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THE OPTIMISATION OF THE ENERGY LOSSES THROUGH JOULE EFFECT BY MEANS OF A MINIMUM ENERGETICAL COMPONENT OF AN OPTIMISATION CRITERION IN CASE OF THE REVERSE OF AN ENGINE WITH CONTINUOUS CURRENT WITH SEPARATE EXCITATION AND CONSTANT FLOW

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Abstract: In the paper the reduction of energy input is made throughout the minimisation of energy losses wheu the electic action machine is a continuous current engine with separate excitation and constant flow. The problem brought about is to minimize these energy losses wheu the engine changes its rotation sense and this is the case when the kinetic energy of the moving bodies as well as the electric one introduced in the network twin into warmth. In order to realise this, the analythic expression of the optimum tension of loadind due to wich a programme of conducting according to traiectories cau be realised and wich cau be implemented on a numerical equipment of conducting. The optimum tension of loading is got by replacing in the circuit of the induce, the optimal command or the optimal current and the optimum traiectoire or the optimum speed got by the aplication of au optimisation criterion with minimum energy compound during engine reversal. The calculi made in this paper are based on the fallowing hipotheses: the time of engine reversal is considered to be arbitrary, the resistent moment is constant and the electromagnetic inertion is neglected

Key words: optimisation, energy, continuous current engine, separate excitation, constant flow

1. Introduction

Multiplying the equation of the inducted voltages when the motor changes direction with the inducted current neglecting the value of the total inductance from the circuit of the inductee we have :

$$R_a \cdot i^2 = -(u_a + u_e) \cdot i \tag{1}$$

where : R_a - the total rezistence of the circuit;
I - armature current;
u_a - circuit driving voltage;
u_e - the electromotive voltage generated by the rotation.

Considering as the optimization criteria the energy loss by Joule effect in the inductee when the motor changes its direction of rotation we obtain :

$$W = \int_0^{t_r} R_a \cdot i^2 dt = -\int_0^{t_r} (u_a + u_e) \cdot i dt \tag{2}$$

where t_r - the duration in which the motor changes direction.

If we substitute $T = J \cdot \frac{w_N}{M_N}$, $t = \frac{t}{T}$ and $i = \frac{i}{i_N}$ we have :

$$W = \int_0^{t_r \cdot T} R_a \cdot i_N^2 \cdot T \cdot i^2 dt = \int_0^{t_r \cdot T} r \cdot i^2 dt \tag{3}$$

where τ_r - the relative duration time which the motor changes direction of rotation.

Due to the fact that ρ has a constant value it means that it does not influence the extreme value and we can study the energy loss by Joule effect in the following equation :

$$W = \int_0^{t_r \cdot T} i^2 dt \tag{4}$$

2. The mathematical model

To establish the mathematical model we shall start with the following relation :

$$M_e - M_r = M_J \tag{5}$$

keeping in mind the angle α of the rotor and that $\frac{da}{dt} = w$:

$$M_J = J \cdot \frac{dw}{dt} + w \cdot \frac{dJ}{da} \cdot \frac{da}{dt} \tag{6}$$

or $$M_J = J \cdot \frac{dw}{dt} + w^2 \cdot \frac{dJ}{da} \tag{7}$$

Because J is independent of α :

$$M_J = J \cdot \frac{dw}{dt} \tag{8}$$

Introducing relation (8) in (5) and considering that M_r is constant we have :

$$M_e = M_0 + J \cdot \frac{dw}{dt} \tag{9}$$

From (9) we can very easily switch to relative coordinates because (9) is equivalent with :

$$M_e = M_0 + J \cdot \frac{dw}{dt} \cdot \frac{dt}{dt} \tag{10}$$

Now substituting the relative coordinates: $m = \frac{M_e}{M_N}$; $u = \frac{w}{w_N}$ we have :

$$\frac{M_e}{M_N} \cdot M_N = \frac{M_0}{M_n} \cdot M_N + J \cdot \frac{d}{dt}\left(\frac{w}{w_N} \cdot w_N\right) \cdot \frac{d}{dt} \cdot \left(\frac{t}{T}\right) \tag{11}$$

or $$m \cdot M_N = m_0 \cdot M_N + J \cdot w_N \cdot \frac{1}{T} \cdot \frac{du}{dt} \tag{12}$$

Because : $J \cdot w_N \cdot \frac{1}{T} = M_N$ and from equation (12) we may eliminate M_N we have :

$$m = m_0 + \frac{du}{dt} \tag{13}$$

Because equation (13) is a liniar one we consider the following conditions :

$$u(0) = o \quad \text{and} \quad u(t_r \cdot T) = 0 \tag{14}$$

On the other hand , the electromagnetive voltage is :

$$u_e = \frac{1}{2p} \cdot \frac{p}{a} \cdot N \cdot \Phi \cdot w = k \cdot \Phi \cdot w \tag{15}$$

where : p - the number of pole pairs;
a - the number of ways of coiling pairs;
N - the total number of conducting wires of the inductee;
$k = \frac{1}{2p} \cdot \frac{p}{a} \cdot N$ - the electrical constant of the motor.

Because the magnetic flux is constant $\Phi = \Phi_0$ equation (15) becomes :

$$u_e = k \cdot \Phi_0 \cdot w = k_e \cdot w \tag{16}$$

On the other hand the electromagnetive couple of the motor can be calculated considering the electromagnetive power $p = u_e \cdot i$ we have :

$$M_e = \frac{P}{w} = \frac{u_e \cdot i}{w} = \frac{k_e \cdot w \cdot i}{w} = k_e \cdot i \tag{17}$$

From practical experience we know that it’s best to use an electromotive couple like :

$$M_e = k_m \cdot i \tag{18}$$

where k_m is empirical constant : $k_m = 0.97 \cdot k_e$.

Swithing now to relative coordinates :

$$\frac{M_e}{M_N} \cdot M_N = k_m \cdot \frac{i}{i_N} \cdot i_N \tag{19}$$

or $$m = k_a \cdot i \tag{20}$$

where $k_a = k_m \cdot \frac{i_N}{M_N}$ is constant.

Considering relation (20), (13) and (14) we obtain the following mathematical model in relative coordinates :

$$i = \frac{i}{k_a} \cdot m = \frac{1}{k_a} \cdot (m_0 + \dot{u}) \tag{21}$$

But because when the motor changes direction, the resistant couple also chages direction when the otation speed reaches 0, by using the signum function we can capture this moment :

$$signu = \begin{cases} 1 & \text{if } u \geq 0 \\ -1 & \text{if } u < 0 \end{cases} \tag{22}$$

In this case relation (21) becomes :

$$i = \frac{1}{k_a} \cdot [m_0 \cdot signu + \dot{u}] \tag{23}$$

Substituting (23) in (4) and considering that $\frac{1}{k_a^2}$ is a constant value, we can minimize the energy loss by Joule effect using an optimal criteria with dynamical programming and variational calculus :

$$W = \min \int_0^{t_r \cdot T} [m_0 \cdot signu + \dot{u}]^2 \, dt \tag{24}$$

3. Establishing the optimizatin issue

This is formulated as follows :

“One has to determine the optimal control $i(\tau)$ which can transport the linear system described by equation (23) from its primary state (0,0) in the final state $(\tau_r \cdot T, 0)$ througn an optimal trajectory $\nu(\tau)$ which ensures the minimization of the energy loss by Joule effect, meaning to minimize the performance index :

$$W = \min \int_0^{t_r \cdot T} [m_0 \cdot signu + \dot{u}]^2 \, dt \tag{25}$$

restricting the angle of the rotor in the stator :

$$a_0 = \int_0^{t_r \cdot T} u dt \tag{26}$$

where τ_r is free".

Because the system's conditions are restricted by (26) the optimization issue changes an the new performance index is obtained by integrating the Lagrange function:

$$L = [m_0 \cdot signu + \dot{u}]^2 + l \cdot u \tag{27}$$

where λ - Lagrange multiplyer.

So the new performance index is :

$$W = \min \int_0^{t_r \cdot T} \{[m_0 \cdot signu + \dot{u}]^2 + l \cdot u\} dt \tag{28}$$

In this case the optimal trajectory is $\nu(\tau)$ which optimizes the performance index, will by given as an equation to the Euler equation :

$$\frac{JL}{Ju} - \frac{d}{dt}\left(\frac{JL}{J\dot{u}}\right) = 0 \tag{29}$$

The optimal trajectory $\nu(\tau)$ will reach a relative minimum of the performance index if it verifies Legendre's formula :

$$\frac{J^2 L}{J\dot{u}^2} \geq 0 \tag{30}$$

A similar optimization criteria is given in [4] where the duration in which the motor changes direction of rotation is given. In this paper we bring something new by considering this duration arbitrary.

4. Calculating the optimal trajectory and the optimal control

Due to the fact that the optimal trajectory $\nu(\tau)$ is calculated from Euler equation (29) we shall have to calculate the solution of diferential equation (29) :

$$\frac{JL}{Ju} = l \;\; ; \;\; \frac{JL}{J\dot{u}} = 2 \cdot (m_0 \cdot \mathrm{sign} u + \dot{u}) \;\; ; \;\; \frac{d}{dt}\left(\frac{J\mathrm{L}}{J\dot{u}}\right) = 2 \cdot \ddot{u} \tag{31}$$

With relation (31), relation (29) becomes :

$$l - 2 \cdot \ddot{u} = 0 \tag{32}$$

On the other hand because :

$$\frac{J^2 L}{J\dot{u}^2} = 2 \geq 0 \tag{33}$$

the optimal trajectory $\nu(\tau)$ will achieve a relative minimum of the performance index W.

With the help of relation (32) integrating it we obtain the following relations :

$$\ddot{u} = \frac{1}{2} \cdot l \;\; ; \;\; \dot{u} = \frac{1}{2} \cdot l \cdot t + c_1 \;\; ; \;\; u = \frac{1}{4} \cdot l \cdot t^2 + c_1 \cdot t + c_2 \tag{34}$$

Considering for the last equation in (34) $\nu(0)=0$ we obtain $c_2=0$.Than we have :

$$u = \frac{1}{4} \cdot l \cdot t^2 + c_1 \cdot t \tag{35}$$

In relation (35) considering $u(t_1 \cdot T) = 0$ to we have :

$$\frac{1}{4} \cdot l \cdot t^2 \cdot T^2 + c_1 \cdot t_r \cdot T = 0 \tag{36}$$

With (35) relation (26) becames :

$$a_0 = \frac{1}{2} \cdot c_1 \cdot t_r^2 \cdot T^2 + \frac{1}{12} \cdot l \cdot t_r^3 \cdot T^3 \tag{37}$$

Grouping relations (36) and (37) we obtain a system with three variables: τ_2, c_1 and λ but we need one mor equation to be able to solve this.

Let's assume that the optimal trajectory $\nu(\tau)$ must reach a given curve $\nu = g(\tau)$. In this case the optimal trajectory musn't have a variation of the Lagrange's function L when the final point moves are the curve .

The condition suported to the final point (τ_r , T , 0) is :

$$L + (\dot{g} - \dot{n}) \cdot \frac{JL}{J\dot{u}} = 0 \tag{38}$$

Because $g(t_r \cdot T) = 0$ and $\dot{g}(t_r \cdot T) = 0$ equation (38) becomes :

$$\begin{aligned} &(m_0 \cdot signu + \frac{1}{2} \cdot l \cdot t_r \cdot T + c_1)^2 + l \cdot (\frac{1}{4} \cdot l \cdot t_r^2 \cdot T^2 + c_1 \cdot t_r \cdot T) \\ &- 2 \cdot (\frac{1}{2} \cdot l \cdot t_r \cdot T + c_1) \cdot (m_0 \cdot signu + \frac{1}{2} \cdot l \cdot t_r \cdot T + c_1) = 0 \end{aligned} \tag{39}$$

Finishing the calculs for (39) and remembering that $sign^2(n) = 1$ we have :

$$m_0^2 - c_1^2 = 0 \tag{40}$$

With equation (40) when $m_0 = c_1$ the system we talked about has the following solutions :

$$t_r = \frac{1}{T} \cdot \sqrt{\frac{6 \cdot a_0}{m_0}} \quad ; \quad l = -4 \cdot m_0 \cdot \sqrt{\frac{m_0}{6 \cdot a_0}} \tag{41}$$

Substituting these results in relation (35) we get the optimal trajectory :

$$u(t) = -(m_0 \cdot \sqrt{\frac{m_0}{6 \cdot a_0}}) \cdot t^2 + m_0 \cdot t \tag{42}$$

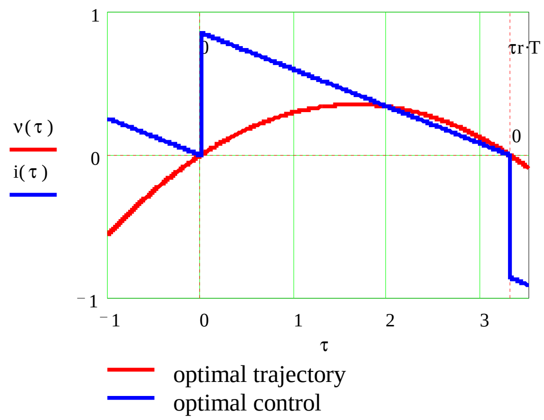
We shall obtain the optimal control if we substitute relation (42) in (23):

$$i = \frac{1}{k_a}[m_0 \cdot signu - 2 \cdot m_0 \sqrt{\frac{m_0}{6 \cdot a_0}} \cdot t + m_0] \tag{43}$$

Thus the performance index if we neglect the value of $\frac{1}{k_a^2}$ is :

$$W = \frac{4\sqrt{6}}{3} \cdot m_0^2 \cdot \sqrt{\frac{a_0}{m_0}} \tag{44}$$

The optimal control and trajectory can be reprezented in a graphical way as shown in fig.1 considering that:M_N =7Nm; M_0=3 Nm; J=0,01 kgm^2; $\alpha_0 = 45 \cdot \frac{p}{180}$ rad; $w_N = 1000 \frac{2p}{60}$ rad/s.



The variation of the current must be discontinued when the speed of rotation reaches 0. Except for this moment the optimal control function is linear and continuous .

5. Conclusions

The two optimal solutions are similar with onen in (4) but the duration is a lot shorter .

An increase of the speed can be done with a combined optimization criteria with minimal energy loss and minimal duration of the change of direction . Using this type of criteria we could increas the speed .

Using the minimal energy loss criteria (Joule effect) electrical drives like the to one we studied must establish speed variations wich must follow a parabolic trajectory.

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