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DEVICES STABILITY DEPENDING ON THE EUCLIDEAN CONDITION NUMBER

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Abstract This paper considers the problem of devices which are described with liniar systems equations. The sources or the structure of devices are affected by perturbations. In the first case the study of stability is made by using the Wielandt and Kantorovitch inequality and in the second case it is made by applying the spectral norm of the inverse matrix of coefficients. This study is made at the variation of a parameter. The examples are selected from the domain of electric circuits, continuous and alternative, with real and complex matrix of coefficients.

Key words: spectral norm, condition number, perturbed matrix, stability.

1.INTRODUCTION

Let be the complex vector **z**, the components $z_1, z_2 \ldots z_n$, with Hölder p-norm [1]:

$$\|\mathbf{z}\|_p = \left(\sum_{i=1}^{n} |z_i|^n\right)^{\frac{1}{n}} \qquad p=1,2,\infty \tag{1}$$

The euclidean norm, for p=2, is defined by

$$\|\mathbf{z}\|_2^2 = \mathbf{z}^H\mathbf{z} \tag{2}$$

where $\mathbf{z}^H$ is the Hermitian of the **z** vector, so the transjugate of this.

The acute angle θ between the vectors **x** and **y** is calculated by [2]

$$|\,\mathbf{x}^H\,\mathbf{y}\,| = \|\mathbf{x}\|_2 \cdot \|\mathbf{y}\|_2 \ \cos\theta \tag{3}$$

In the case of real components the Hermitian is the same with the transposed and the absolute value is not necessary.

For n x n matrix **A** the following induced matrix norm is defined

$$\|\mathbf{A}\|_\mathbf{p} = \max\frac{\|\mathbf{Az}\|_\mathbf{p}}{\|\mathbf{z}\|_\mathbf{p}} \qquad \mathbf{z} \neq \mathbf{0} \tag{4}$$

The euclidean norm (the spectral norm) is [3]

$$\|\mathbf{A}\|_2 = \sigma_1 \tag{5}$$

where σ_1 is the largest of the singular values of **A** matrix, or the radical of the largest eigenvalue of the matrix

$$\mathbf{B} = \mathbf{A}^H\,\mathbf{A} \tag{6}$$

If the inverse $\mathbf{A}^{-1}$ exists the euclidean condition number is defined [3] by

$$k(\mathbf{A})=\|\mathbf{A}\|_2\|\mathbf{A}^{-1}\|_2=\frac{\sigma_1}{\sigma_n} \quad (7)$$

where σ_n is the smallest of the singular values of **A** matrix.

The condition number and the stability are tight bonded. Two cases are presented.

2.THE STABILITY WHEN THE SOURCES ARE PERTURBED

We consider that the device is described by the equation

$$\mathbf{Az}=\mathbf{u} \quad (8)$$

If the sources are perturbed, the right member (the source vector) will be $\mathbf{u}'$ and the solution of the equation (the effect vector) won't be **z** but $\mathbf{z}'$.

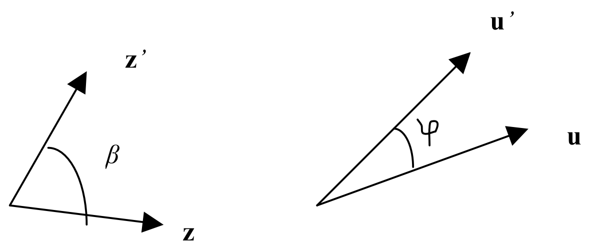


Fig.1. The angles between the source vectors and the effect vectors

Let be β the angle between the vectors **z** and $\mathbf{z}'$ and φ the angle between **u** and $\mathbf{u}'$. The Wielandt and Kantorovitch [4] inequality is

$$\mathrm{tg}\frac{\beta}{2}\le k(\mathbf{A})\ \mathrm{tg}\frac{\varphi}{2} \quad (9)$$

In the case of complex component (3) is used for made the calculus.

2.1. *The algorithm for the stability*

1.We establish the physical and the mathematical model.

2.We choose a method in order to solve the mathematical model and the (8) equation results.

3.We define the perturbation p of the source vector and then we calculate $\mathrm{tg}\frac{\varphi}{2}$.

4.We choose the m parameter of **A** matrix. Then we present the variation of euclidean condition number (depending on m) and the variation of the right hand member of inequalities (9) (depending on m).

5.The described variation is the maximum limits for the variation of the left hand member of (9).

2.2.*Example for algorithm use. Real component*

The physical model is a continuous electric circuit with three parallel branches, every branch having an electrical source and an electric resistance.

E_1=15V, R_1=x=1, 2, 3 …50Ω
E_2=5V, R_2=20Ω
E_3=10V, R_3=30Ω

The E_3 source is perturbed so that E_3=10+p. In example p=1V.

The parameter for stability study is the resistance R_1=x.

The effect vector is the vector of current.

The MathCad is used for mathematical computation. The matrix method is used in order to solve the electrical circuit

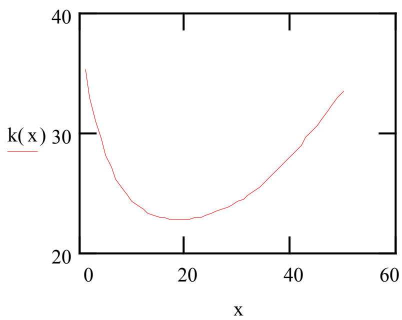
$$A(x) := \begin{bmatrix} 1 & 1 & 1 \\ x & -R_2 & 0 \\ 0 & -R_2 & R_3 \end{bmatrix} \qquad z(x,p) := \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad u(p) := \begin{bmatrix} 0 \\ E_1 - E_2 \\ E_3 + p - E_2 \end{bmatrix}$$

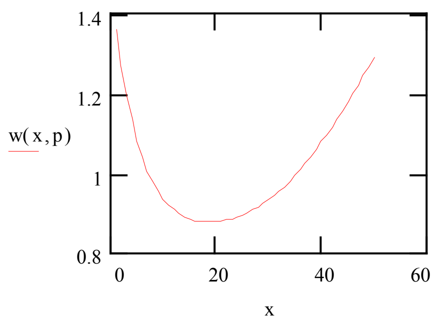
The euclidean condition number is k(x)

$$k(x) := \mathrm{cond2}(A(x))$$

The right hand member of inequality (9) is

$$w(x,p) := k(x)\cdot \mathrm{tgfipedoi}(p)$$





The minimum of euclidean condition number is for x=R_1=19Ω, k(19)=22,85, and the minimum of the right hand member of (9) inequalities is 0,877.

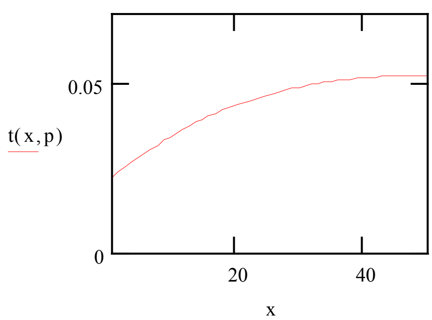
For this value the circuit stability is maximum.

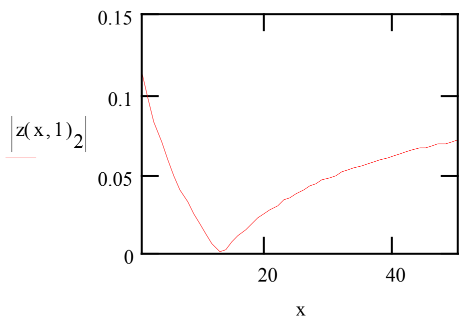
The angle between effect vectors is

$$z(x,p) := \left[A(x)^{(-1)}\cdot u(p)\right] \qquad \mathrm{cosb}(x,p) := \frac{\overrightarrow{z(x,0)\cdot z(x,p)}}{|z(x,0)|\cdot|z(x,p)|}$$

$$\mathrm{tgbepedoi}(x,p) := \left(\frac{1-\mathrm{cosb}(x,p)}{1+\mathrm{cosb}(x,p)}\right)^{\frac{1}{2}} \qquad t(x,p) := \mathrm{tgbepedoi}(x,p)$$

t(x,p) is the left hand member of Wielandt and Kantorovitch inequality. This variation doesn't follow the w(x,p)





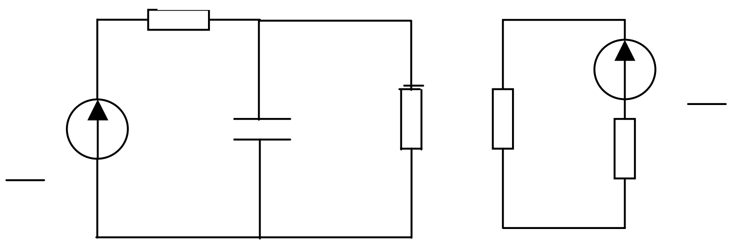
For great values of euclidean condition number [4] t(x,p) and w(x,p) are tight bonded.

$z(x,1)_2$ is the value of I_3, the current in the perturbed branch.

It is possible to do a study of stability for two parameters, $x=R_1, y=R_3$ for 0-30Ω. The minimum of euclidean condition number is k=17,98 for $R_1=R_3=9Ω$.

2.3. *Example for algorithm use. Complex component*

We consider a power transformer, Fig.2, with duplicate supply and shunt capacitor. The corresponding terminals of the coils are in the top of the drawing.



Numerical dates are

$\omega L_1=25\Omega$, $\dfrac{1}{\omega C_2}=20\Omega$,

$\omega L_3=40\Omega$, $\omega L_4=10\Omega$,

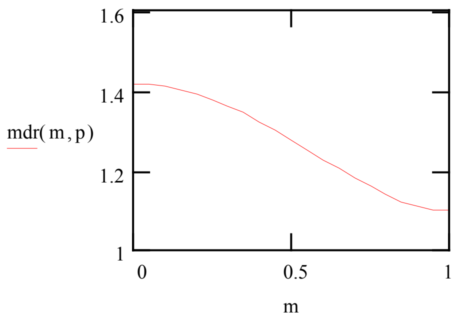
$\omega L_{34}=16\Omega$, $R_4=10\Omega$, $\underline{E_1}=50V$, $\underline{E_4}=10+j17{,}3V$

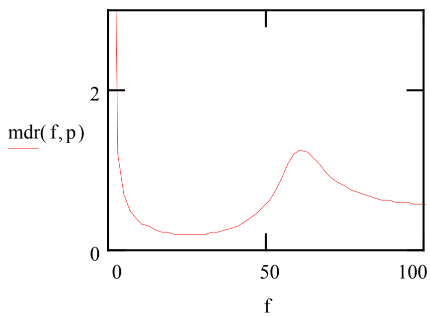
It is applied the matrix form of Kirchhoff teorems. It is presented **A** matrix and **u** source vector (p is the perturbation in the source vector)

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ j\omega L_1 & -\dfrac{1}{j\omega C_2} & 0 & 0 \\ 0 & \dfrac{1}{j\omega C_2} & j\omega L_3 & j\omega L_{34} \\ 0 & 0 & j\omega L_{34} & R+j\omega L_4 \end{bmatrix} \qquad u(p) := \begin{bmatrix} 0 \\ 50+p \\ 0 \\ 10+17.3j \end{bmatrix}$$

We have the following cases

1)The turns ratio of the transformer's coils is the parameter m so that $\omega L_{34}=m\sqrt{\omega L_3 \omega L_4}$ =m 20Ω, where m ∈ (0,1). The variation of the right hand side of Wielandt and Kantorovitch inequality mdr(m,p) (depending on m) for p=2, is demonstrating that the stability is directly proportional with the turn ratio.





2)The right hand side of Wielandt and Kantorovitch inequality mdr(f,p) depending of frequency $f \in (0,100)$ has a minimum for $f \in (0,50)$Hz where the stability is maximum For high frequency the stability falls.
3)If the inductive reactance is growing $\omega L_1 \in (1,30)\Omega$, the stability of circuit falls.
If the $\underline{E}_4$ source is perturbed, there won't be any variation of the current in the third branch, for $\omega L_1=20\Omega$ (Boucherot circuit).
4) If the capacitive reactance is growing between 0 and 50Ω, the stability of circuit has a minimum for 15Ω.
5)If the resistance R_4 is growing between 0 and 50Ω the stability of circuit falls.

3.THE STABILITY WHEN THE STRUCTURE IS PERTURBED

We are considering that (8) is describing the device. The structure of the device is perturbed so the matrix of coefficients is **A**+**P**, where **P** is the perturbation matrix.

The system solution (the effect vector) is not **z**, but **z′**. We write

$$\frac{\|\mathbf{z}-\mathbf{z}'\|_2}{\|\mathbf{z}\|_2} = \text{er} \tag{10}$$

When the structure is perturbed, this relation is proved [3]

$$\|\mathbf{A}^{-1}\|_2 \|\mathbf{P}\|_2 \geq \frac{\text{er}}{\text{er}+1} \tag{11}$$

with

$$\|\mathbf{A}^{-1}\|_2 \|\mathbf{P}\|_2 \leq 1 \tag{12}$$

3.1 *The algorithm for the stability study*

1.We made the physical and the mathematical model.

2.We choose the method for solving the mathematical model and the (8) equation results.

3.The perturbation matrix **P** is settled.

4.The maximum of the error is settled, er. This is the error (10) after the action of perturbation.

5.We choose the m parameter of **A** matrix. We get m_0 the value of parameter that fulfils (11).

6.We verify (12) for the m_0 value.

If (11) and (12) are true for m_0 then the new structure of device, with m_0, will have the er error.

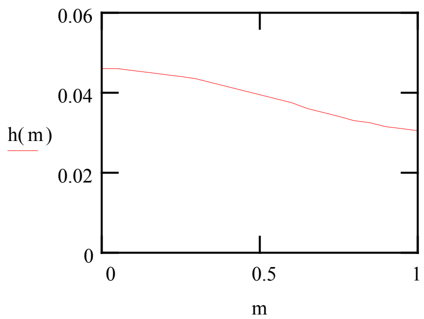
3.2. *Example for algorithm use*

We consider the same circuit, Fig.2. The perturbation matrix appears on the fact that the resistance R_4 is modified with 0.01Ω.

The turns ratio of the transformer's coils is the parameter m. We consider that $\omega L_{34}=m\sqrt{\omega L_3 \omega L_4}=m\,20\Omega$.

The error er=4% is settled.

We consider $\mathbf{A}^{-1}=\mathbf{A}^{-1}(m)$ and we write $h(m)=\left\|\mathbf{A}^{-1}\right\|_2\left\|\mathbf{P}\right\|_2$.



From the table (MathCad) we get $m \in [0{,}55;\ 1]$ so that the error $er \le 4\%$.

4.CONCLUSIONS

The paper demonstrates the possibility to use the euclidean condition number for the study of stability. The advantage is that one single number, depending on a parameter from the device structure, characterizes the stability of the device.

The uses of d.c. and a.c. circuits for example is an original aspect.

There is a great difference between the two members of Wielandt and Kantorovitch inequality because the real and complex matrix of examples is well-conditioned.

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