \begin{cases} 1 \; then \; \nu \geq 0 \\ -1 \; then \; \nu < 0 \end{cases} \qquad (8)$$

In these conditions relation (7) becomes:

$$i = \frac{1}{K_a} \cdot \left((\mu_0 + k_1 \cdot \nu) \cdot sign(\nu) + \dot{\nu} \right) \qquad (9)$$

Replacing relation (9) in relation (3) and knowing that $1/K_a^2$ is a constant, which doesn't influence the extreme the energy losses through Joule effect can be decreased by the following performance index:

$$W = \int_0^T \left[(\mu_0 + k_1 \cdot \nu) \cdot sign(\nu) + \dot{\nu} \right] d\tau \qquad (10)$$

Applying the dynamic optimization method that uses variation calculation, the optimization problem will be as follows:

" Determine optimal command i(τ) that will transporte the linear system described by (7) from initial state (0,0) to final state (T,0) through an optimal trajectory ν(τ) that will assure the energy losses through Joule effect minimisation through performance index given by:

$$W(\tau) = \int_0^T \left[(\mu_0 + k_1 \cdot \nu(\tau)) \cdot sign(\nu(\tau)) + \dot{\nu}(\tau) \right] d\tau \qquad (11)$$

imposing a izoperimetrical restriction of the system sizes given by a certain distance fixation:

$$\alpha_0 = \int_0^T \nu(\tau) d\tau \qquad (12)$$

fixation that represents a certain value of rotor angler position inside stator."

The performance new index given by the integral applied to Lagrange function:

$$L = \left[(\mu_0 + k_1 \cdot \nu) \cdot sign(\nu) + \dot{\nu} \right]^2 + \lambda_0 \cdot \nu \qquad (13)$$

meaning

$$W = \int_0^T L d\tau \qquad (14)$$

Optimal trajectory ν(τ) is obtained as an Euler-Lagrange equation solution:

$$\frac{\partial L}{\partial \nu} - \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{\nu}} \right) = 0 \qquad (15)$$

Optimal trajectory ν(τ) will determine a performance index relative minimum (14) if will verify Legendre minimum condition:

$$\frac{\partial^2 L}{\partial \dot{v}^2} \geq 0 \qquad (16)$$

The same optimization criteria is given in relation [3] where reversing time is fixed but the resistant moment is constant.

3. THE DETERMINATION OF THE OPTIMAL TRAJECTORY AND OPTIMAL COMMAND

Knowing that the optimal trajectory is obtained from the Euler- Lagrange (15) equation solution, we will determine the differential equation solution. While:

$$\begin{cases} \dfrac{\partial L}{\partial v} = 2 \cdot \left[(\mu_0 + k_1 \cdot v) \cdot sign\ (v) + \dot{v} \right] \cdot k_1 \cdot sign\ (v) + \lambda_0 \\ \dfrac{\partial L}{\partial \dot{v}} = 2 \cdot \left[(\mu_0 + k_1 \cdot v) \cdot sign\ (v) + \dot{v} \right] \qquad (17) \\ \dfrac{d}{d\tau}\left(\dfrac{\partial L}{\partial \dot{v}} \right) = 2 \cdot k_1 \cdot \dot{v} \cdot sign\ (v) + 2 \cdot \ddot{v} \end{cases}$$

relation (15) becomes

$$2 \cdot \ddot{v} - 2 \cdot k_1^2 \cdot v = 2 \cdot \mu_0 \cdot k_1 + \lambda_0 \qquad (18)$$

and (18) is a differential equation with constant coefficients.
General solution of the differential equation (18) is:

$$v = c_1 \cdot e^{k_1 \cdot \tau} + c_2 \cdot e^{-k_1 \cdot \tau} - \frac{2 \cdot \mu_0 \cdot k_1 + \lambda_0}{2 \cdot k_1^2} \qquad (19)$$

will impose to the relation (19) the initial condition$v(0)=0$ and will have:

$$c_1 = \frac{2 \cdot \mu_0 \cdot k_1 + \lambda_0}{2 \cdot k_1^2} - c_2 \qquad (20)$$

With relation (20), relation (19) becomes:

$$v = c_2 \cdot \left(e^{-k_1 \cdot \tau} - e^{k_1 \cdot \tau} \right) + \frac{2 \cdot \mu_0 \cdot k_1 + \lambda_0}{2 \cdot k_1^2} \cdot e^{k_1 \cdot \tau} - \frac{2 \cdot \mu_0 \cdot k_1 + \lambda_0}{2 \cdot k_1^2} \qquad (21)$$

If applying the finale condition$v(T)=0$ to relation (21) in the izoperimetric restriction (12), will have:

$$c_2 \cdot \left(e^{-k_1 \cdot T} - e^{k_1 \cdot T} \right) + \lambda_0 \cdot \left(e^{k_1 \cdot T} - 1 \right) \cdot \frac{1}{2 \cdot k_1^2} + \frac{\mu_0}{k_1} \cdot \left(e^{k_1 \cdot T} - 1 \right) = 0 \qquad (22)$$

Replacing relation (21) in the isopermetric restriction (12) will have:

$$c_2 \cdot \left[2 - \left(e^{k_1 \cdot T} + e^{-k_1 \cdot T} \right) \right] \cdot \frac{1}{k_1} + \lambda_0 \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1 \right) - T \right] \cdot \frac{1}{2 \cdot k_1^2} + \frac{\mu_0}{k_1} \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1 \right) - T \right] - \alpha_0 = 0 \qquad (23)$$

With (22) and (23) will obtain a two equations system with two unknowns: C_2 and λ_0 . solving this two equations system will be obtained the expressions for the two unknowns as follows:

$$c_2 = \frac{G1}{G2} \quad and \quad \lambda_0 = \frac{H_1}{H_2} \qquad (24)$$

where:

$$G1 = \left(e^{k_1 \cdot T} - 1\right) \cdot \left\{ \frac{\mu_0}{k_1} \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) - T \right] - \alpha_0 \right\} - \frac{\mu_0}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) - T \right]$$

$$G2 = \left(e^{-k_1 \cdot T} - e^{k_1 \cdot T}\right) \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) - T \right] - \frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) \cdot \left[2 - \left(e^{k_1 \cdot T} + e^{-k_1 \cdot T}\right) \right]$$

$$H1 = \left(e^{-k_1 \cdot T} - e^{k_1 \cdot T}\right) \cdot \left\{ \frac{\mu_0}{k_1} \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) - T \right] - \alpha_0 \right\} - \frac{\mu_0}{k_1^2} \cdot \left(e^{k_1 \cdot T} - 1\right) \cdot \left[2 - \left(e^{k_1 \cdot T} + e^{-k_1 \cdot T}\right) \right]$$

$$H2 = \frac{1}{2 \cdot k_1^3} \cdot \left(e^{k_1 \cdot T} - 1\right) \cdot \left[2 - \left(e^{k_1 \cdot T} + e^{-k_1 \cdot T}\right) \right] - \frac{1}{2 \cdot k_1^2} \cdot \left(e^{-k_1 \cdot T} - e^{k_1 \cdot T}\right) \cdot \left[\frac{1}{k_1} \cdot \left(e^{k_1 \cdot T} - 1\right) - T \right]$$

After replacing relations (24) in relation (20) will obtain the C_1 expression; having C_1, C_2, λ_0 coefficients the optimal trajectory and optimal command will be completely determined .

The optimal command and trajectory can be represented in a graphical way as shown in fig.2 considering that: c_1= -2.406; c_2= -2.62; λ_0= -3.772;
μ_0=0.429; k1=0.571; k_a=0.035; α_0=0.785 [rad].

The variation of the current must be discontinued when the rotation speed reaches zero.Except for this moment the optimal control function is linear and continuous.

The only undetermined size is the optimal supplying voltage, which will be obtained by replacing the optimal trajectory and optimal command in voltages equation from induced circuit in the reversing moment.

$$u_a(\tau) = -\left[R \cdot i_N \cdot i(\tau) + \frac{L \cdot i_N}{T} \cdot \frac{d(\tau)}{d\tau} + k_e \cdot \omega_N \cdot \nu(\tau) \right] \qquad (26)$$

The optimal supplyng voltage can be represented in a graphical way as shown in fig.1.considering that:
R=0.25[Ω]; i_N=16[A]; L=0.011[H]; ke=0.016;
T=0.15[s]

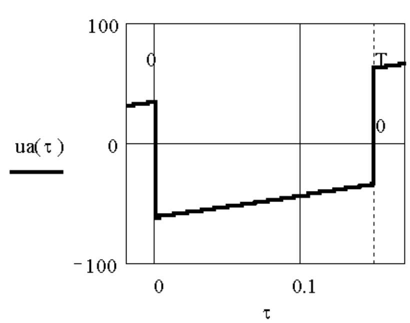

Figure 1 The optimal supplying voltage courve

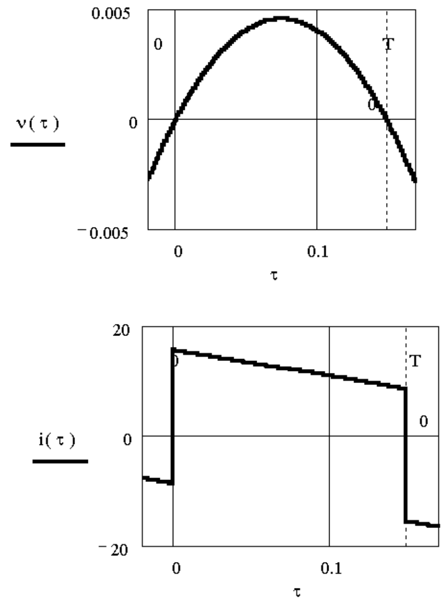

Figure 2 The optimal command and trajectory courve

4. CONCLUSIONS

Using the minimal energy loss criteria (Joule effect) electrical drives like the one we studied must establish speed variations which must follow a exponential trajectory.

An increase of the speed can be done with a combined optimization criteria with minimal energy loss and minimal duration of the change of dinction.

5. BIBLIOGRAPHY

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